$$S_t = S_{t+3} = S_{t-3}\ , \quad S_{t+1} = S_{t-2}$$
$$S_{t-1} = S_{t+2}$$

$$\text{et } S_{t+1} + S_{t+2} + S_{t+3} = 0, \quad \forall t$$

en particulier $S_t + S_{t+1} + S_{t+2} = 0$.

et $S_{t-1} + S_t + S_{t+1} = 0$.

D'où $3S_t + 4S_{t+1} + 4S_{t-1} = S_{t-1} + S_{t+1}$

$$= S_{t+2} + S_{t-2}$$

D'où $\sum_{j=-2}^{2} a_j S_{t-j} = 0$.

Ce filtre élimine bien la saisonnalité de période 3.

b/ Calcul de $\sum_{j=-2}^{2} a_j T_{t-j}$ avec $T_t = a_0 + a_1 t + a_2 t^2 + a_3 t^3$.
$= m_t$

$$T_{t-j} = a_0 + a_1 (t-j) + a_2 (t-j)^2 + a_3 (t-j)^3$$

$$\sum_{j=-2}^{2} a_j T_{t-j} = -\frac{1}{9}\Big\{ a_0 + a_1 (t+2) + a_2 (t+2)^2 + a_3 (t+2)^3 + a_0 + a_1 (t-2) + a_2 (t-2)^2 + a_3 (t-2)^3 \Big\}$$

$$+ \frac{4}{9}\Big\{ a_0 + a_1 (t+1) + a_2 (t+1)^2 + a_3 (t+1)^3 + a_0 + a_1 (t-1) + a_2 (t-1)^2 + a_3 (t-1)^3 \Big\}$$

$$+ \frac{3}{9}\Big\{ a_0 + a_1 t + a_2 t^2 + a_3 t^3 \Big\} = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

- $(t+1)^3 = t^3 + 3t^2 + 3t + 1$; $(t-1)^3 = t^3 - 3t^2 + 3t - 1$

$(t+2)^3 = t^3 + 6t^2 + 12t + 8$; $(t-2)^3 = t^3 - 6t^2 + 12t - 8$.