Ezz Aboulezz
260677463
ECSE 443 Assignment 4

Q1.a)

In part a we compute the integral using the mid-point rule. The value computed for the integral was 4.355199572285354 with number of steps = 7.

This value was compared with the MATLAB function value. Absolute and Relative errors we found to be:
absolute error = 2.739167815057897e-05
relative error = 6.289459294525523e-06

Q1.b)

In part b we compute the integral using the trapezoidal rule. The value computed for the integral was 4.355199572285354 with number of steps = 8.

This value was compared with the MATLAB function value. Absolute and Relative errors we found to be:
absolute error = 2.739335005674803e-05
relative error = 6.289843184323615e-06

Q1.c)

In part c we compute the integral using the Simpson’s rule. The value computed for the integral was 4.355199572285354 with number of steps = 13.

This value was compared with the MATLAB function value. Absolute and Relative errors we found to be:
absolute error = 4.259497382808064e-05
relative error = 9.780319138184335e-06

Q2.a)

In part a we compute the integral using the mid-point rule. The value computed for the integral was 7.905347558749129e+02 with number of steps = 371.

This value was compared with the MATLAB function value. Absolute and Relative errors we found to be:
absolute error = 9.584108014450976e-04
relative error = 1.212356107542929e-06

Q2.b)

In part b we compute the integral using the trapezoidal rule. The value computed for the integral was 7.905365739128109e+02 with number of steps = 554.

This value was compared with the MATLAB function value. Absolute and Relative errors we found to be:
absolute error = 8.596270965881558e-04
relative error = 1.087398179555833e-06

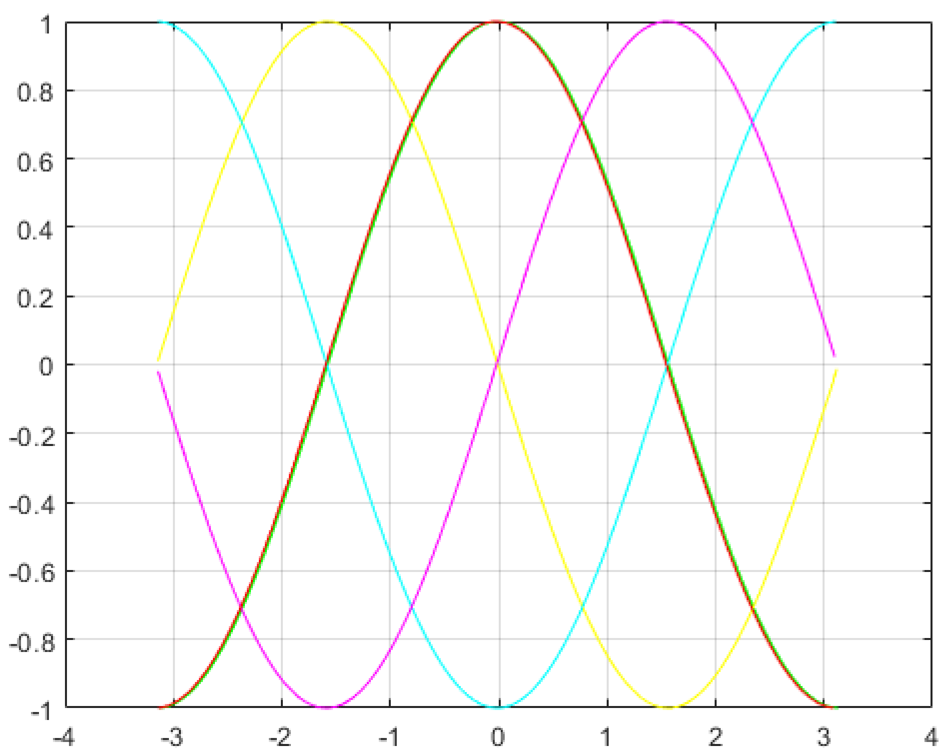
Q2.c)

In part c we compute the integral using the Simpson's rule. The value computed for the integral was 7.905364603760823e+02 with number of steps = 14.

This value was compared with the MATLAB function value. Absolute and Relative errors we found to be:
absolute error = 7.460903680112096e-04
relative error = 9.437781931020496e-07

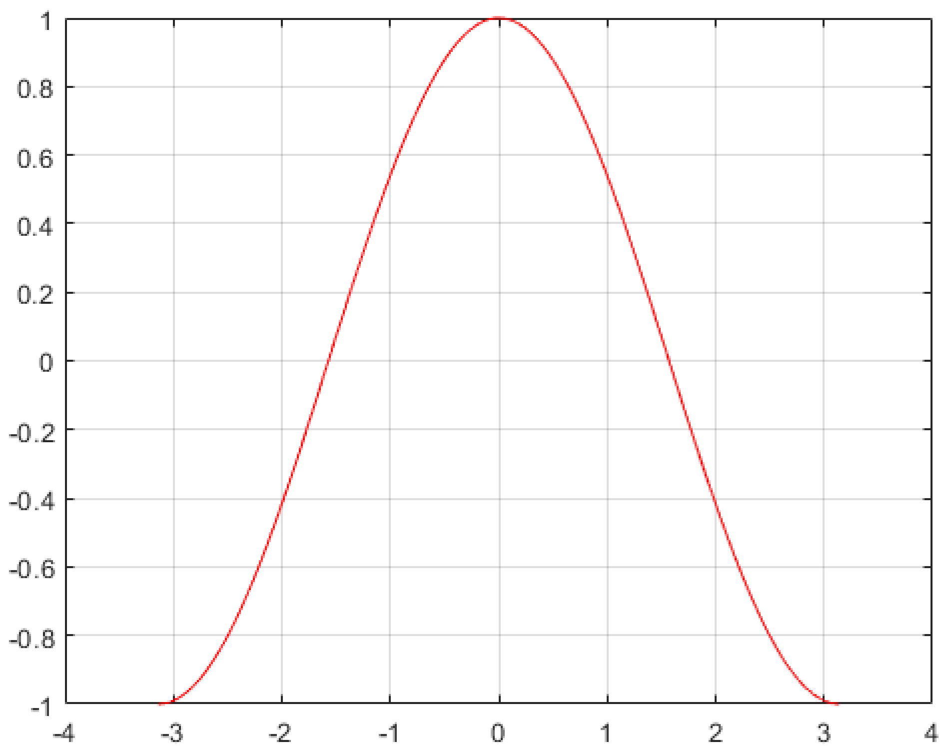
Q3.a)

In part a we compute the fifth backward difference and plot all 5 of them**.**



Q3.b)

In part b we compute the forward difference representation and plot it.



Q4.

f'(0) = 7, f'(2) = -7, f'(4) = -43, f''(0) = -5.

Appendix: Code used for all questions

Q1.a)

```
%function, limits, true value, and # of
steps
format long;
f = @(x) log(5-4*cos(x));
LL = 0;
UL = pi;
true = integral(f, LL, UL);
n = 0;
%relative error and midpoint sum
absoluteError = 0;
relativeError = 0;
errorPower = 0;
midSum = 0;
while ~(errorPower == -6)
    midSum = 0;
    n = n + 1;
    LL = 0;
    %delta x is difference between
bounds divided n
    deltaX = (UL - LL)/n;
    %compute the integral for n using
midpoint rule
    for i=1:n
        %compute x and y
        x = LL + i*(deltaX) -
(deltaX/2);
        y = log(5-4*cos(x));
        %multiply y and delta x, and
store in sum
        midSum = midSum + y*deltaX;
    end
    %error calculation
    absoluteError = abs(midSum - true);
    relativeError = absoluteError/true;
    errorPower =
ceil(log10(relativeError)-1);
end
disp(n);
disp(midSum);
disp(relativeError);
```

Q1.b)

```
%function, limits, true value, and # of
steps
f = @(x) log(5-4*cos(x));
LL = 0;
UL = pi;
true = integral(f, LL, UL);
n = 0;
%relative error and trapezoidal sum
absoluteError = 0;
relativeError = 0;
errorPower = 0;
trapSum = 0;
while ~(errorPower == -6)
    trapSum = 0;
    n = n + 1;
    LL = 0;
    %delta x is difference between
bounds divided n-1
    deltaX = (UL - LL)/(n-1);
    for i=1:n
        %compute x and y
        x = LL + ((i-1)*(deltaX));
        y = log(5-4*cos(x));
        if (i == 0)||(i == n)
            coeff = 0.5;
        else
            coeff = 1;
        end
        %store in sum
        trapSum = trapSum +
(coeff*deltaX*y);
    end
    %error calculation
    absoluteError = abs(trapSum - true);
    relativeError = absoluteError/true;
    errorPower =
ceil(log10(relativeError)-1);
end
disp(n);
disp(trapSum);
disp(relativeError);
```

Q1.c)

```
%function, limits, true value, and # of
steps
f = @(x) log(5-4*cos(x));
LL = 0;
UL = pi;
true = integral(f, LL, UL);
n = 0;
%relative error and Simpson sum
absoluteError = 0;
relativeError = 0;
errorPower = 0;
simSum = 0;
while ~(errorPower == -6)
    simSum = 0;
    n = n + 1;
    LL = 0;
    %delta x is difference between
bounds divided n-1
    deltaX = (UL - LL)/(n-1);
    %compute the integral for n using
Simpson's rule
    for i=1:n
        %compute x and y
        x = LL + ((i-1)*(deltaX));
        y = log(5-4*cos(x));
        %coefficient we multiply y with
based on parity
        if (mod(i,2) == 0)
            coeff = 4;
        else
            coeff = 2;
        end
        if (i == 0)||(i == n)
            coeff = 1;
        end
        %store in sum
        simSum = simSum +
(coeff*deltaX*(1/3)*y);
    end
    %error calculation
    absoluteError = abs(simSum - true);
    relativeError = absoluteError/true;
    errorPower =
ceil(log10(relativeError)-1);
end
disp(n);
disp(simSum);
disp(relativeError);
```

Q2.a)

```
%function, limits, true value, and # of
steps
syms x y;
f = x^2 + y;
LLy = x;
ULy = 2*x^3;
LLx = 2;
ULx = 3;
trueY = int(f, y, LLy, ULy);
trueX = int(trueY, x, 2, 3);
trueX = double(trueX);
n = 370;
%absolute error
absoluteError = 0;
errorPower = 0;
while (errorPower > -4)
    n = n + 1;
    LLy = x;
    %dyIntegral stores the first
integration
    dyIntegral = 0;
    deltaX = (ULy - LLy)/n;
    %compute integral if upper and lower
limits not equal
    while(LLy ~= ULy)
        %mid point rule
        dyIntegral = dyIntegral +
subs(f, y, LLy + (deltaX/2))*(deltaX);
        LLy = LLy + deltaX;
    end
    LLx = 2;
    deltaX = (ULx - LLx)/n;
    %dxIntegral stores the final value
    dxIntegral = 0;
    %compute the integral if lower limit
of x is less than upper limit
    while(LLx < ULx)
        %mid point rule
        dxIntegral = dxIntegral +
subs(dyIntegral, x, LLx +
(deltaX/2))*(deltaX);
        LLx = LLx + deltaX;
    end
    %error calculation
    dxIntegral = double(dxIntegral);
    absoluteError = abs(dxIntegral -
trueX);
    errorPower =
ceil(log10(absoluteError)-1);
end
disp(dxIntegral);
disp(n);
disp(absoluteError);
```

Q2.b)

```
%function, limits, true value, and # of
steps
syms x y;
f = x^2 + y;
LLy = x;
ULy = 2*x^3;
LLx = 2;
ULx = 3;
trueY = int(f, y, LLy, ULy);
trueX = int(trueY, x, 2, 3);
trueX = double(trueX);
n = 550;
%absolute error
absoluteError = 0;
errorPower = 0;
while (errorPower > -4)
    n = n + 1;
    LLy = x;
    %dyIntegral stores the first
integration
    dyIntegral = 0;
    deltaX = (ULy - LLy)/n;
    %compute integral if upper and lower
limits are not equal
    while(LLy ~= ULy)
        %trapezoidal rule
        y1 = subs(f, y, LLy);
        y2 = subs(f, y, LLy + deltaX);
        dyIntegral = dyIntegral + (y1 +
y2)*(deltaX/2);
        LLy = LLy + deltaX;
    end
    LLx = 2;
    %dxIntegral stores the final value
    dxIntegral = 0;
    deltaX = (ULx - LLx)/n;
    %compute the integral if lower limit
of x is less than upper limit
    while(LLx < ULx)
        %trapezoidal rule
        y3 = subs(dyIntegral, x, LLx);
        y4 = subs(dyIntegral, x, LLx +
deltaX);
        dxIntegral = dxIntegral + (y3 +
y4)*(deltaX/2);
        LLx = LLx + deltaX;
    end
    %error calculation
    dxIntegral = double(dxIntegral);
    absoluteError = abs(dxIntegral -
trueX);
    errorPower =
ceil(log10(absoluteError)-1);
end
disp(dxIntegral);
disp(n);
disp(absoluteError);
```

Q2.c)

```
%function, limits, true value, and # of
steps
syms x y;
f = x^2 + y;
LLy = x;
ULy = 2*x^3;
LLx = 2;
ULx = 3;
trueY = int(f, y, LLy, ULy);
trueX = int(trueY, x, 2, 3);
trueX = double(trueX);
n = 0;
%absolute error
absoluteError = 0;
errorPower = 0;
while (errorPower > -4)
    n = n + 1;
    LLy = x;
    t1 = subs(f, y, LLy);
    t2 = subs(f, y, ULy);
    %dyIntegral stores the first
integration
    dyIntegral = t1 + t2;
    deltaX = (ULy - LLy)/n;
    %Simpson rule
    for i=1:2:n-1
        dyIntegral = dyIntegral +
4*subs(f, y, LLy + i*deltaX);
    end
    for j=2:2:n-2
        dyIntegral = dyIntegral +
2*subs(f, y, LLy + j*deltaX);
    end
    dyIntegral = (deltaX/3)*dyIntegral;
    LLx = 2;
    t3 = subs(dyIntegral, x, LLx);
    t4 = subs(dyIntegral, x, ULx);
    %dxIntegral stores the final value
    dxIntegral = t3 + t4;
    deltaX = (ULx - LLx)/n;
    %Simpson rule
    for i=1:2:n-1
        dxIntegral = dxIntegral +
4*subs(dyIntegral, x, LLx + i*deltaX);
    end
    for j=2:2:n-2
        dxIntegral = dxIntegral +
2*subs(dyIntegral, x, LLx + j*deltaX);
    end
    dxIntegral = (deltaX/3)*dxIntegral;
    %error calculation
    dxIntegral = double(dxIntegral);
    absoluteError = abs(dxIntegral -
trueX);
    errorPower =
ceil(log10(absoluteError)-1);
end
disp(dxIntegral);
disp(n);
disp(absoluteError);
```

Q3.

```
%step size, range, function
h = 0.01;
x = -pi:h:pi;
f = sin(x);
%compute derivatives using finite
difference
y1 = diff(f)/h;
y2 = diff(y1)/h;
y3 = diff(y2)/h;
y4 = diff(y3)/h;
y5 = diff(y4)/h;
plot(x(:,1:length(y1)),y1,'g',
x(:,1:length(y2)),y2,'y',
x(:,1:length(y3)),y3,'c',
x(:,1:length(y4)),y4,'m',
x(:,1:length(y5)),y5,'r')
grid
%b
%step size, range, function
h = 0.001;
x = -pi:h:pi;
f = sin(x);
sum = zeros(1, length(f)-3);
%compute forward difference
representation
for i=1:(length(f)-3)
    sum(i) = ((-11/6)*f(i) +
(3)*(f(i+1)) - (1.5)*f(i+2) +
(1/3)*f(i+3))/h;
end
figure;
plot(x(:,1:length(sum)), sum, 'r');
grid
```

Q4.

```
x = [0, 1, 2, 3, 4];
f = [30, 33, 28, 12, -22];
%step size
h = 1;

f0FirstOrder = (-f(3) + 4*f(2) -
3*f(1))/2*h;
f2FirstOrder = (-f(5) + 4*f(4) -
3*f(3))/2*h;
f4FirstOrder = (3*f(5) - 4*f(4) +
f(3))/(2*h);
f0SecondOrder = (-f(4) + 4*f(3) - 5*f(2)
+ 2*f(1))/(h^2);

disp(f0FirstOrder);
disp(f2FirstOrder);
disp(f4FirstOrder);
disp(f0SecondOrder);
```