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ECSE 443 Final Project

Q1.

In question 1 we compute the integral using Simpson's rule. The value computed for the integral was 0.785325349762923. This answer matches the true value within 4 decimals.

Q2.

In question 2 we compute the integral using the midpoint method. The value computed for the integral was 0.623775746323755.

Q3. (unsolved)

Q4.

In question 4 the values of the coefficients were determined using forward finite difference. We assumed `y(a) => y(2), y(b) => y(3), and y(c) => y(4)`

The coefficients were found to be:
1.0e+02 * -0.139547265702589
1.0e+02 * -0.861509728326910
1.0e+02 * -2.369463437976441

Q5.

In question 5 we compute the answer and the square error of the solution using LU factorization.

The solution was: 18.000000000000000
25.999999999999986
33.999999999999901
82.000000000000000
And the square error was 2.524354896707238e-27. This was very reasonable.

Q6.

In question 6 we compute the answer and the square error of the solution using fixed point iteration.

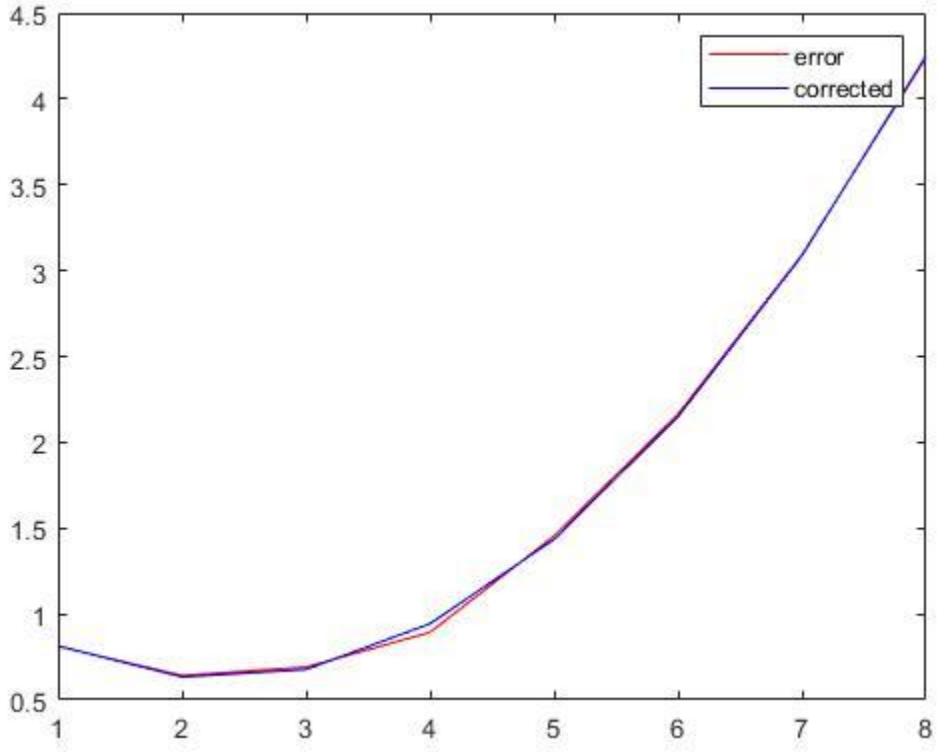
The solution was: 18.000000000000000
26.000000000000004
33.999999999999993
82.000000000000000
And the square error was 1.577721810442024e-29. This was very reasonable.

Q7.

In question 7 we find the smallest possible root using the secant method. The root was found to be 4.730040769668035.

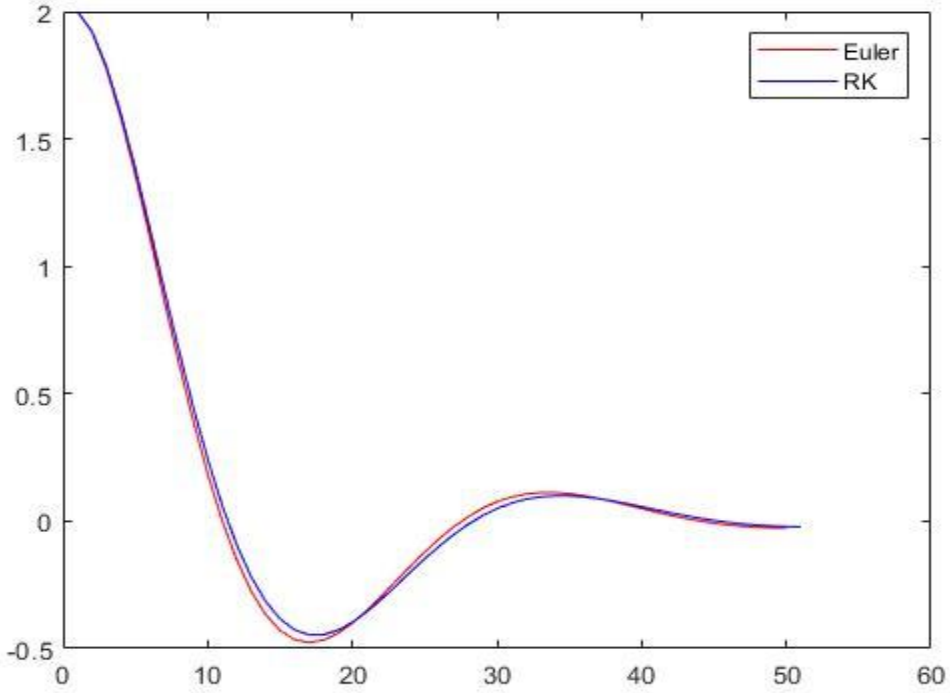
Q8.

In this question the 4th point was found to be erroneous and both the error and the corrected graphs were plotted. The point was corrected to 0.943642857142857.



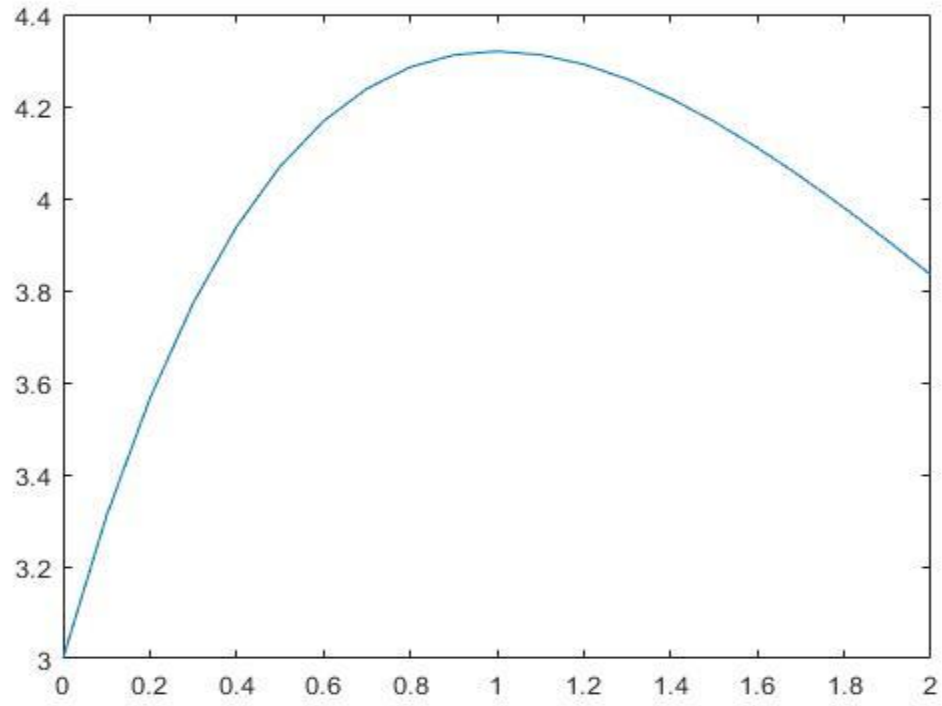
Q9.

In this question both Euler and RK methods were used and the results were plotted.
The resulting plots were very close to being identical.



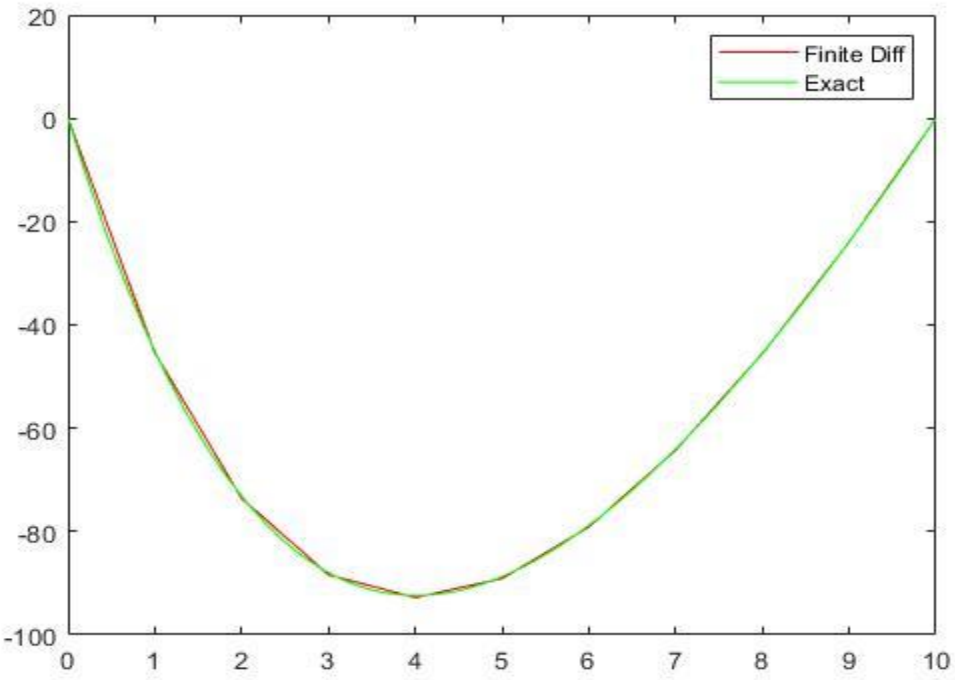
Q10.

In this question the second order RK was used to solve the ODE and plot the result.



Q11.

In this question we solve the boundary value problem using finite difference method. Both the solution and the exact graphs were plotted.



Q12. (unsolved)

Appendix: Code used for all questions

Q1

```
%function, limits and true value
format long;
syms x;
f = @(x) 1/(exp(x) + exp(-x));
LL = 0;
UL = inf;
trueValue = int(f(x), LL, UL);
%number of steps, step size, accuracy,
relative error and sum
n = 0;
s = 0;
acc = 0.0001;
relativeError = 100;
simSum = 0;
while(relativeError > acc)
    %large upper limit = 100
    s = (100 - LL)/n;
    simSum = 0;
    for i=1:n
        %simpson's rule
        add = (s/6)*(f(LL + (i-1)*s) +
f((LL + (i-1)*s) + s) + 4*f((LL + (i-
1)*s) + s/2));
        simSum = simSum + add;
    end
    relativeError = abs(simSum -
trueValue)/trueValue;
    n = n + 1;
end
display(simSum);
display(trueValue);
```

Q2

```
%limits, step size, number of steps and
sum
format long;
LL = 0;
UL = 1;
s = 0.01;
n = (UL - LL)/s;
midSum = 0;
x = 0;
deltaX = (UL - LL)/100;
for i=1:n  %100
    %midpoint rule
    x = LL + (i*(deltaX)) -
((deltaX)/2);
    %function
    y = (x^3)/((1 - x^2)^0.5);
    midSum = midSum + y*deltaX;
end
display(midSum);
```

Q3 (unsolved)

Q4

```
%output and step size
format long;
y = [1, 0.1160, -0.1084, -0.0409,
0.0052, 0.0067, 0.009];
s = 0.25;
%get matrix -> [y''(a), y'(a), y(a);
                %y''(b), y'(b), y(b);
                %y''(c), y'(c), y(c)]
%y(a) => y(2), y(b) => y(3), and y(c) =>
y(4)
yA = y(2);
yB = y(3);
yC = y(4);
%forward finite difference
%for y''(a), y'(a)
y1stDerA = ((-11/6)*y(2) + 3*y(3) -
1.5*y(4) + (1/3)*y(5))/s;
y2ndDerA = (2*y(2) - 5*y(3) + 4*y(4) -
y(5))/(s*s);
%for y''(b), y'(b)
y1stDerB = ((-11/6)*y(3) + 3*y(4) -
1.5*y(5) + (1/3)*y(6))/s;
y2ndDerB = (2*y(3) - 5*y(4) + 4*y(5) -
y(6))/(s*s);
%for y''(c), y'(c)
y1stDerC = ((-11/6)*y(4) + 3*y(5) -
1.5*y(6) + (1/3)*y(7))/s;
y2ndDerC = (2*y(4) - 5*y(5) + 4*y(6) -
y(7))/(s*s);
%assemble answer
matrix = [y2ndDerA, y1stDerA, yA;
          y2ndDerB, y1stDerB, yB;
          y2ndDerC, y1stDerC, yC];
answer = matrix\[1, 1, 1]';
display(answer);
```

Q5

```
%matrix A, vector b and number of rows
A = [3 -5 47 20; 11 16 17 10; 56 22 11 -
18; 17 66 -12 7];
b = [18; 26; 34; 82;];
r = 4;
%U and L
U = A;
L = eye(4);
%doolittle algorithm to get U & L
for i = 1:r-1
    for j = i+1:r
        L(j,i) = U(j,i)/U(i,i);
        U(j,i:r) = U(j,i:r) -
(U(i,i:r)*L(j,i));
    end
end
%solve for y using y = b/L
y = L^(-1)*b;
%solve for x using x = y/U
x = U^(-1)*y;
%verify
s = zeros(4,1);
for i=1:4
    s(i,1) = A(i,1)*x(1,1) +
A(i,2)*x(2,1) + A(i,3)*x(3,1) +
A(i,4)*x(4,1);
end
%get square error
squareError = 0;
e = zeros(4,1);
for i=1:4
    e(i,1) = (b(i,1) - s(i,1))^2;
end
for i=1:4
    squareError = squareError + e(i,1);
end
squareError = squareError/4;
display(squareError);
disp(s);
```

Q6

```
%matrix A, vector b and x (Ax = b)
format long;
A = [3 -5 47 20; 11 16 17 10; 56 22 11 -
18; 17 66 -12 7];
b = [18; 26; 34; 82;];
x = [0; 0; 0; 0];
for i=1:1000 %large number to decrease
error
  x(4) = (b(2) - A(2)*x(1) - A(2,2)*x(2)
- A(2,3)*x(3))/A(2,4);
  x(3) = (b(1) - A(1)*x(1) - A(1,2)*x(2)
- A(1,4)*x(4))/A(1,3);
  x(2) = (b(4) - A(4)*x(1) - A(4,3)*x(3)
- A(4,4)*x(4))/A(4,2);
  x(1) = (b(3) - A(3,2)*x(2) -
A(3,3)*x(3) - A(3,4)*x(4))/A(3);
  x = x';
end
%verify
s = zeros(4,1);
for i=1:4
    s(i,1) = A(i,1)*x(1,1) +
A(i,2)*x(2,1) + A(i,3)*x(3,1) +
A(i,4)*x(4,1);
end
%get square error
squareError = 0;
e = zeros(4,1);
for i=1:4
    e(i,1) = (b(i,1) - s(i,1))^2;
end
for i=1:4
    squareError = squareError + e(i,1);
end
squareError = squareError/4;
display(squareError);
disp(s);
```

Q7

```
%function, guesses and relative error
syms x;
f = @(x) cos(x)*cosh(x) - 1;
g1 = 4;
g2 = 6;
relativeError = 10^(-4);
%function at guess
fg1 = f(g1);
deltaX = 100;
%secant method
while(abs(deltaX) > relativeError)
    fg2 = f(g2);
    deltaX = (g2-g1)*fg2/(fg2-fg1);
    %update
    fg1 = fg2;
    g1 = g2;
    g2 = g2 - deltaX;
end
answer = g2;
display(answer);
```

Q8

```
%error at x = 4
%input and output
x = [1, 2, 3, 4, 5, 6, 7, 8];
y = [0.812, 0.642, 0.691, 0.893, 1.454,
2.164, 3.092, 4.24];
%coefficients and corrected function
z = ones(size(x));
A = [z', x', (x.*x)'];
nA = A'*A;
nB = A'*y';
coeff = nA\nB;
correct = coeff(1) + coeff(2).*x +
coeff(3).*x.^2;
%display corrected point
display(correct(4));
%display both plots
plot(x, y, 'red')
hold on;
plot(x, correct, 'blue')
hold off;
legend('error', 'corrected')
```

Q9

```
%initial conditions, t's, step size and
number of steps
y0 = 2;
dy0 = 0;
t0 = 0;
t5 = 5;
s = 0.1;
n = (t5 - t0)/s;
%Euler
for i=1:n
    %#ok<*SAGROW>
    y(i) = y0 + (dy0*s);
    dy(i)= dy0 + (-2*dy0 - 4*y0)*s;
    %update IC's
    y0 = y(i);
    dy0 = dy(i);
end
%reset initial conditions
y0 = 2;
dy0 = 0;
g = dy0;
f = @(g,y) -2*g - 4*y;
%R-K
RK = zeros(1,10);
i = 1;
for t = t0:s:t5
    K1 = s*g;
    R1 = s*f(g, y0);
    K2 = s*(g + K1/2);
    R2 = s*f(g + R1/2, y0 + K1/2);
    K3 = s*(g + K2/2);
    R3 = s*f(g + R2/2, y0 + K2/2);
    K4 = s*(g + K3/2);
    R4 = s*f(g + R3/2, y0 + K3/2);
    Ky = (K1 + 2*K2 + 2*K3 + K4)/6;
    Kg = (R1 + 2*R2 + 2*R3 + R4)/6;
    y0 = y0 + Ky;
    RK(i) = y0;
    i = i + 1;
    g = g + Kg;
end
plot(y,'red')
hold on
plot(RK,'blue')
hold off
legend('Euler', 'RK')
```

Q10

```
%x's, initial condition, step size,
number of steps
x0 = 0;
x2 = 2;
y0 = 3;
s = 0.1;
n = (x2-x0)/s;
x = (x0:s:x2)';
y = zeros(10, 1);
%second-order R-K
for i=1:n
    y(1) = y0;
    y1 = -1.2*y(i) + 7*exp(-0.3*x(i));
    y2 = -1.2*(y(i) + s*y1) + 7*exp(-
0.3*(x(i) + s));
    y(i+1) = y(i) + (0.5*s)*(y1 + y2);
end
plot(x,y);
```

Q11

```
syms x
syms y(x)
%exact using dsolve and diff
%function and conditions
fnc = diff(y, x, 2) + (1/4)*diff(y, x,
1) == 8;
c1 = y(0) == 0;
c2 = y(10) == 0;
exactF = dsolve(fnc, [c1 c2]);
%finite difference method
%step size, t's, t range, number of
steps, and equ
eq = 8;
s = 1;
t0 = 0;
t10 = 10;
t = t0:s:t10;
n = 1 + (t10 - t0)/s;
%prepare equations
%yh = y + h, y_h = y - h
syms y
syms yh
syms y_h
dydt = (yh - y_h)/2*s;
d2ydt2 = (yh - 2*y + y_h)/s^2;
%finite difference equation
fnc(y_h, y, yh) = dydt/4 + d2ydt2;
%set A and B
A(1:n,1:n)=0;
A(1,1)=1;
A(n,n)=1;
%fill A coefficients
for i=2:n-1
    A(i,i-1) = fnc(1,0,0);
    A(i,i) = fnc(0,1,0);
    A(i,i+1) = fnc(0,0,1);
end
B(1:n,1)=0;
B(2:n-1,1)=eq;
finiteD = (A'*A)\(A'*B);
plot(t,finiteD,'red');
hold on
fplot(exactF,[0,10],'green')
hold off
legend('Finite Diff', 'Exact')
```