Problem Sheet for Exponential and Poisson Processes

1. The time T required to repair a machine is an exponentially distributed random variable with mean $\frac{1}{2}$ hour.

   (a) What is the probability that a repair time exceeds $\frac{1}{2}$ hour?

   (b) What is the probability that a repair takes at least 12.5 hours given that its duration exceeds 12 hours?

2. Suppose you arrive at the Post to find five customers before you, one being served and four waiting in line. There is only one desk in service. You join the end of the line. If the service times are all exponential with rate μ, what is the expected amount of time you will spend in the bank?

3. Consider a post office with two clerks. Three people, A, B, C enter at the same time. A and B go directly to the clerks, and C waits until either A or B leaves before he begins service. What is the probability that A is still in the post office after the two other two have left when:

   (a) the service time for each clerk is exactly ten minutes?

   (b) the service times are i with probability $\frac{1}{3}$, $i = 1, 2, 3$?

   (c) the service times are exponential with mean $\frac{1}{\mu}$?

4. If X_i, $i = 1, 2, 3$ are independent exponential random variables with rates λ_i, find

   (a) $P[X_1 < X_2 < X_3]$

   (b) $P[X_1 < X_2 | \max(X_1, X_2, X_3) = X_3]$

   (c) $\mathrm{E}[\max X_i | X_1 < X_2 < X_3]$

   (d) $\mathrm{E}[\max X_i]$

5. Consider a two-server system in which a customer is served first by server 1, then by server 2 and then departs. The service times at server i exponential random variables with rates μ_i, $i = 1, 2$. When you arrive, you find a server 1 free and two customers at server 2 ; customer A in service and customer B waiting in line.

   (a) Find P_A, the probability that A is still in service when you move over to server 2.

   (b) Find P_B, the probability that B is still in the system when you move over to server 2.

   (c) Find $\mathrm{E}[T]$, where T is the time that you spend in the system.
   Hint: Write
   $$T = S_1 + S_3 + W_A + W_B$$
   where S_i is your service time at server i, W_A is the amount of time you wait in queue while A is being served, and W_B is the amount of time you wait in queue while B is being served.