# Model Preprocessing

## Handling of Categorical Variables

To incorporate the categorical variables into the regression model, each categorical variable was converted into a set of binary (dummy) variables.

## Aggregation to the Weekly Level

Variables such as Spend and GRP were summed across all instances within the same week for each brand.

For dummy variables representing categories like Informational, ENT (Entertainment), and other program categories, averages were computed to reflect the proportion of ads that fell into each category within a week. For instance, if 70% of a brand's ads in a given week were informational, the Informational variable would have an average of 0.7 for that week.

## Transformation

In the analysis, we applied the Box-Cox transformation to both the GRP and revenue variables to address potential issues of non-normality and heteroscedasticity in the data.

# Model 1: Base Model

Results

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Variable | Estimate | Std. Error | t value | Pr(>|t|) |
| GRP\_boxcox | 3815.9 | 187.2 | 20.385 | < 2e-16 \*\*\* |
| inf\_avg | -1481888.2 | 187678.5 | -7.896 | 4.83e-14 \*\*\* |
| inf\_ENT\_avg | 1481344.9 | 187556.0 | 7.898 | 4.76e-14 \*\*\* |
| inf\_INF\_avg | 1496945.3 | 187823.8 | 7.970 | 2.94e-14 \*\*\* |
| inf\_SPC\_avg | 1520778.5 | 188336.2 | 8.075 | 1.45e-14 \*\*\* |
| inf\_SPR\_avg | 1482701.5 | 187441.9 | 7.910 | 4.39e-14 \*\*\* |

**Significance Codes:**

\*\*\*: p < 0.001

\*\*: p < 0.01

\*: p < 0.05

. : p < 0.1

' ' : p ≥ 0.1

**Notes:**

* Residual standard error: 14180 on 315 degrees of freedom
* Multiple R-squared: 0.6705, Adjusted R-squared: 0.6642
* F-statistic: 106.8 on 6 and 315 DF, p-value: < 2.2e-16

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# Model 2: Lagged Variables Model

Revenue\_boxcox ~ lag(GRP\_boxcox, 1) + lag(GRP\_boxcox, 2) + lag(GRP\_boxcox, 3) + lag(GRP\_boxcox, 4) + lag(inf\_avg, 1) + lag(inf\_avg, 2) + lag(inf\_avg, 3) + lag(inf\_avg, 4) +

lag(inf\_ENT\_avg, 2) + lag(inf\_INF\_avg, 2) + lag(inf\_SPC\_avg, 2) + lag(inf\_SPR\_avg, 2) + lag(inf\_ENT\_avg, 3) + lag(inf\_INF\_avg, 3) + lag(inf\_SPC\_avg, 3) + lag(inf\_SPR\_avg, 3) + lag(inf\_ENT\_avg, 4) + lag(inf\_INF\_avg, 4) + lag(inf\_SPC\_avg, 4) + lag(inf\_SPR\_avg, 4)

Results

Residuals:

Min 1Q Median 3Q Max

-30042 -6640 1910 10485 38264

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Coefficient | Estimate | Std. Error | t value | Pr(>t) | Sign. Level |
| lag(GRP\_boxcox, 1) | 1193.9 | 367.2 | 3.251 | 0.001281 | \*\* |
| lag(GRP\_boxcox, 2) | 900.3 | 385.7 | 2.334 | 0.02024 | \* |
| lag(GRP\_boxcox, 3) | 923.4 | 379.9 | 2.431 | 0.015656 | \* |
| lag(GRP\_boxcox, 4) | 1307 | 363.9 | 3.591 | 0.000385 | \*\*\* |
| lag(inf\_avg, 1) | -202.2 | 3148.9 | -0.064 | 0.948836 |  |
| lag(inf\_avg, 2) | -796378 | 306220.1 | -2.601 | 0.00977 | \*\* |
| lag(inf\_avg, 3) | -428416 | 362686.3 | -1.181 | 0.238457 |  |
| lag(inf\_avg, 4) | -712601 | 307399.1 | -2.318 | 0.02112 | \* |
| lag(inf\_ENT\_avg, 2) | 794107.4 | 306158.1 | 2.594 | 0.009963 | \*\* |
| lag(inf\_INF\_avg, 2) | 796076.1 | 306187.6 | 2.6 | 0.00979 | \*\* |
| lag(inf\_SPC\_avg, 2) | 816623.1 | 306604.4 | 2.663 | 0.008157 | \*\* |
| lag(inf\_SPR\_avg, 2) | 796093.8 | 306194.3 | 2.6 | 0.009789 | \*\* |
| lag(inf\_ENT\_avg, 3) | 428133.2 | 362499.2 | 1.181 | 0.238524 |  |
| lag(inf\_INF\_avg, 3) | 425901.2 | 362539.8 | 1.175 | 0.241027 |  |
| lag(inf\_SPC\_avg, 3) | 447270.3 | 362868 | 1.233 | 0.218701 |  |
| lag(inf\_SPR\_avg, 3) | 426914.1 | 362618.4 | 1.177 | 0.240015 |  |
| lag(inf\_ENT\_avg, 4) | 716033 | 307363.7 | 2.33 | 0.020498 | \* |
| lag(inf\_INF\_avg, 4) | 716876.6 | 307424 | 2.332 | 0.020375 | \* |
| lag(inf\_SPC\_avg, 4) | 731850.9 | 307767.8 | 2.378 | 0.018043 | \* |
| lag(inf\_SPR\_avg, 4) | 712432.2 | 307404.5 | 2.318 | 0.021152 | \* |

**Significance Codes:**

\*\*\*: p < 0.001

\*\*: p < 0.01

\*: p < 0.05

. : p < 0.1

' ' : p ≥ 0.1

**Notes:**

* Residual standard error: 12630 on 297 degrees of freedom
* Multiple R-squared: 0.7535, Adjusted R-squared: 0.7369
* F-statistic: 45.39 on 20 and 297 DF, p-value: < 2.2e-16

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# Model 3: GLS AR(1)

Results

**Generalized Least Squares Fit by REML**

**Model:**  
Revenue\_boxcox ~ GRP\_boxcox + inf\_avg + inf\_ENT\_avg + inf\_INF\_avg + inf\_SPC\_avg + inf\_SPR\_avg + 0

**Data:**  
df\_weekly

|  |  |  |  |
| --- | --- | --- | --- |
|  | AIC | BIC | logLik |
|  | 6385.592 | 6460.644 | -3172.796 |

**Correlation Structure:**  
AR(1)

**Formula:**  
~1 | week\_end

**Parameter estimate(s):**  
Phi: 0.9552295

**Variance Function:**  
Structure: Different standard deviations per stratum  
Formula: ~1 | brand\_code\_uc

**Parameter estimates:**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 544907 | 548436 | 560405 | 710197 | 617480 | 569063 | 599631 | 558929 |
| 1.0000000 | 1.0930385 | 0.2688458 | 0.1343923 | 0.4235435 | 0.6222469 | 0.3774674 | 0.7100098 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 581335 | 500426 | 545320 | 635032 | 617497 |
| 0.7841691 | 0.2577837 | 0.3054187 | 0.4665658 | 0.4554535 |

**Coefficients:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Value | Std. Error | t-value | p-value |
| GRP\_boxcox | 438.30 | 34.835 | 12.582016 | 0.000 |
| inf\_avg | 79192.54 | 22807.978 | 3.472142 | 0.0006 |
| inf\_ENT\_avg | -81364.26 | 22886.791 | -3.555075 | 0.0004 |
| inf\_INF\_avg | -78585.70 | 22878.461 | -3.434921 | 0.0007 |
| inf\_SPC\_avg | -66352.74 | 22897.008 | -2.897878 | 0.004 |
| inf\_SPR\_avg | -82260.16 | 22749.567 | -3.615900 | 0.0003 |

**Correlation:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | GRP\_boxcox | inf\_avg | inf\_ENT\_avg | inf\_INF\_avg | inf\_SPC\_avg |
| inf\_avg | -0.701 |  |  |  |  |
| inf\_ENT\_avg | 0.698 | -1.000 |  |  |  |
| inf\_INF\_avg | 0.689 | -0.996 | 0.995 |  |  |
| inf\_SPC\_avg | 0.696 | -0.996 | 0.996 | 0.991 |  |
| inf\_SPR\_avg | 0.701 | -1.000 | 1.000 | 0.996 | 0.996 |

**Standardized Residuals:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Min | Q1 | Med | Q3 | Max |
| 0.4437053 | 0.8455107 | 0.9511092 | 1.0664801 | 2.3214105 |

**Residual Standard Error:**  
32339.54

**Degrees of Freedom:**  
321 total; 315 residual

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# Model 4: GLS ARMA(2,2)

**Model:**  
Revenue\_log ~ GRP\_log + inf\_avg + inf\_ENT\_avg + inf\_INF\_avg + inf\_SPC\_avg + inf\_SPR\_avg

**Data:**  
df\_weekly

|  |  |  |  |
| --- | --- | --- | --- |
|  | AIC | BIC | logLik |
|  | 435.4978 | 525.4832 | -193.7489 |

**Correlation Structure:**  
ARMA(2,2)

**Formula:**  
~1 | week\_end

**Parameter estimate(s):**  
Phi1: 0.94156896  
Phi2: 0.05609251  
Theta1: -0.46627929  
Theta2: -0.09985474

**Variance Function:**  
Structure: Different standard deviations per stratum  
Formula: ~1 | brand\_code\_uc

**Parameter estimates:**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 544907 | 548436 | 560405 | 710197 | 617480 | 569063 | 599631 | 558929 |
| 1.0000000 | 0.8814988 | 2.9817324 | 3.1937603 | 2.6943469 | 1.8422222 | 3.5628313 | 1.5209606 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 581335 | 500426 | 545320 | 635032 | 617497 |
| 1.6671183 | 3.2983408 | 3.4233790 | 2.0781895 | 2.6889955 |

**Coefficients:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Value | Std. Error | t-value | p-value |
| (Intercept) | 16.770301 | 0.053033 | 316.22158 | 0.0000 |
| GRP\_log | 0.026312 | 0.011830 | 2.22415 | 0.0268 |
| inf\_avg | -15.746169 | 6.877993 | -2.28935 | 0.0227 |
| inf\_ENT\_avg | 15.656741 | 6.876108 | 2.27698 | 0.0235 |
| inf\_INF\_avg | 15.693825 | 6.862343 | 2.28695 | 0.0229 |
| inf\_SPC\_avg | 16.675235 | 6.875172 | 2.42543 | 0.0159 |
| inf\_SPR\_avg | 15.802214 | 6.865419 | 2.30171 | 0.0220 |

**Correlation:**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | (Intercept) | GRP\_log | inf\_avg | inf\_ENT\_avg | inf\_INF\_avg | inf\_SPC\_avg |
| GRP\_log | -0.850 |  |  |  |  |  |
| inf\_avg | 0.363 | -0.280 |  |  |  |  |
| inf\_ENT\_avg | -0.363 | 0.280 | -1.000 |  |  |  |
| inf\_INF\_avg | -0.361 | 0.277 | -0.999 | 0.999 |  |  |
| inf\_SPC\_avg | -0.368 | 0.288 | -0.999 | 0.999 | 0.999 |  |
| inf\_SPR\_avg | -0.364 | 0.282 | -1.000 | 1.000 | 0.999 | 0.999 |

**Standardized Residuals:**

| **Min** | **Q1** | **Med** | **Q3** | **Max** |
| --- | --- | --- | --- | --- |
| -1.4618217 | -1.0883974 | -1.0042634 | -0.9225285 | -0.2637588 |

**Residual Standard Error:**  
1.022698

**Degrees of Freedom:**  
321 total; 314 residual

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# Model 5: GAM

**Family:**  
Gaussian

**Link Function:**  
Identity

**Formula:**  
Revenue\_log ~

    s(GRP\_log) +

    s(inf\_avg) +

    te(inf\_avg, ENT\_avg) +

    te(inf\_avg, INF\_avg) +

    te(inf\_avg, SPC\_avg) +

    te(inf\_avg, SPR\_avg) +

    s(lag(GRP\_log, 1)) +

    s(lag(inf\_avg, 1)) +

    te(lag(inf\_avg, 1), lag(ENT\_avg, 1)) +

    te(lag(inf\_avg, 1), lag(INF\_avg, 1)) +

    te(lag(inf\_avg, 1), lag(SPC\_avg, 1)) +

    te(lag(inf\_avg, 1), lag(SPR\_avg, 1)) +

    te(lag(inf\_avg, 2), lag(ENT\_avg, 2)) +

    te(lag(inf\_avg, 2), lag(INF\_avg, 2)) +

    te(lag(inf\_avg, 2), lag(SPC\_avg, 2)) +

    te(lag(inf\_avg, 2), lag(SPR\_avg, 2))

**Parametric Coefficients:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Estimate | Std. Error | t value | Pr(>|t|) |
| (Intercept) | 14.66438 | 0.02817 | 520.6 | <2e-16 \*\*\* |

**Approximate Significance of Smooth Terms:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Term | edf | Ref.df | F | p-value |
| s(GRP\_log) | 5.2396 | 6.3700 | 3.863 | 0.000907 \*\*\* |
| s(inf\_avg) | 0.9985 | 0.9985 | 20.906 | 8.77e-06 \*\*\* |
| te(inf\_avg, ENT\_avg) | 20.7819 | 21.9375 | 4.308 | < 2e-16 \*\*\* |
| te(inf\_avg, INF\_avg) | 6.0456 | 7.3837 | 7.101 | < 2e-16 \*\*\* |
| te(inf\_avg, SPC\_avg) | 7.2724 | 7.4887 | 11.050 | < 2e-16 \*\*\* |
| te(inf\_avg, SPR\_avg) | 3.5326 | 4.3704 | 10.580 | < 2e-16 \*\*\* |
| s(lag(GRP\_log, 1)) | 1.0000 | 1.0000 | 2.665 | 0.104016 |
| s(lag(inf\_avg, 1)) | 0.9985 | 0.9985 | 21.974 | 5.34e-06 \*\*\* |
| te(lag(inf\_avg, 1), lag(ENT\_avg, 1)) | 3.0296 | 3.4278 | 13.278 | < 2e-16 \*\*\* |
| te(lag(inf\_avg, 1), lag(INF\_avg, 1)) | 13.9575 | 16.0988 | 4.830 | < 2e-16 \*\*\* |
| te(lag(inf\_avg, 1), lag(SPC\_avg, 1)) | 7.3075 | 7.5170 | 9.306 | < 2e-16 \*\*\* |
| te(lag(inf\_avg, 1), lag(SPR\_avg, 1)) | 5.2086 | 6.1420 | 8.644 | < 2e-16 \*\*\* |
| te(lag(inf\_avg, 2), lag(ENT\_avg, 2)) | 4.0674 | 4.7667 | 12.559 | < 2e-16 \*\*\* |
| te(lag(inf\_avg, 2), lag(INF\_avg, 2)) | 11.9367 | 14.0166 | 5.154 | < 2e-16 \*\*\* |
| te(lag(inf\_avg, 2), lag(SPC\_avg, 2)) | 3.2567 | 3.7663 | 11.047 | 2.60e-07 \*\*\* |
| te(lag(inf\_avg, 2), lag(SPR\_avg, 2)) | 4.4540 | 4.9609 | 15.608 | < 2e-16 \*\*\* |

**Significance Codes:**  
0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

**Model Summary:**

|  |  |
| --- | --- |
|  | Value |
| Rank | 309/314 |
| R-sq.(adj) | 0.778 |
| Deviance explained | 84.7% |
| GCV | 0.36884 |
| Scale est. | 0.25312 |
| n | 319 |

A collage of graphs and diagrams

Description automatically generated

# Model 6: DLM

A group of graphs showing the different state

Description automatically generated with medium confidence

A group of graphs showing different types of state

Description automatically generated with medium confidence