Drexel University

Office of the Dean of the College of Engineering

ENGR 232 – Dynamic Engineering Systems

Week 1 – Pre Lab

Review ENGR 231 material – example with equilibrium points, d-field, stability, and solutions in MATLAB

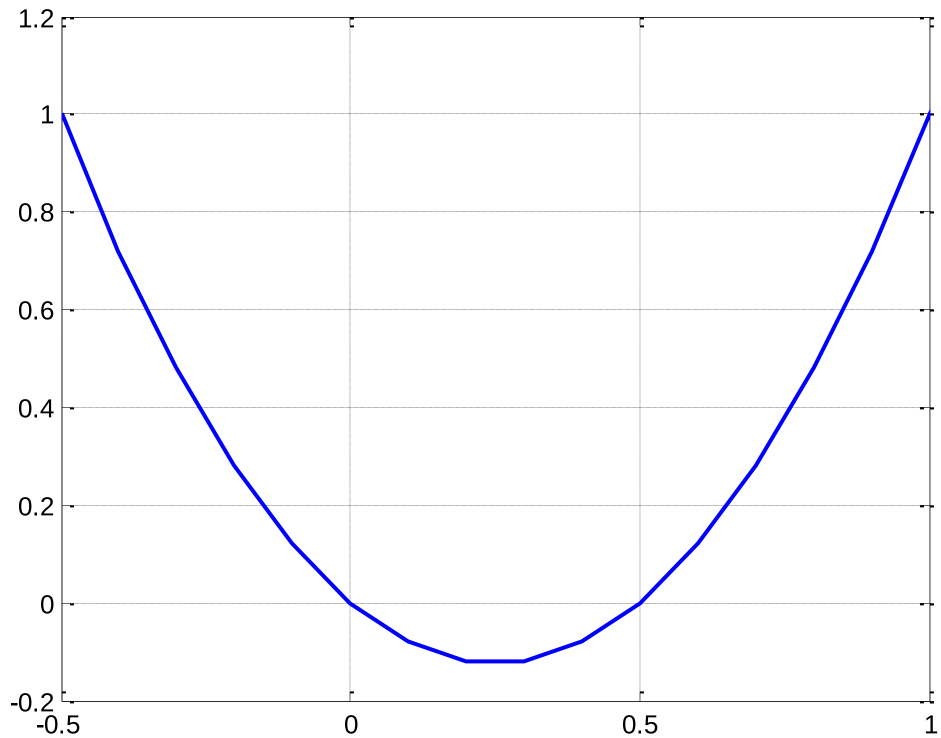
Stability of equilibrium points in first order equations:

For first order autonomous systems, the derivative does not change with time, so we can plot dy/dt vs. y and use this plot to determine the stability of the equilibrium points.

Recall: Stable, unstable and semi-stable equilibrium points in ENGR 231.

Example:

$$\frac{dy}{dt} = 2y^2 - y$$



This is a graph of $\frac{dy}{dt}$ $vs\ y$. Here, we can clearly see the equilibrium points at 0 and $1/2$ which can be verified by setting $dy/dt = 0$ and solving for y. The equilibrium point 0 is stable and the equilibrium point ½ is unstable.

Region of attraction of the stable point is: $(-\infty, 0.5)$ and the unstable point is [0.5]

Direction fields:

Given a first order ordinary differential equation,

$$\frac{dy}{dt} = f(t, y),$$

we have an expression for the derivative of the variable for each value t and y. Recall what the derivative is however, it is slope of the line tangent to the curve at that point. One can construct a figure on which the value of the derivative is used to draw an arrow of corresponding slope over a grid of points t, y.

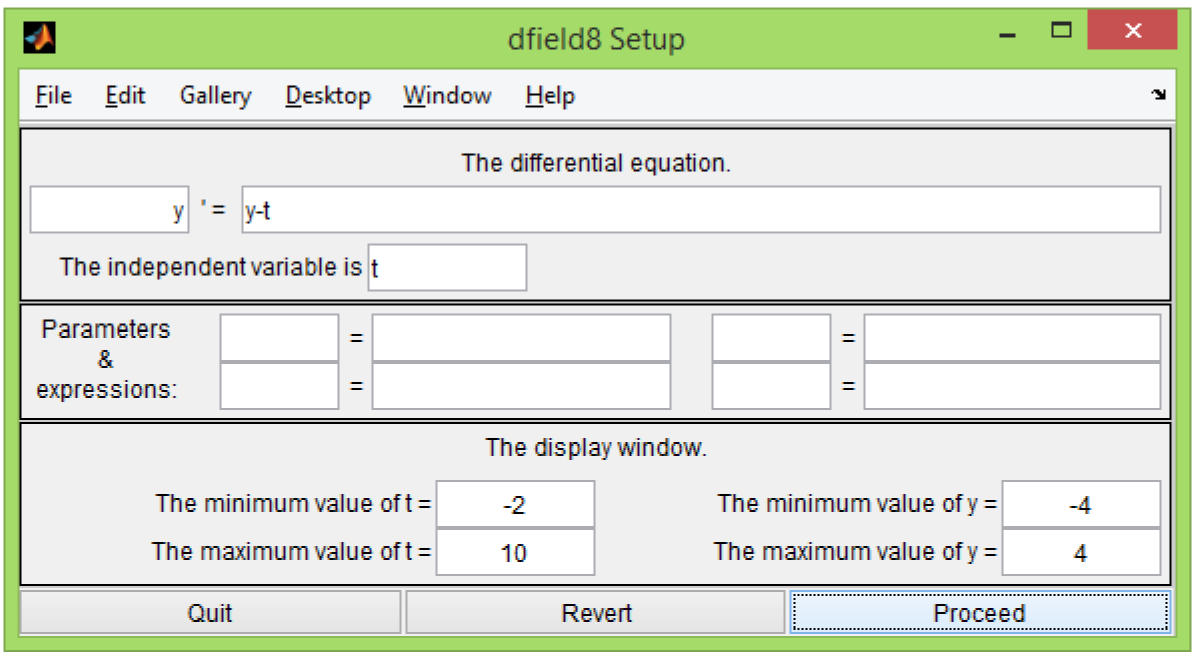
For example:

$$\frac{dy}{dt} = y - t$$

On a set of axes with t on the x-axis and y on the y-axis, the value of the derivative can be determined at each point by calculating dy/dt at that point. Representing this derivative as a directional arrow of slope = dy/dt yields a direction field plot.

We will not do this by hand, but rather using the Rice University's d-field tool.

You can download the m-file from the course website under the MATLAB Resources Folder.

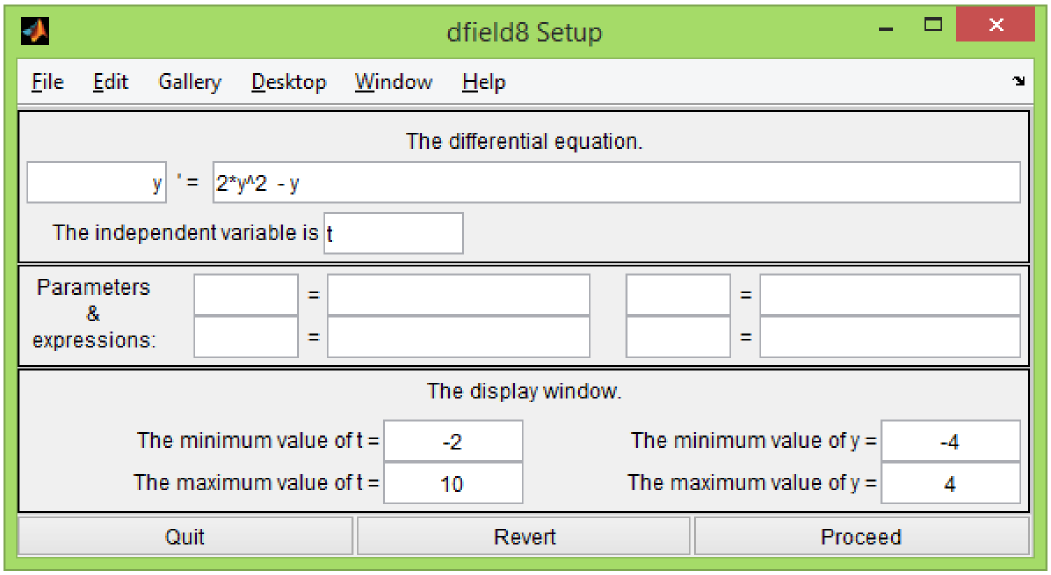


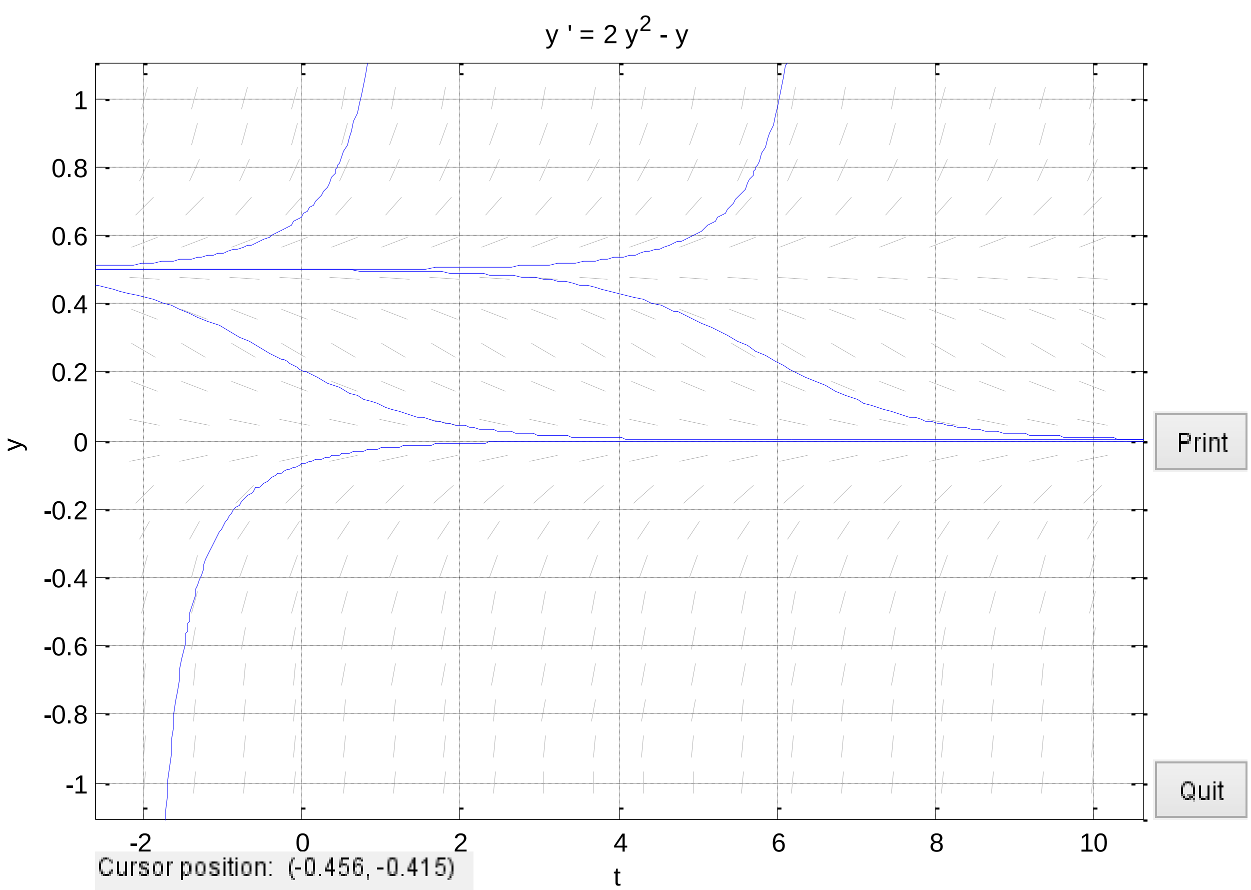
Note: You can type in the differential equation (MATLAB syntax needed) and set the time-scale and the limits for the y-axis.

The online applet uses arrows, the MATLAB version does not have arrows – we assume time is moving forward so all arrows point accordingly.

Previous Example:

$$\frac{dy}{dt} = 2y^2 - y$$





By clicking on the graph, you are asking MATLAB to generate a solution trajectory which will correspond to a unique set of initial conditions. Notice how the lines all move away from 0.5. The ROA of equilibrium point 0 is $(-\infty, 0.5)$ and so any lines that pass through this region will be attracted to the equilibrium point 0 – as seen.

Recognize that we have not solved the differential equation. We can see what the solution will look like simply based on knowing the function (derivative). This is a numerical solution and gives us quantitative values for a solution. While we cannot write down the equation for the solution based solely on this approach, we can certainly study the behaviour of the system by analyzing the solution trajectories.