$$v_3 = a_3 + \mathrm{sgn}(a_{11}) \|a_3\| e_1$$

$$= (-5) - 5(1) = (-10)$$

$$H_3 = I - 2\frac{v_3 {}^t v_3}{{}^t v_3 \, v_3} = (1) - \frac{2}{100}(-10)(-10) = (-1)$$

$$H_3 A_3 = (-1)(-5) = (5).$$

1

$$R = H_3 H_2 H_1 A.$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 0 & 5 \\ 0 & 3 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -2 & -4 \\ 0 & -3 & -6 \\ 0 & 0 & 5 \end{pmatrix}.$$

1,5

$$Q = H_1 H_2 H_3$$

$$= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

1,5

$$Q \cdot R = A.$$

$$v_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} + 1\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}2\\0\\0\end{pmatrix}$$

$$H_1 = I - 2\,\frac{v_1\,{}^t v_1}{{}^t v_1\, v_1} = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} - \frac{2}{4}\begin{pmatrix}2\\0\\0\end{pmatrix}(2\;0\;0)$$

$$= \begin{pmatrix}-1&0&0\\0&1&0\\0&0&1\end{pmatrix} \quad 1{,}5$$

$$H_1A_1 = \begin{pmatrix}-1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}1&2&4\\0&0&5\\0&3&6\end{pmatrix} = \begin{pmatrix}-1&-2&-4\\0&0&5\\0&3&6\end{pmatrix}$$

$$A_2 = \begin{pmatrix}0&5\\3&6\end{pmatrix} ; \quad a_2 = \begin{pmatrix}0\\3\end{pmatrix}$$

~~review~~

$$\|a_2\|_2 = \sqrt{9} = 3.$$

$$v_2 = a_2 + \text{sign}(a_{11})\,\|a_2\|\,e_1$$

$$= \begin{pmatrix}0\\3\end{pmatrix} + 3\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}3\\3\end{pmatrix}.$$

$$H_2 = I - 2\,\frac{v_2\,{}^t v_2}{{}^t v_2\, v_2} = \begin{pmatrix}1&0\\0&1\end{pmatrix} - \frac{2}{18}\begin{pmatrix}3\\3\end{pmatrix}(3\;\;3)$$

$$= \begin{pmatrix}0&-1\\-1&0\end{pmatrix} \quad 1{,}5$$

$$H_2A_2 = \begin{pmatrix}0&-1\\-1&0\end{pmatrix}\begin{pmatrix}0&5\\3&6\end{pmatrix} = \begin{pmatrix}-3&-6\\0&-5\end{pmatrix}.$$

$$A_3 = (-5) \qquad a_3 = -5$$

$$\|a_3\| = 5$$

$(2,1)$ $\begin{pmatrix} -1 & 1 & 1 \\ -6 & -\cancel{2} & 0 \\ 16 & \cancel{14} & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 11 \\ 14 \\ -52 \end{pmatrix}$; $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -6 \\ 11 \\ -6 \end{pmatrix}$

$P(\lambda) = \lambda^3 - 6\lambda^2 + 11\lambda - 6.$

Solution $\lambda_1 = 3$, $\lambda_2 = 2$, $\lambda_3 = 1$.

$\lambda = 1$, $\frac{P(\lambda)}{\lambda - 1} = \lambda^2 - 5\lambda + 6.$

(3)

$V_{\lambda=1} = y^{(2)} - 5y^{(1)} + 6y^{(0)} = \begin{pmatrix} 0 \\ 4 \\ -4 \end{pmatrix} = 4\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}.$

$\lambda = 2$ $\frac{P(\lambda)}{\lambda - 2} = \lambda^2 - 4\lambda + 3.$

$V_{\lambda=2} = y^{(2)} - 4y^{(1)} + 3y^{(0)} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} = 2\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

$\lambda = 3$. $\frac{P(\lambda)}{\lambda - 3} = \lambda^2 - 3\lambda + 2\cancel{8}.$

$V_{\lambda=3} = y^{(2)} - 3y^{(1)} + 2y^{(0)} = \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}.$

Ex 3: Householder.

$A_1 = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 0 & 5 \\ 0 & 3 & 6 \end{pmatrix}$

$a_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\|a_1\| = 1$

$v_1 = a_1 + \mathrm{sgn}(a_{11}) \|a_1\| e_1$

Correction de l'examen

Analyse numérique Matricielle.

Exo 1 :

$$A = \begin{pmatrix} 3 & 1 & 0 \\ -4 & -1 & 0 \\ 4 & 8 & -2 \end{pmatrix}$$

$\|A\|_1 = |3| + |1| + |0| + |-4| + |-1| + |0| + |4| + |8| + |-2| = 23$ 1

$\|A\|_\infty = \max |a_{ij}| = 8.$

$|||A|||_1 = \max(11, 10, 2) = 11$ 2

$|||A|||_\infty = \max(4, 5, 14) = 14.$

$$B = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}.$$

$\|B\|_2 = \sqrt{1^2 + 0^2 + (-1)^2 + 1^2} = \sqrt{3}.$ 1

$|||B|||_2 = \sqrt{\rho({}^tA\,A)} =$

$${}^tA\,A = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$ 2

$\phi(\lambda) = \lambda^2 - 3\lambda + 1$ les v. p.

$\lambda_1 = \frac{3+\sqrt{5}}{2}$, $\lambda_2 = \frac{3-\sqrt{5}}{2}.$

$|||B|||_2 = \sqrt{\frac{3+\sqrt{5}}{2}}.$

Exo 2.

$y^{(0)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$; $y^{(1)} = A\,y^{(0)} = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$ 0,5

$y^{(2)} = A\,y^{(1)} = \begin{pmatrix} -1 \\ -6 \\ 16 \end{pmatrix}$ 0,5 , $y^{(3)} = A\,y^{(2)} = \begin{pmatrix} -11 \\ -14 \\ 52 \end{pmatrix}$ 0,5

$$\begin{pmatrix} y^{(2)} & y^{(1)} & y^{(0)} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = -\left(y^{(3)} \right).$$