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$$1.1 \quad Y(s) = \frac{s-14}{(s+2)(s-3)}$$

$$Y(s) = \frac{4}{s+2} + \frac{B-3}{s-3}$$

$$\frac{s-14}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$$

$$y(t) = 4e^{-2t} - 3e^{3t}$$

$$s-18 = A(s+3) + B(s+2)$$

$$s-18 = As + 3A + Bs + 2B$$

$$s = As + Bs$$

$$1 = A + B \quad A = 4$$

$$-18 = -3A + 2B \quad B = -3$$

$$1.2 \quad Y(s) = \frac{-3s^2 - 14s + 32}{(s+4)(s^2+4)} = \frac{A}{s+4} + \frac{Bs+C}{s^2+4}$$

$$Y(s) = \frac{2}{s+4} + \frac{-5s+6}{s^2+4}$$

$$-3s^2 - 14s + 32 = A(s^2+4) + (Bs+C)(s+4)$$

$$Y(s) = \frac{2}{s+4} + \frac{-5s}{s^2+4} + \frac{6}{s^2+4}$$

$$-3s^2 - 14s + 32 = As^2 + 4A + Bs^2 + Cs + 4Bs + 4C$$

$$y(t) = 2e^{-4t} - 5\cos 2t + 3\sin 2t$$

$$-3s^2 = As^2 + Bs^2$$

$$\begin{bmatrix} 1 & 1 & 0 & -3 \\ 0 & 4 & 1 & -14 \\ 0 & 0 & 4 & 32 \end{bmatrix} \begin{array}{l} A=2 \\ B=-5 \\ C=6 \end{array}$$

$$-3 = A + B$$

$$-14s = Bs + Cs + 4Bs$$

$$-14 = C + 4B$$

$$32 = 4C + 4A$$

$$1.3 \quad Y(s) = \frac{2s-3}{s^2+2s+10} = \frac{2s-3}{(s+1)^2+9}$$

$$(2s-3)[(s+1)^2+9] = (2s-3)(s^2+2s+10)$$

$$s^2 + 2s = -10$$

$$s^2 + 2s + 1 = -10 + 1$$

$$(s+1)(s+1) = -9$$

$$(s+1)^2 + 9 = 0$$

$$\frac{2(s-\frac{3}{2})}{(s+1)^2+9} = \frac{2s+15}{(s+1)^2+9} - \frac{15}{(s+1)^2+9}$$

$$2s+2 = \frac{2(s+1)}{(s+1)^2+9}$$

$$= 2e^{-1t} \cos 3t$$

$$\frac{2s-3}{(s+1)^2+9} = \frac{2s+2}{(s+1)^2+9} + \frac{-5}{(s+1)^2+9} \Rightarrow \frac{2s+3}{(s+1)^2+9} + \frac{5}{(s+1)^2+9} - \frac{5}{(s+1)^2+9}$$

*(**)

$$= \frac{2(s+1)}{(s+1)^2+9}$$

$$y(t) = 2e^{-it} \cos 3t + -\frac{5}{9} e^{-it} \sin 3t$$

$$1.4 \quad Y(s) = \frac{a_0}{(s+5)(s+2)^2} = \frac{A}{(s+5)} + \frac{B}{(s+2)} + \frac{Cs+D}{(s+2)^2}$$

$$a_0 = A(s+2)(s+2)^2 + B(s+5)(s+2)^2 + (Cs+D)(s+5)(s+2)$$

$$a_0 = As^3 + 16s^2 + 44s + 48 + Bs^3 + 9Bs^2 + 24Bs + 20B + (Cs+D)(s^2 + 7s + 10)$$

$$a_0 = [As^3 + 6As^2 + 12As + 8A] + [Bs^3 + 9Bs^2 + 24Bs + 20B] + Cs^3 + 7Cs^2 + 10Cs + Ds^2 + 7Ds + 10D$$

$$Os^3 = As^3 + Bs^3 + Cs^3$$

$$Os^2 = 6As^2 + 9Bs^2 + 7Cs^2 + Ds^2$$

$$Os = 12As + 24Bs + 10Cs + 7Ds$$

$$a_0 = 8A + 20B + 10C + 7D$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 6 & 9 & 7 & 1 & 0 \\ 12 & 24 & 10 & 7 & 0 \\ 8 & 20 & 0 & 10 & 90 \end{bmatrix}$$

$$A = \frac{a_0}{(-5+2)^2} = \frac{a_0}{49} = 10$$

$$C = \frac{a_0}{(-2+5)} = \frac{a_0}{3} = 30$$

$$\frac{a_0}{(s+5)(s+2)^2} = \frac{a_0/49}{(s+5)} + \frac{\frac{30}{s+2}}{(s+2)} + \frac{\frac{30}{s+5}}{(s+2)^2}$$

$$a_0 = \frac{a_0}{49}(s+2)^2 + \frac{30}{s+2}(s+2)(s+5) + \frac{30}{s+5}(s+5)$$

$$90 = 10 \frac{90}{49} (s^2 + 4s + 4) + 30(s^2 + 7s + 10) + \frac{30Cs + 5C \cdot 30}{Cs^2 + 5Cs + Ds + 5D}$$

$$90 = \frac{90}{49}(4) + 30(10) + 5D$$

$$D = \frac{90}{49}(1) + 30(1) + C(1)$$

$$C = -\frac{1560}{49}$$

$$90 = 40 + 10B + 30 \cdot 5 \quad B = -10$$

$$Q = \frac{90}{49}(4) + 30(7) + -\frac{1560}{49}(5) + -\frac{2130}{49}(1)$$

$$Y(s) = \frac{90}{(s+5)(s+2)^2} = \frac{10}{s+5} + \frac{-10}{s+2} + \frac{30}{(s+2)^2}$$

$$y(t) = 10e^{-5t} + -10e^{-2t} + 30t e^{-2t}$$

$$1.5 \quad y'' + 6y' + 8y = 0 \quad y(0) = -4, \quad y'(0) = 1$$

$$L\{y''\} + 6L\{y'\} + 8L\{y\} = 0$$

$$[s^2 y(s) - s y(0) - y'(0)] + 6[s y(s) - y(0)] + 8[y(s)] = 0$$

$$Y(s) [s^2 - 5 + 4] + Y(s) []$$

$$s^2 y(s) - s + 4 + 6s y(s) - 6 + 8y(s) = 0$$

$$s^2 y(s) - s + 4 + 6s y(s) - 6 + 8y(s) = 0$$

$$y(s) [s^2 + 6s + 8] = s + 2$$

$$y(s) = \frac{s+2}{(s+4)(s+2)} = \frac{1}{s+4} \quad y(t) = e^{-4t}$$

$$1.6 \quad y'' + 6y' + 8y = 0 \quad y'(0) = 1, \quad y(0) = 1$$

$$L\{y''\} + 6L\{y'\} + 8L\{y\} = 0$$

$$y(s) = \frac{s+7}{(s+4)(s+2)} = \frac{-1.5}{s+4} + \frac{2.5}{s+2}$$

$$y(t) = -1.5e^{-4t} + 2.5e^{-2t}$$

$$[s^2 y(s) - s y(0) - y'(0)] + 6[s y(s) - y(0)] + 8[y(s)] = 0$$

$$s^2 y(s) - s + 4 + 6s y(s) - 6 + 8y(s) = 0$$

$$y(s) [s^2 + 6s + 8] = s + 7$$

$$y(s) = \frac{s+7}{(s+4)(s+2)} = \frac{A}{s+4} + \frac{B}{s+2}$$

$$s+7 = A(s+2) + B(s+4)$$

$$1s = As + Bs \quad A = -1.5$$

$$7 = 2A + 4B \quad B = 2.5$$

$$1.7 \quad y'' + 6y' + 8y = 5 \quad y(0) = 1, \quad y'(0) = 1$$

$$\mathcal{L}\{y''\} + 6\mathcal{L}\{y'\} + 8\mathcal{L}\{y\} = 5$$

$$[s^2 y(s) - sy(0) - y'(0)] + 6[sy(s) - y(0)] + 8[y(s)] = \frac{5}{s}$$

$$s^2 y(s) - s - 1 + 6sy(s) - 6 + 8y(s) = \frac{5}{s}$$

$$y(s)[s^2 + 6s + 8] = s + 7 + \frac{5}{s}$$

$$y(s) = \frac{s+7}{s^2 + 6s + 8} + \frac{5}{s(s^2 + 6s + 8)} = \frac{s^2 + 7s + 5}{(s^2 + 6s + 8)s} = \frac{A}{(s+4)} + \frac{B}{(s+2)} + \frac{C}{s}$$

$$s^2 + 7s + 5 = A(s)(s+2) + B(s)(s+4) + C(s+2)(s+4)$$

$$= As^2 + A2s + Bs^2 + B4s + Cs^2 + C8s + C8$$

$$s^2 = As^2 + Bs^2 + Cs^2 \quad A = -7/8$$

$$7s = 2As + 4Bs + 6Cs \quad B = 5/4$$

$$5 = 0A + 0B + 8C \quad C = 5/8$$

$$y(s) = \frac{s^2 + 7s + 5}{s(s^2 + 6s + 8)} = \frac{-7/8}{s+4} + \frac{5/4}{s+2} + \frac{5/8}{s}$$

$$y(t) = \frac{5}{8} + \frac{5}{4}e^{-2t} + \frac{-7}{8}e^{-4t}$$

$$1.8 \quad y'' + 5y' + 6y = 5e^{-5t} \quad y'(0) = 0, \quad y(0) = 0$$

$$[s^2 y(s) - sy(0) - y'(0)] + 5[sy(s) - y(0)] + 6[y(s)] = \frac{5}{s+5}$$

$$y(s)[s^2 + 5s + 6] + 0 = \frac{5}{s+5}$$

$$y(s) = \frac{5}{(s+5)(s+3)(s+2)} = \frac{A}{s+5} + \frac{B}{s+3} + \frac{C}{s+2}$$

$$5 = A(s+3)(s+2) + B(s+5)(s+2) + C(s+5)(s+3)$$

$$5 = As^2 + A5s + A6 + Bs^2 + B7s + B10 + Cs^2 + C8s + C15$$

$$A = 5/6$$

$$B = -5/2$$

$$C = 5/3$$

$$y(s) = \frac{5/6}{s+5} + \frac{-5/2}{s+3} + \frac{5/3}{s+2} \quad \text{Same setup as 1.7.}$$

$$y(t) = \frac{5}{6}e^{-5t} + \frac{-5}{2}e^{-3t} + \frac{5}{3}e^{-2t}$$

$$1.9 \quad y' + 6y = t \quad y(0) = 1$$

$$[sy(s) - y(0)] + 6[y(s)] = t$$

$$y(s) [s+6] + 1 = \frac{1}{s^2}$$

$$y(s) = \frac{1}{s^2(s+6)} + \frac{1}{s+6}$$

$$= \frac{1+s^2}{s^2(s+6)} = \frac{A}{s^2} + \frac{B}{s+6} + \frac{C}{s}$$

cover up

$$s=0$$

$$s = -6$$

$$B = \frac{1+0}{6} = \frac{1}{6}$$

$$A = \frac{1+36}{36} = \frac{37}{36}$$

$$y(s) = \frac{37/36}{s^2} + \frac{1/6}{s+6} + \frac{5/6}{s}$$

$$y(t) = \frac{37}{36}t + \frac{1}{6}e^{-6t} + \frac{5}{6}$$

$$x \frac{s^2+1}{s^2(s+6)} = \frac{37/36}{s^2} + \frac{1/6}{s+6} + \frac{C}{s}$$

$$s^2+1 = \frac{37}{36}(s+6) + \frac{1}{6}(s^2) + C(s)(s+6)$$

$$s^2+1 = \frac{37}{36}s + \frac{37}{36}(6) + \frac{1}{6}s^2 + Cs^2 + 6Cs$$

$$s^2 = \frac{1}{6}s^2 + Cs^2 \quad C = \frac{5}{6}$$