Drexel University

Office of the Dean of the College of Engineering

ENGR 232 – Dynamic Engineering Systems

Week 1 Laboratory Assignment

In this lab we will investigate the use of direction fields to evaluate solutions to first order differential equations.

For this lab, we will study these qualitative analysis tools via the example of population growth using the Gompertz equation as a model. This equation is used to model the growth of tumor cells, and gives an expression of the dependence of tumor size on time.

The differential equation governing the growth is:

$$\frac{dV}{dt} = \beta V \ln\left(\frac{k}{V}\right)$$

Here β is the growth rate k is a reflection of the carrying capacity of the tumor.

Let $\beta = 0.5$; $k = 150$; and V range from 0 to 170

1. Plot the graph of dV/dt vs. V. Note you may have to use the vectorization operator "." to evaluate the function of the differential equation.

2. Determine the equilibrium points (by-hand or MATLAB) and plot them on the same graph as part (1) using red circles. Use a legend to indicate the equilibrium points and the original plot.

3. Using the gradient() function, plot the derivative of dV/dt versus V on the same plot. That is, plot d^2V/dt^2 vs. V.

4. Determine the stability of each equilibrium point and the region of attraction. Write these answers as comments in your code. Comment on the stability as it related to the graph.

5. From this graph, are we able to tell how long the tumor will take to grow to $V = 80$ units if the initial size is $V(0) = 10$ units? Why? Write your answers as comments in the code.

We will now use the direction field tool to examine the solution.

6. **Download** the file from the course website in the MATLAB Resources folder. You may have to right-click and save the file to your computer. IMPORTANT! Make sure it is in the current directory you are working in.
   OR
   Access the dfield tools at http://math.rice.edu/~dfield/dfpp.html

7. Using the same parameters as above, plot the direction field of this differential equation. Set the time and variable limits appropriately.

8. Compare your results from part (4) to the direction field obtained to check: the stability of each equilibrium point and the region of attraction. Include these as comments in your code.

9. Estimate how long it will take to get $V = 140$ if $V(0) = 20$. Hint: You can go to: Options → Keyboard Input to enter the initial conditions.

10. Repeat the direction field plot with $B = 0.2$ and estimate again how long it takes for $V = 140$ if $V(0) = 20$. Comment on your results as they compare to part (9).

Note: Submit a published pdf file of your script with convention **lastname_initials_lab1.m** The published document must include all functions used (if any). All figures must be annotated (labels, legends, markers, title, etc. Answers to questions asked should be printed as an output.