Low Algebraic Matrix Completion

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1 Idea About Symmetric Treatment

Suppose $\Theta \in \mathbb{R}^{n_1 \times n_2}$ is a matrix whose rows and columns lie in low-dimensional algebraic variety $\mathcal{V}_1$ and $\mathcal{V}_2$. Assume that the SVD of Θ is

$$\Theta = U \Sigma V^\top,$$

where $U \in \mathbb{R}^{n_1 \times n_1}$ and $V \in \mathbb{R}^{n_2 \times n_2}$. Since for any invertible matrix $\Gamma \in \mathbb{R}^{n_1 \times n_1}$, the columns of $\Gamma\Theta$ also lie in a low-dimensional algebraic variety, we have that $U^\top \Theta = \Sigma V^\top$ is a LAD matrix. Let r be the rank of Θ and Σ_r corresponds to the nonzero diagonal submatrix of Σ. We also write $V = (V_r, \tilde{V}_r)$ so that

$$\Sigma V^\top = \begin{pmatrix} \Sigma_r V_r^\top \\ 0 \end{pmatrix}.$$

We see that the columns of $\Sigma_r V_r^\top$ are also in a low algebraic variety, and so do $V_r^\top$. Similar argument applies to $U_r^\top$. Thus, one rough approach is to consider the optimization problem

$$\begin{aligned} \min_{\substack{V_r \in \mathbb{R}^{n_2 \times r}, U_r \in \mathbb{R}^{n_1 \times r} \\ \sigma \in \mathbb{R}^r}} \ & \frac{1}{2} \| \mathcal{R}_\Omega (\Theta - U_r \operatorname{diag}(\sigma) V_r^\top \|_F^2 + \lambda_1 \| \left(V_r^\top \right)^{\otimes p} \|_* + \lambda_2 \| \left(U_r^\top \right)^{\otimes p} \|_* \\ & + \frac{\lambda_3}{2} \| U_r^\top U_r - I_r \|_F^2 + \frac{\lambda_4}{2} \| V_r^\top V_r - I_r \|_F^2. \end{aligned}$$

There are many problems related to this approach, for example, how to choose r and how to solve it.

2 Simulation

I have finished nearly everything of the code, but still have several steps away from reproducing the results. I have some troubles figuring out how to choose the step size in the gradient decent step. Also, from my current simulation, I have not seen any significant improvement over LRMC. I am still working on it.