Corrigé-type du TD N° 03 (ANOVA)

Exercice 1.

1) Effet des traitements sur les rendements : $p = 4, n_1 = 7, n_2 = 8, n_3 = 9, n_4 = 8,$

$n = \sum_{j=1}^{p} n_j = 7 + 8 + 9 + 8 = 32, SCT = 1105, MCF = 349.5$

On a : $MCF = \dfrac{SCF}{p-1} \Rightarrow SCF = (p-1)\, MCF = 3 \times 349.5 = 1048.5$

et $SCT = SCF + SCR \Rightarrow SCR = SCT - SCF = 56.5$

$MCR = \dfrac{SCR}{n-p} = \dfrac{56.5}{28} = 2.02$

$F = \dfrac{MCF}{MCR} = \dfrac{349.5}{2.02} = 173.02$

Variation	**ddl**	SC	MC	F
Factorielle	$p-1=3$	1048.5	349.5	173.02
Résiduelle	$n-p=28$	56.5	2.02	
Totale	$n-1=31$	1105		

2) Test d'influence des trimestres sur les rendements

$$\begin{cases} H_0 : & \mu_1 = \mu_2 = \mu_3 = \mu_4, \\ H_1 : & \exists j, k, \mu_j \neq \mu_k, \end{cases} \quad j \neq k.$$

$F = 173.02 > f_{0.95}(p-1, n-p) = f_{0.95}(3, 28) = 2.95$, on rejete H_0 alors il y a une influence des trimestres sur les rendements, il existe au moins deux trimestres de moyennes différentes.

3) Comparaison entre les effets :

$$\text{T}_2 \text{ vs } \text{T}_4 : \begin{cases} H_0 : & \mu_2 = \mu_4 \\ H_1 : & \mu_2 \neq \mu_4 \end{cases} ; \quad \begin{matrix} \bar{y}_2 = \dfrac{1}{n_2} \sum_{i=1}^{n_2} y_{i1} = 6.625, \\ \bar{y}_4 = \dfrac{1}{n_4} \sum_{i=1}^{n_4} y_{i4} = 5. \end{matrix}$$

$$T = \frac{\bar{y}_2 - \bar{y}_4}{\sqrt{\left(\dfrac{1}{n_2} + \dfrac{1}{n_4}\right) MCR}} = \frac{6.625 - 5}{\sqrt{\left(\dfrac{1}{8} + \dfrac{1}{8}\right) 2.02}} = 2.29$$

$m = C_4^2 = \dfrac{4!}{2!2!} = 6; t_{1-\frac{\alpha}{2m}}(n-p) = t_{1-\frac{0.05}{12}}(28) = 2.7633$

$|T| < t \Rightarrow$ on accepte H_0, les deux trimestres T_2 et T_4 ont le même effet.

$$\text{T}_3 \text{ vs } \text{T}_4 : \begin{cases} H_0 : & \mu_3 = \mu_4 \\ H_1 : & \mu_3 \neq \mu_4 \end{cases} ; \bar{y}3 = \frac{1}{n_3} \sum_{i=1}^{n_3} y_{i3} = 8.11$$

$$T = \frac{\bar{y}_3 - \bar{y}_4}{\sqrt{\left(\dfrac{1}{n_3} + \dfrac{1}{n_4}\right) MCR}} = \frac{8.11 - 5}{\sqrt{\left(\dfrac{1}{9} + \dfrac{1}{8}\right) 2.02}} = 4.51$$

$|T| > t \Rightarrow$ on rejete H_0, les deux trimestres T_3 et T_4 n'ont pas le même effet.

Exercice 2. Efficacité d'un médicament

$p = 3$

1) Calcule de la table d'AV(1)

$$\left.\begin{array}{l} n_1 = 4 \\ n_2 = 6 \\ n_3 = 8 \end{array}\right\} \Rightarrow n = \sum_{j=1}^{p} n_j = 18$$

$$\bar{y}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} y_{i1} = \frac{1}{4}(0 + 1 + 0.2 + 0.3) = 0.375$$

$$\bar{y}_2 = \frac{1}{n_2}\sum_{i=1}^{n_2} y_{i2} = \frac{1}{6}(3.1 + 2.7 + 2.6 + 2.8 + 3 + 3.2) = 2.9$$

$$\bar{y}_3 = \frac{1}{n_3}\sum_{i=1}^{n_3} y_{i3} = \frac{1}{8}(2.3 + 3.5 + \ldots + 3.2) = 2.6375$$

$$\bar{y} = \frac{1}{n}\sum_{j=1}^{p}\sum_{i=1}^{n_j} y_{ij} = \frac{40}{18} = 2.22$$

$$SCT = \sum_{j=1}^{p}\sum_{i=1}^{n_j} y_{ij}^2 - n\bar{y}^2 = 109.48 - 18\,(2.22)^2 = 20.7688$$

$$SCF = \sum_{j=1}^{p} n_j\bar{y}_j^2 - n\bar{y}^2 = n_1\bar{y}_1^2 + n_2\bar{y}_2^2 + n_3\bar{y}_3^2 - n\bar{y}^2 = 4\,(0.375)^2 + 6\,(2.9)^2 + 8\,(2.6375)^2 - 18\,(2.22)^2 = 17.9625$$

$SCR = SCT - SCF = 20.7688 - 17.9625 = 2.8063$

$MCF = \dfrac{SCF}{p-1} = \dfrac{17.9625}{2} = 8.9813$, $MCR = \dfrac{SCR}{n-p} = \dfrac{2.8063}{15} = 0.1871$

$F = MCF/MCR = 48.0024$

Variation	**ddl**	SC	MC	F
Factorielle	$p - 1 = 2$	17.9625	8.98125	48.0024
Résiduelle	$n - p = 15$	2.8063	0.1871	
Totale	$n - 1 = 17$	20.7688		

$$\left\{\begin{array}{ll} H_0: & \mu_1 = \mu_2 = \mu_3, \\ H_1: & \exists j, k, \mu_j \neq \mu_k, \end{array}\right. \quad j \neq k.$$

$F = 48.0024 > f_{0.95}(2, 15) = 3.68$, on rejete H_0 alors les traitements ne sont pas équivalents.

2) Test de l'efficacité du nouveau médicament

$$\left\{\begin{array}{ll} H_0: & \mu_1 = \mu_3 \\ H_1: & \mu_1 \neq \mu_3 \end{array}\right. ; T_1 = \frac{\bar{y}_1 - \bar{y}_3}{\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_3}\right) MCR}} = \frac{0.375 - 2.6375}{\sqrt{\left(\frac{1}{4} + \frac{1}{8}\right) 0.1871}} = -8.54.$$

$$\left\{\begin{array}{ll} H_0: & \mu_2 = \mu_3 \\ H_1: & \mu_2 \neq \mu_3 \end{array}\right. ; T_2 = \frac{\bar{y}_2 - \bar{y}_3}{\sqrt{\left(\frac{1}{n_2} + \frac{1}{n_3}\right) MCR}} = \frac{2.9 - 2.6375}{\sqrt{\left(\frac{1}{6} + \frac{1}{8}\right) 0.1871}} = 1.1237.$$

$m = C_3^2 = 3; t_{1-\frac{\alpha}{2m}}(n-p) = t_{1-\frac{0.05}{6}}(15) = t_{0.99}(15) = 2.6025.$

$|T_1| > t \Rightarrow$ on rejete $H_0 \Rightarrow$ il y a une différence entre A et C.

$|T_2| < t \Rightarrow$ on accepte $H_0 \Rightarrow B$ et C ont la même efficacité.

Exercice 3.

1) AV(1)

a)

Variation	**ddl**	SC	MC	F
Factorielle	$p-1=2$	72	36	0.22
Résiduelle	$n-p=9$	1454	161.56	
Totale	$n-1=11$	1526		

$\bar{y}_1 = 20, \bar{y}_2 = 17, \bar{y}_3 = 23, \bar{y} = 20$
$n_1 = n_2 = n_3 = 4, p = 3, n = 12$

$$SCT = \sum_{j=1}^{p}\sum_{i=1}^{n_j} y_{ij}^2 - n\bar{y}^2 = 6326 - 12\,(20)^2 = 1526.$$

$$SCF = \sum_{j=1}^{p} n_j \bar{y}_j^2 - n\bar{y}^2 = n_1\bar{y}_1^2 + n_2\bar{y}_2^2 + n_3\bar{y}_3^2 - n\bar{y}^2 = 4 \times \left(20^2 + 17^2 + 23^2\right) - 12 \times 20^2 = 72.$$

$SCR = SCT - SCF = 1526 - 72 = 1454.$

$$MCF = \frac{SCF}{p-1} = \frac{72}{2} = 36\ ,\ MCR = \frac{SCR}{n-p} = \frac{1454}{9} = 161.56$$

$F = MCF/MCR = 36/161.56 = 0.22$

$$\begin{cases} H_0: & \mu_1 = \mu_2 = \mu_3, \\ H_1: & \exists j, k, \mu_j \neq \mu_k, \end{cases} \quad j \neq k.$$

$F < f_{0.95}\,(2, 9) = 4.26 \Rightarrow$ on accepte $H_0 \Rightarrow$ on ne peut pas affirmer l'existence d'une différence entre les performance globales des équipes.

b)

Variation	**ddl**	SC	MC	F
Factorielle	$p-1=3$	1362	454	22.15
Résiduelle	$n-p=8$	164	20.5	
Totale	$n-1=11$	1526		

$\bar{y}_1 = 25, \bar{y}_2 = 18, \bar{y}_3 = 33, \bar{y}_4 = 4, \bar{y} = 20$
$n_1 = n_2 = n_3 = n_4 = 3, p = 4, n = 12$

$$SCF = \sum_{j=1}^{p} n_j \bar{y}_j^2 - n\bar{y}^2 = n_1\bar{y}_1^2 + n_2\bar{y}_2^2 + n_3\bar{y}_3^2 + n_4\bar{y}_4^2 - n\bar{y}^2 = 3 \times \left(25^2 + 18^2 + 33^2 + 4^2\right) - 12 \times 20^2 = 1362.$$

$SCR = SCT - SCF = 1526 - 1362 = 164.$

$$MCF = \frac{SCF}{p-1} = \frac{1362}{3} = 454\ ,\ MCR = \frac{SCR}{n-p} = \frac{164}{8} = 20.5.$$

$F = MCF/MCR = 454/20.5 = 22.15$

$$\begin{cases} H_0: & \mu_1 = \mu_2 = \mu_3 = \mu_4, \\ H_1: & \exists j, k, \mu_j \neq \mu_k, \end{cases} \quad j \neq k.$$

$F > f_{0.95}\,(3, 8) = 4.07 \Rightarrow$ on rejete H_0 alors on peut conclure que les postes présentent des difficultés de montage inégales.

2) AV(2) sans interaction

Variation	**ddl**	**SC**	**MC**	F
Fact. A	2	72	36	2.35
Fact. B	3	1362	454	29.61
Résiduelle	6	92	15.33	
Totale	11	1526		

$$SCA = q\sum_{i=1}^{p} \bar{y}_{i*}^2 - n\bar{y}^2 = 4 \times \left(20^2 + 17^2 + 23^2\right) - 12 \times 20^2 = 72$$

$$SCB = p\sum_{j=1}^{q} \bar{y}_{*j}^2 - n\bar{y}^2 = 3 \times \left(25^2 + 18^2 + 33^2 + 4^2\right) - 12 \times 20^2 = 1362$$

$SCR = SCT - SCA - SCB = 1526 - 1362 - 72 = 92$

$MCA = \dfrac{SCA}{p-1} = \dfrac{72}{2} = 36$, $MCB = \dfrac{SCB}{q-1} = \dfrac{1362}{3} = 454$, $MCR = \dfrac{SCR}{(p-1)(q-1)} = \dfrac{92}{6} = 15.33$

$F_A = \dfrac{MCA}{MCR} = 2.35; F_B = \dfrac{MCB}{MCR} = 29.61$

$F_A < f_{0.95}(2,6) = 5.14 \Rightarrow$ on accepte $H_0\left(\alpha_i = 0, i = \overline{1,3}\right) \Rightarrow$ Il n'y a pas un effet équipe.

$F_B > f_{0.95}(3,6) = 4.76 \Rightarrow$ on rejete $H_0\left(\beta_j = 0, j = \overline{1,q}\right)$ alors on peut conclure qu'il y a un effet postes de travail.

3) AV(2) avec interaction

$p = 3, q = 4, r = 2$ observations par case, $n = pqr = 24$

	A	B	C	D	$\bar{y}_{i**}$
du matin	13	6.5	17.5	3	10
du soir	9	8.5	15.5	1	8.5
du nuit	15.5	12	16.5	2	11.5
$\bar{y}_{*j*}$	12.5	9	16.5	2	$\bar{y} = 10$

$$SCT = \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{r} y_{ijk}^2 - n\bar{y}^2 = 3196 - 24\left(10^2\right) = 796$$

$$SCA = qr\sum_{i=1}^{p} \bar{y}_{i**}^2 - n\bar{y}^2 = 4 * 2\left(10^2 + 8.5^2 + 11.5^2\right) - 24\left(10^2\right) == 36$$

$$SCB = pr\sum_{j=1}^{q} \bar{y}_{*j*}^2 - n\bar{y}^2 = 3 \times 2\left(12.5^2 + 9^2 + 16.5^2 + 2^2\right) - 24\left(10^2\right) = 681$$

$$SCAB = r\sum_{i=1}^{p}\sum_{j=1}^{q} (\bar{y}_{ij*} - \bar{y}_{i**} - \bar{y}_{*j*} + \bar{y})^2 = 2\times\Big((13 - 10 - 12.5 + 10)^2 + \cdots + (2 - 11.5 - 2 + 10)^2\Big) =$$

$2 \times 23 = 46$

$SCR = SCT - SCA - SCB - SCAB = 796 - 36 - 681 - 46 = 33$

Variation	**ddl**	SC	MC	F
Fact. A	2	36	18	6.55
Fact. B	3	681	227	82.55
Fact. A,B	6	46	7.67	2.79
Résiduelles	12	33	2.75	
Totale	23	796		

$MCA = SCA/(p-1) = 36/2 = 18$

$MCB = SCB/(q-1) = 681/3 = 227$

$MCAB = SCAB/(p-1)(q-1) = 46/6 = 7.67$

$MCR = SCR/pq(r-1) = 681/3 = 33/12 = 2.75$

$F_A = MCA/MCR = 18/2.75 = 6.55$

$F_B = MCB/MCR = 227/2.75 = 82.55$

$F_{AB} = MCAB/MCR = 7.67/2.75 = 2.79$

$F_A > f_{0.95}(2,12) = 3.89 \Rightarrow H_0\left(\alpha_i = 0, i = \overline{1,3}\right)$ est rejeté⇒Il y a un effet équipe.

$F_B > f_{0.95}(3,12) = 3.49 \Rightarrow H_0\left(\beta_j = 0, j = \overline{1,4}\right)$ est rejeté⇒ Il y a un effet postes.

$F_{AB} < f_{0.95}(6,12) = 3.00 \Rightarrow H0\left(\gamma_{ij} = 0, i = \overline{1,3}, j = \overline{1,4}\right) \Rightarrow$ Il n'y a plus d'effet d'interaction entre A et B