Université Hassiba Benbouali Chlef
Faculté des Sciences exactes et informatique
Département de Mathématiques

Année universitaire 2021/2022
Master I: Math.Appli & Stat
Module : Analyse de données

Hom work 01

Exercice 01: nonparametric test

Appearance transit times for 11 patients with significantly occluded right coronary arteries are given below:

Subject	1	2	3	4	5	6	7	8	9	10	11
Transit time (in sec)	1.80	3.30	5.65	2.25	2.50	3.50	2.25	3.10	2.70	2.70	3.00

Can we conclude, at the 0.05 level of significance, that the median appearance transit time in the population from which the data were drawn, is different from 3.50 seconds?

Exercice 02: nonparametric test- Large sample approximation

Based on these data, is there sufficient evidence to conclude that the median age of students from Besease Senior High School is smaller than 22 years?

9	13	16	16	16	17	18	19	19	19
19	20	20	21	21	23	24	25	25	27

Exercice 03: nonparametric test- Large sample approximation

The exercise capability of people suffering chronic airflow obstruction (CAO) is severely limited. In order to determine maximum exercise ventilation under two different experimental conditions, 21 patients suffering from CAO exercised to exhaustion under each condition. Ventilation was then measured. The accompanying data are from

Patient	1	2	3	4	5	6	7	8	9	10	11
Condition 1	62	57	56	55	50.5	50	47.2	43.5	40	40	41
Condition 2	52	46	51	52.4	55	51	43	40	34.2	34	33
Difference	10	11	5	2.6	-4.5	-1	4.2	3.5	5.8	6	8
Patient	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	
Condition 1	33	31	28	27.1	27.5	27	25	19.2	17.5	12	
Condition 2	32	38	26	28	28	18	21	18	16	15	
Difference	1	-7	2	-0.9	-0.5	9	4	1.2	1.5	-3	

Do these data suggest that the mean ventilation is different for the two experimental conditions? Let's analyze the data using a level .05 signed-rank test.

Exercice 04:

L'échantillon suivant de n = 10 observations est la réalisation d'une variable aléatoire réelle dont les valeurs sont tabulées ci-dessous.

x1	x2	x3	x4	x5	x6	x7	x8	x9	x10
2.57	3.64	5.97	3.54	1.59	1.63	2.80	3.77	1.79	2.34

1. Rappeler l'expression de la fonction de répartition empirique $\widehat{F}_n(x)$.
2. À partir du graphe de la fonction de répartition empirique, déterminer les quartiles notés respectivement Q_1 pour le premier quartile, Q_3 pour le troisième quartile et Me pour la médiane.

Exercice 05:

Let $(X_1, ..., X_n)$ be a random sample from a distribution on $\mathbb{R}$ with Lebesgue density $f_X(x) = 2^{-1}(1-\theta^2)e^{\theta x - |x|}$, where $\theta \in (-1, 1)$ is unknown.

1. Show that the median of the distribution of X_1 is given by $m(\theta) = (1-\theta)^{-1}\log(1+\theta)$ when $\theta \geq 0$ and $m(\theta) = -m(-\theta)$ when $\theta < 0$.

2. Show that the mean of the distribution of X_1 is $\mu(\theta) = 2\theta/(1-\theta^2)$.

Exercice 06:

The cumulative distribution function of the continuous random variable V is

$$x = \begin{cases} 0 \quad \text{if} \quad v < -5 \\ c(v+5)^2 \quad \text{if} \quad -5 \leq v < 7 \\ 1 \quad \text{if} \quad v \geq 7 \end{cases}$$

(a) What is c?
(b) What is $P[V > 4]$? and $P[-3 < V \leq 0]$?
(c) What is the value of a such that $P[V > a] = 2/3$?

Exercice 07:
Soit X une variable aléatoire qui suit la loi suivante : $P(X = 1) = 0.5$; $P(X = 3) = 0.3$; $P(X = 4) = 0.2$
(a) Dessinez le diagramme en bâtons.
(b) Dessinez le graphe de la fonction de répartition.

Exercice 08:
Quel résultat illustre le code qui suit :

```
n = 100;x = rnorm(n);m = cumsum(x)/(1:n);v = 1;q = qnorm(0.975)
ic1 = m+q*sqrt(v)/sqrt(1:n);ic2 = m-q*sqrt(v)/sqrt(1:n)
plot(m,type="l");abline(h=0,col="red")
lines(ic1,col="blue",lty="dotted");lines(ic2,col="blue",lty="dotted")

xc = rcauchy(n,0,1);mc = cumsum(xc)/(1:n)
plot(mc,type="l");abline(h=0,col="red")

med = rep(0,n);for (i in 1:n) med[i] = quantile(x[1:i],probs=0.5)
p = 0.5;v = p*(1-p)/(dnorm(qnorm(p)))^2
ic1 = med+q*sqrt(v)/sqrt(1:n);ic2 = med-q*sqrt(v)/sqrt(1:n)
plot(med,type="l");abline(h=0,col="red")
lines(ic1,col="blue",lty="dotted");lines(ic2,col="blue",lty="dotted")
for (i in 1:n) med[i] = quantile(xc[1:i],probs=0.5)
v = p*(1-p)/(dcauchy(qcauchy(p)))^2
# les calculs de ic1 et ic2 et le code de plot pour la loi de Cauchy
# sont identiques que ceux dans le cas gaussien
```