Problem Sheet for Branching Process

1. For a branching process with the family size distribution given by $p_0 = 1/6$, $p_2 = 1/3$, $p_3 = 1/2$. Calculate
   (a) the probability generating function of Z_2 given $Z_0 = 2$, where Z_2 is the population of the second generation.
   (b) the mean and variance of Z_2 and the probability of extinction.
2. Consider a branching process in which the family size distribution is Poisson with mean λ.
   (a) Under what condition will the probability of extinction of the process be less than 1?
   (b) Show that when $\lambda = 2$ the extinction probability is 0.2032.
   (c) When $\lambda = 2$ find the expected size of the 10th generation, and the probability of extinction by the 3rd generation.
3. Let $\{Z_n\}_{n=0}^{\infty}$ be a branching process with $Z_0 = 1$. It is known that $p_0 = p$ and $p_2 = 1 - p$, where $p \in [0, 1]$.
   (a) Find a condition on the size of p such that the probability of extinction is 0.
   (b) Find the range of p such that the probability of extinction is smaller than 1. Calculate the probability of extinction as a function of p.
   (c) Calculate the mean and the variance of Z_n when $p = \frac{1}{3}$.