STA457 Fall 2001

Test #1 - Solutions

(5) **1.** True or False

a. A white noise process is a weakly stationary process.

b. A random walk is a weakly stationary process.

c. There exists a stationary process which satisfies $\rho(3) \neq \rho(-3)$

d. Assume the population Toronto is continually increasing. If Toronto's population is measured monthly for 10 years the 120 figures are considered one time series observation.

e. The situation in part (d) is an example of a stationary process.

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(7) **2.** Let $\{X_t\}$ be a sequence of independent random variables with each with pmf

$$p_{X_t}(x) = \begin{cases} -1 & \text{with probability } .75 \\ 1 & \text{with probability } .25 \end{cases}$$

Also define the sequence $\{S_t\}$

$$S_t = \sum_{i=1}^{t} X_i$$

a. What is $E(S_t)$? You can just state the answer.

b. What is $Var(S_t)$? You can just state the answer.

c. What is $Cov(S_4, S_8)$? You can just state the answer.

a. $-t/2$ (2 marks)
b. $3t/4$ (2 marks)
c. 3 (3 marks)

(13) **3.** $\{Z_t\}$ is a sequence of $iidN(0,4)$ random variables. Define $X_t = Z_t Z_{t-1}$.

a. What is $\mu_X(t)$?

b. What is the variance of X_t at any time t?

c. What is the covariance of X_t and X_{t-1}?

d. What is the covariance of X_t and X_{t-2}?

e. Is this a stationary process? Explain. If the process is stationary state the ACVF and ACF.

a. $E(Z_t Z_{t-1}) = E(Z_t)E(Z_{t-1}) = 0$ (1 mark)

b. $Var(X_t) = E((Z_t Z_{t-1})^2) = E(Z_t^2)E(Z_{t-1}^2) = \sigma^2 \times \sigma^2 = 16$ (2 marks)

c. $Cov(X_t, X_{t-1}) = E(Z_t Z_{t-1} Z_{t-1} Z_{t-2}) = E(Z_t)E(Z_{t-1}^2)E(Z_{t-2}) = 0 \times \sigma^2 \times 0 = 0$ (2 marks)

d. 0 like part c (1 mark)

e. yes it is stationary (1 mark)

the expected value at any time t and the covariance at any times t and $t+h$ is free of t (2 marks)

ACVF $\gamma_X(t) = \begin{cases} 16 & t = 0 \\ 0 & t = \pm 1, \pm 2, \dots \end{cases}$ (2 marks)

ACF $\rho_X(t) = \begin{cases} 1 & t = 0 \\ 0 & t = \pm 1, \pm 2, \dots \end{cases}$ (2 marks)

(5) **4.** Hand in the Splus plot of the simulated AR(1) process.