3) $Var(A_t) = \sum_{j=-q}^{q} a_j^2 \; Var(Y_{t-j})$

puisque les Y_t, $t \in \mathbb{Z}$ sont i.i.d.

$$= \frac{1}{2q+1} \sigma^2 < \sigma^2 = Var(Y_t)$$

Donc la suite $\{A_t\}$ est moins dispersée que la suite $\{Y_t\}$.

4) On cherche α, β et γ tels que

$$(1) \begin{cases} (1+\alpha B+\beta B^2+\gamma B^3)(at+b) = at+b \\ (1+\alpha B+\beta B^2+\gamma B^3)\, S_t = 0 \end{cases}$$

$S_t = S_{t+2}$ et $\sum_{j=1}^{2} S_{t+j} = 0, \; \forall t$

$= S_{t-2}$

$$(1) \iff \begin{cases} at+b+\alpha(a(t-1)+b)+\beta(a(t-2)+b)+\gamma(a(t-3)+b) = at+b \\ S_t + \alpha S_{t-1} + \beta S_{t-2} + \gamma S_{t-3} = 0. \end{cases}$$

$$\iff \begin{cases} at(1+\alpha+\beta+\gamma) + b(1+\alpha+\beta+\gamma) + a(-\alpha-2\beta-3\gamma) = at+b \\ S_t - \alpha S_t + \beta S_t - \gamma S_t = 0 \end{cases}$$

$$\Rightarrow \begin{cases} 1+\alpha+\beta+\gamma = 1 \\ \alpha+2\beta+3\gamma = 0 \\ 1-\alpha+\beta-\gamma = 0 \end{cases} \Rightarrow \begin{array}{l} \alpha = \gamma = 1/4 \\ \beta = -1/2 \end{array}$$

Ce filtre est donc $\varphi(B) = 1 + \frac{B}{4} - \frac{B^2}{2} + \frac{B^3}{4}$