Solutions of Options, Futures and Other Derivatives (Hull)

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Contents

1 Chapter One

1.1 Practice Questions

What is the difference between a long forward position and a short forward position?

By definition, a forward contract is an agreement between two parties to buy or sell the underlying asset at an certain price on a future date. If one of the parties assumes the long position, it agrees to buy the asset. It implies that he believes that the price of the asset at contract maturity will be greater than the delivery price. The other party assumes the short position, it agrees to sell the underlying asset. It implies that he believes that the price of the asset at contract maturity will be less than the delivery price. Note that the payoff functions of them are symmetric.

1.2 Practice Questions

Explain carefully the difference between hedging, speculation and arbitrage.

Hedging: The process of using derivatives to mitigate risks which investors face from adverse movements in the price of an asset.

Speculation: The process of betting the direction of the price of an asset.

Arbitrage: It is an strategy that market participants use to get profit without risk by simultaneously buying and selling securities.

1.3 Practice Questions

What is the difference between entering into a long forward contract when the forward price is \$50 and taking a long position in a call option with a strike price of \$50?

The main difference between these two operations is that potential gain (or loss) of the former is not limited since he is supposed to buy the security for \$50. By contrast, an investor, who takes a long position in a call option, has his loss limited to the amount paid for the security due to the fact that exercising the right of buying an option is not an obligation.

1.4 Practice Questions

Explain carefully the difference between selling a call option and buying a put option.

A call option gives the holder the right to buy an asset by a certain date at a certain price. The seller of a call option expects the buyer to not use the right during the period of analysis. In terms of payoff, this derivatives gives: $\min\{K - S_T, 0\} = -\max\{S_T - K, 0\}$.

A put option gives the holder the right to sell an asset by a certain date at a certain price. The purchaser of a put option expects an decrease of the stock price since his payoff is: $\max\{K - S_T, 0\}$.

Note, however, that only the buyer of the option has the right to exercise it.

1.5 Practice Questions

An investor enters into a short forward contract to sell 100,000 British pounds for US dollars at an exchange rate of 1.400 US dollars per pound. How much does the investor gain or lose if the exchange rate at the end of the contract is (a) 1.3900 (b) 1.4200

(a) $-[1.39 - 1.4] \times 10^5 = 10^3$

(b) $-[1.42 - 1.4] \times 10^5 = -2 \times 10^3$

1.6 Practice Questions

A trader enters into a short cotton futures contract when the futures price is 50 cents per pound. The contract is for the delivery of 50,000 pounds. How much does the trader gain or lose if the cotton price at the end of the contract is (a) 48.20 cents per pound (b) 51.30 cents per pound

(a) $-[0.4820 - 0.50] \times 50,000 = 900$

(b) $-[0.5130 - 0.50] \times 50,000 = 650$

1.7 Practice Questions

Suppose that you write a put contract with a strike price of \$40 and an expiration date in 3 months. The current stock price is \$41 and the contract is on 100 shares. What have you committed yourself to? How much could you gain or lose?

I have committed myself to buy the underlying asset by \$40 at the exercise date, regardless the current price of the stock. The profit is based on possible outcomes for the stock price at the date on which the party on the other side of the contract decides to exercise the option. For example:

Stock Price	Profit
$S_T = 30$	(30-40)×100 + premium
$S_T = 0$	(0-40)×100 + premium

It is important to notice that the right of selling will only be exercised, if and only, the stock price is below $40. The worst case scenario is represented above when the stock price drops to zero.

1.8 Practice Questions

What is the difference between the over-the-counter market and the exchange-traded market? What are the bid and offer quotes of a market maker in the over-the-counter market?

A key advantage of the over-the-counter market is that its contract terms are not imposed by an exchange. Market participants are free to trade any mutually attractive deal.

Bid price represents the maximum price that a buyer is willing to pay for the asset.

Offer (Ask) represents the minimum price that a seller is willing to accept for the asset.

1.9 Practice Questions

You would like to speculate on a rise in the price of a certain stock. The current stock price is $29 and a 3-month call with a strike price of $30 costs $2.90. You have $5,800 to invest. Identify two alternative investment strategies, one in the stock and the other in an option on the stock. What are the potential gains and losses from each?

Assuming that the speculator is willing to invest $5800. One alternative would be to purchase 200 shares, the other involves the purchase of 2000 call options. Both alternatives will yield the same profit if the stock price is:

$$200(P - 29) = 2000(P - 30) - 5800$$
$$200P - 5800 = 2000P - 60000 - 5800$$
$$P = 33 + \frac{1}{3}$$

Hence, the profit is roughly $866. Now, putting some numbers.

First of all, let's suppose that the price of the stock rises to \$36. The alternative of putting all investment into stocks yields a profit of:

$$200 \times (36 - 29) = 1400$$

However, the total profit from the 2000 call options that are purchased as the second alternative:

$$2000 \times (36 - 30) - 5800 = 12000 - 5800 = 6200$$

The option strategy is more profitable than directly buying stocks.Suppose now that the price falls to \$26. The first alternative of buying stocks would yield a loss of:

$$200 \times (26 - 29) = -600$$

Because the call options expire without being exercised, the option strategy would lead to a loss of \$5800.

1.10 Practice Questions

Suppose that you own 5,000 shares worth \$25 each. How can put options be used to provide you with insurance against a decline in the value of your holding over the next 4 months?

Suppose that put option contracts on the underlying stock are available and each of them has a strike price of \$24 as well as a cost of \$1. I could buy the options in order to protect the total wealth.

The total cost of the strategy:

$$\$1 \times \text{The number of shares} = \$5000$$

There are two possibilities:

[A] Stock price < Strike:

Since it is a put option contract, it will give the holder the right to sell the asset for \$25. Therefore, it will be exercised if the market price falls bellow the strike price.

In this particular case, the amount realized is:

$$25 \times 5000 - 5000 = 120000 - 5000 = 120000$$

[B] Stock price > Strike:

The options are not exercised and expire worthless.

In this particular case, the amount realized is:

$$25 \times 5000 - 5000 = 120000$$

1.11 Practice Questions

When first issued, a stock provides funds for a company. Is the same true of a stock option? Discuss.

A stock option does not provide funds for a company. It is an agreement made between investors, therefore a company is not directly involved.

1.12 Practice Questions

Explain why a futures contract can be used for either speculation or hedging

The mechanism of future contracts are similar to forward contracts. Hedgers and speculators can use these derivatives to meet their demands.

Hedging:

Consider that an investor will gain when the price of an underlying asset decreases and lose when it increases. Hedgers can reduce the risk by entering into the opposite position, it means that a long position in a futures contract would be the best choice.

Similarly, if the investor wants protect himself against price drops, a short position in a futures contract would minimize his market risk exposure.

Speculation:

However, investors might be better off if they choose not to hedge. For example: A company will have to pay £1 Million for goods it has purchased from a supplier. The company could hedge its risk by fixing the foreign exchange at $\bar{y}$. Hence, the price to be paid would be \$ $10^3\bar{y}$. If the exchange rate is lower than the expected, $\bar{y} - c$ (for $c > 0$), then the company would regret the hedging strategy because the speculation would lead additional profit of \$ $c \times 10^3$.

1.13 Practice Questions

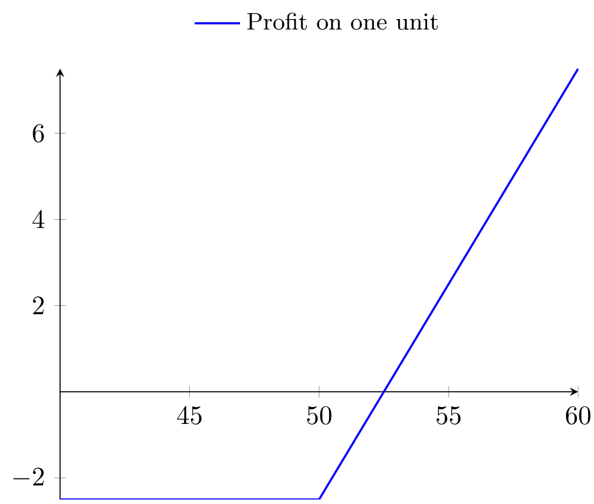
Suppose that a March call option to buy a share for \$50 costs \$2.50 and is held until March. Under what circumstances will the holder of the option make a profit? Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a long

position in the option depends on the stock price at maturity of the option.

The holder of the option will make a profit, if and only if, $S_T - 50 - 2.50 > 0$

The holder of the option will exercise the option, if and only if, $S_T - 50 > 0$

Where S_T is the stock price at the exercise date.



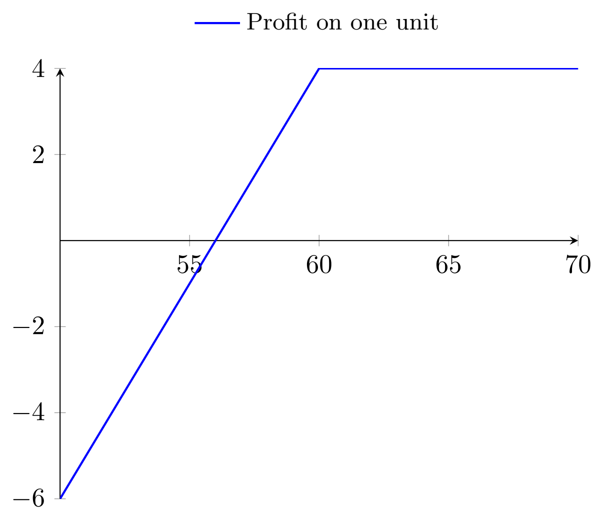
1.14 Practice Questions

Suppose that a June put option to sell a share for \$60 costs \$4 and is held until June. Under what circumstances will the seller of the option (i.e., the party with the short position) make a profit? Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a short position in the option depends on the stock price at maturity of the option.

The holder of the option will make a profit, if and only if, $(S_T - 60) + 4 > 0$

The holder of the option will exercise the option, if and only if, $(60 - S_T) > 0$. Note that the investor who has long position is the only one able to exercise it.

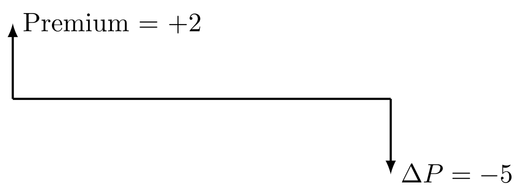
Where S_T is the stock price at the maturity date.



1.15 Practice Questions

It is May and a trader writes a September call option with a strike price of \$20. The stock price is \$18 and the option price is \$2. Describe the trader's cash flows if the option is held until September and the stock price is \$25 at that time.

Under the assumption that the trader holds the option until the maturity date, the cash flow diagram is:



In other words, the writer receives the premium of \$2 received from the sale of the option. Although, he has an outflow due to the fact that the option was exercised (stock price above the strike price).

1.16 Practice Questions

A trader writes a December put option with a strike price of \$30. The price of the option is \$4. Under what circumstances does the trader make a gain?

Selling an option is also known as *writing the option.*

The payoff function follows the same pattern as mentioned on the previous questions.

The function can be defined as: $f(S) = I_{(S<30)} * (S - 30) + 4$

Alternatively,

$$f(S) = \left\{ \begin{array}{ll} S - 26, & \text{if } S \leq 30 \\ 4, & \text{if } S > 30 \end{array} \right\}$$

Where S represents the price of asset at maturity. If $S > 30$, the payoff function is constant at $f(S) = 4$. Therefore, we need to evaluate the other case. The trader makes a gain if the price of asset at maturity is above \$26.

1.17 Practice Questions

A company knows that it is due to receive a certain amount of a foreign currency in 4 months. What type of option contract is appropriate for hedging?

A long position in a put option such that the underlying asset is the foreign currency.

1.18 Practice Questions

"Options and futures are zero-sum games". What do you think is meant by this?

One could argue that options and futures are zero-sum games because each participant's gain of profit is equivalent to the loss obtained by their counterpart. The total profit of the participants sum up to zero.

1.19 Practice Questions

On July 1, 2011, a company enters into a forward contract to buy 10 million Japanese yen on January 1, 2012. On September 1, 2011, it enters into a forward contract to sell 10 million Japanese yen on January 1, 2012. Describe the payoff from this strategy.

Let:

- F_B be the forward rate defined on July 1, 2011
- F_S be the forward rate defined on September 1, 2011
- S_T be price of asset at the contract maturity

Note that both of them have the same maturity date. Therefore, we can express the payoff function as:

$$\text{Payoff}(S_T, F_B, F_S) = 10^6[(S_T - F_B) + (F_S - S_T)]$$

1.20 Practice Questions

Suppose that USD/sterling spot and forward exchange rates are as follows:

Spot	1.5580
90-day forward	1.5556
180-day forward	1.5518

What opportunities are open to an arbitrageur in the following situations?

(a)**A 180-day European call option to buy £1 for $1.42 costs 2 cents**

- Buy the European call option

Profit:

$$\max\{S_T - 1.42, 0\} - 0.02$$

- Take the short position in the forward contract

Profit:

$$K - S_T$$

Hence, Total Profit is equal to:

$$\max\{S_T - 1.42, 0\} - 0.02 + (1.5518 - S_T)$$

For the particular case which $S_T > 1.42$:

$$1.5318 - 1.42 > 0$$

For the particular case which $S_T < 1.42$:

$$1.5318 - S_T > 0$$

It implies that the profit is positive, for all S_T.

(b)**A 90-day European put option to sell £1 for $1.49 costs 2 cents**

- There is no open arbitrage opportunities

It is straightforward to see that the strategy of buying the put option and selling the exchange rate through the forward contract is not always profitable.

If the trade buys the put option, his payoff is going to be:

$$\max\{1.49 - S_T, 0\} - 0.02$$

If the trade takes the short position in the forward contract, his payoff is:

$$1.5556 - S_T$$

Net Profit:

- For $S_T < 1.49$:

$$1.49 + 1.5556 - 2 \times S_T$$

- For $S_T > 1.49$:

$$-0.02 + 1.5556 - S_T$$

The last case is ambiguous since it leads to negative profit for high S_T.

1.21 Practice Questions

Describe the profit from the following portfolio: a long forward contract on an asset and a long European put option on the asset with the same maturity as the forward contract and a strike price that is equal to the forward price of the asset at the time the portfolio is set up.

Let:

- $F_{price} = K$ be the forward price of the asset;
- c_O be the cost of the European option;
- c_F be the cost of the forward contract

Profit of the option:

$$\max\{F_{price} - S_T, 0\} - c_O$$

Profit of the forward contract:

$$S_T - F_{price} - c_F$$

The total profit:

$$\max\{F_{price} - S_T, 0\} + S_T - F_{price} - (c_O + c_F)$$

$$= \max\{0, S_T - F_{price}\} - (c_O + c_F)$$

Rearranging the terms,

$$\text{Profit}(S_T, F_{Price}) = \left\{ \begin{array}{ll} 0, & \text{if } F_{price} - S_T \geq 0 \\ S_T - F_{price}, & \text{if } S_T - F_{price} > 0 \end{array} \right\} - (c_O + c_F)$$

By building this strategy, the trader could replicate the terminal value of a European call option with the same maturity of the forward contract and strike price equal to delivery price.

1.22 Practice Questions

A trader buys a call option with a strike price of \$30 for \$3. Does the trader ever exercise the option and lose money on the trade? Explain your answer.

Define C as the call cost.

- Condition to exercise the option:

$$S_T - K \geq 0$$

- Condition to lose money (given that the option is exercised):

$$S_T - K < C$$

Hence,

$$S_T - 30 < 3 \Leftrightarrow S_T < 33$$

1.23 Practice Questions

A trader sells a put option with a strike price of \$40 for \$5. What is the trader's maximum gain and maximum loss? How does your answer change if it is a call option?

If the trader is buying a put option, the payoff will be:

$$max\{K - S_T; 0\} - 3$$

If the trader is selling a put option, the payoff will be:

$$-max\{K - S_T; 0\} + 3 = min\{S_T - K; 0\}$$

Substituting the numbers, it gives us:

$$min\{S_T - 30; 0\} + 3$$

It can be seen that the maximum gain for this derivative is + 3, while the maximum loss converges to - 27 ($S_T \to 0$).

1.24 Further Questions

The price of gold is currently \$1,400 per ounce. The forward price for delivery in 1 year is \$1,500 per ounce. An arbitrageur can borrow money at 4% per annum. What should the arbitrageur do? Assume that the cost of storing gold is zero and that gold provides no income.

The present value of the forward contract:

$$FV = 1400 \times (1 + 4\%) = 1456 < 1500$$

Since the forward price is greater than the future value of the price of gold, an investor would borrow \$1,400 per ounce at the interest rate of 4% and sell it forward for \$1,500 per ounce. After paying off the loan, the strategy would give him a profit of \$44 per ounce.

1.25 Further Questions

A trader buys a European call option and sells a European put option. The options have the same underlying asset, strike price, and maturity. Describe the trader's position. Under what circumstances does the price of the call equal the price of the put?

Assumptions:

- Same underlying asset
- $K_c = K_p$ (same strike)
- $T_c = T_p$ (same maturity)

The overall position can be expressed as:

$$= \text{Payoff}_{\text{PUT}} + \text{Payoff}_{\text{CALL}}$$

$$= \max\{S_T - K; 0\} + \min\{S_T - K; 0\}$$

The price of the call is going to be equal to the price of the put, if:

$$C = \max\{S_T - K; 0\} = \min\{S_T - K; 0\} = P \Leftrightarrow S_T - K = 0 \Rightarrow S_T = K$$