Exercice 1.

X : la durée de vie d'une batterie.
$\mu_x = 100$
$S^2 \Rightarrow S = \sqrt{S^2} = \sqrt{141.13} = 11.880$
$n = 5.$

1-$P(\overline{X} > 105) = ?$
population inconnue, σ_x inconnu, $n = 16 < 30 \Rightarrow 3$ $^{\text{ème}}$ cas
$\overline{X} \curvearrowright t(n-1)$ $(student)$
$\mu_{\overline{x}} = \mu_x = 100$
$\sigma_{\overline{x}} = \frac{S}{\sqrt{n}} = \frac{11.880}{\sqrt{5}} = 5.3129$

$P(\overline{X} > 105) = 1 - P(\overline{X} \leq 105) = 1 - P(\frac{\overline{X}-\mu_{\overline{x}}}{\sigma_{\overline{x}}} \leq \frac{105-\mu_{\overline{x}}}{\sigma_{\overline{x}}})$
$= 1 - P(T \leq \frac{105-100}{5.3129}) = 1 - P(T \leq 0.94111) = 1 - 0.80 = 0.20$
(on utilise la table de la loi de student n-1=5-1=4, en suite on cherche la valeur la plus proche à 0.941 dans la ligne de n=4 puis en haut on trouve cum. prob = $t_{.80}$
c.à.d 0.80)

2. $n = 30$

$P(95 \leq \overline{X} \leq 100) = ?$
population inconnue, σ_x inconnu, $n = 30 \Rightarrow 2$ $^{\text{ème}}$ cas
$\overline{X} \curvearrowright N(\mu_{\overline{x}}\,; \sigma_{\overline{x}})$
$\mu_{\overline{x}} = \mu_x = 100$
$\sigma_{\overline{x}} = \frac{S}{\sqrt{n}} = \frac{11.880}{\sqrt{30}} = 2.1690$
$P(95 \leq \overline{X} \leq 100) = P(\frac{95-\mu_{\overline{x}}}{\sigma_{\overline{x}}} \leq \frac{\overline{X}-\mu_{\overline{x}}}{\sigma_{\overline{x}}} \leq \frac{100-\mu_{\overline{x}}}{\sigma_{\overline{x}}}) = P(\frac{95-100}{2.1690} \leq Z \leq \frac{100-100}{2.1690})$
$= P(-2.3 \leq Z \leq 0) = F(0) - F(-2.3) = F(0) - (1 - F(2.3)) = F(0) + F(2.3) - 1$
$= 0.5 + 0.9893 - 1 = 0.4893 = 48.93\%$

Exercice 2

1- $P(\ S^2 \geq 0.195) =?$
$S^2 \curvearrowright \chi^2$
$P(\ S^2 \geq 0.195) = 1 - P(\ S^2 < 0.195) = 1 - P(\ \frac{(n-1)S^2}{\sigma_x^2} < \frac{(n-1)0.195}{\sigma_x^2}) =$
$1 - P(\chi^2 < \frac{(n-1)0.195}{\sigma_x^2})$
$= 1 - P(\ \chi^2 < \frac{(15-1)0.195}{0.4^2}) = 1 - P(\ \chi^2 < 17.1) = 1 - \ \chi^2_{.75} = 1 - 0.75 = 0.25$

2- $P(\ S^2 \leq 0.089) =?$
$S^2 \curvearrowright \chi^2$
$P(\ S^2 \leq 0.089) = P(\ \frac{(n-1)S^2}{\sigma_x^2} \leq \frac{(n-1)0.089}{\sigma_x^2}) = P(\ \chi^2 \leq \frac{(15-1)0.089}{0.4^2})$
$= P(\ \chi^2 \leq 7.79) = \ \chi^2_{.10} = 0.10$