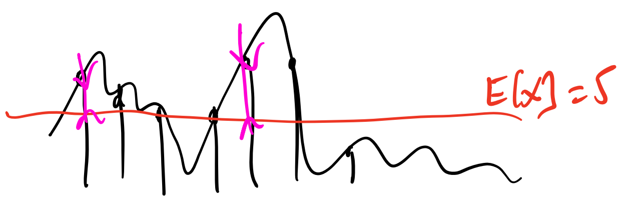
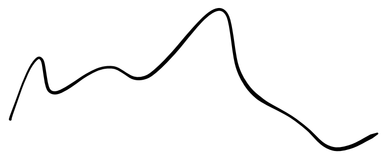
$\boxed{\text{Cours du } 13/09/2022}$

$$X \to \begin{cases} 1 & \frac{1}{6} \\ 2 & \\ \vdots & \\ 6 & \frac{1}{6} \end{cases}$$

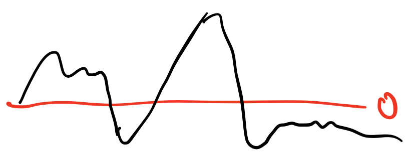
$$E[X] = 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + \dots + 6 \times \frac{1}{6}$$

$$= \frac{1}{6}\Big[\underbrace{1 + 2 \dots + 6}_{\frac{6+7}{2}}\Big] = \frac{7}{2} = \boxed{3.5}$$





$X - E(X)$



$$E\left[\left(X - E(X)\right)^n\right]$$

$\underline{n=1}$ $\quad E[X - E(X)] = E[X] - \underbrace{E[E(X)]}_{E[X]}$

$= \boxed{0}$

$\underline{n=2}$ $\quad \underbrace{E\left[\left(X - E(X)\right)^2\right]}$ $=$ <u>Variance</u>

$$E[X^2] - \underbrace{2E[X E(X)]}_{-2E(X)E(X)} + E[\underbrace{E^2(X)}_{E^2(X)}]$$

$$\boxed{E(X^2) - E^2(X)}$$

$X \sim N(m, \sigma^2)$ $\qquad p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-m)^2}{2\sigma^2}\right] \quad x \in \mathbb{R}$

$m = 0$

$\sigma^2 = 1$ $\qquad \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$

loi normale centrée réduite

$$E[X] = \int_{\mathbb{R}} x \boxed{p(x)dx} = \int_{\mathbb{R}} (\sigma u + m) \underbrace{\frac{1}{\sqrt{2\pi\cancel{\sigma^2}}} \exp\left(-\frac{u^2}{2}\right) \cancel{\sigma}\, du}$$

$$\boxed{u = \frac{x-m}{\sigma}}$$

$x = \sigma u + m$

$$= \sigma \underbrace{\int_{\mathbb{R}} u \frac{1}{\sqrt{2\pi}} e^{-u^2/2}}_{0} + m \underbrace{\int \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du}_{1}$$

$$= \boxed{m}$$

Variance $\qquad E[X^2] - \underbrace{E^2[X]}_{m^2}$

$$E[X^2] = \int x^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-m)^2}{2\sigma^2}} dx$$

$$= \int_{\mathbb{R}} (\sigma^2 u^2 + 2m\sigma u + m^2) \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$

$$u = \frac{x-m}{\sigma} \iff x = \sigma u + m$$

$$= \sigma^2 \underbrace{\int \frac{u^2}{\sqrt{2\pi}} e^{-u^2/2}\, du}_{1} + m^2 \underbrace{\int \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du}_{1}$$

$$E[X^2] = \sigma^2 + m^2$$

$$\boxed{\operatorname{Var} X = \sigma^2}$$

$$\boxed{X \sim N(\underset{E[X]}{m}, \underset{\operatorname{Var} X}{\sigma^2})}$$

$$X \begin{cases} \to 0 & q \\ \to 1 & p \end{cases}$$

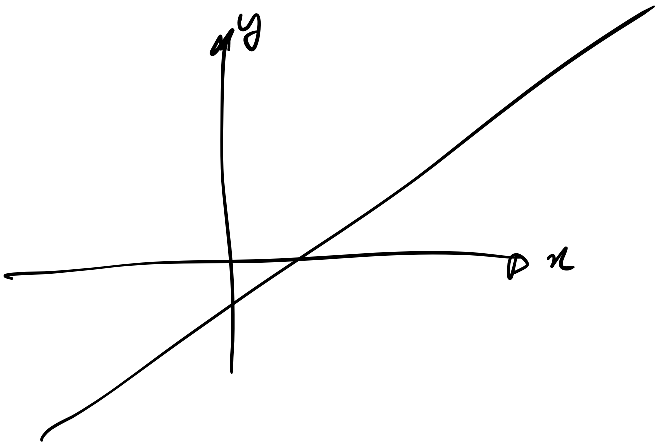
$$E[X] = 0 \times q + 1 \times p = \boxed{p}$$

$$E[X^2] = 0^2 \times q + 1^2 \times p = \boxed{p}$$

$$\operatorname{Var} X = E[X^2] - E^2[X]$$
$$= p - p^2 = p(1-p) = \boxed{pq}$$

<u>Changements de variables</u>

$$X \longrightarrow \boxed{\qquad} \longrightarrow Y = g(X)$$
$$= aX + b$$



$a \neq 0$

Cas discret

$$\boxed{Y = (X-2)^2 \qquad X \sim \mathcal{P}(\lambda)}$$

$$P[X = k] = \frac{\lambda^k}{k!} e^{-\lambda} \quad k \in \mathbb{N}$$

X	Y
0	4
1	1
2	0
3	1
4	4
5	9
6	16
⋮	

- Y est à valeurs dans $\{i^2, \ i \in \mathbb{N}\}$
- $P[Y = i^2] = P[(X-2)^2 = i^2]$

$$= P[X = 2+i \text{ ou } X = 2-i]$$

Cas 1 $\quad 2+i = 2-i \iff i = 0$

$$P[Y=0] = P(X=2) = \boxed{\frac{\lambda^2}{2} e^{-\lambda}}$$

Cas #2 $\quad \boxed{i \neq 0}$

$$P[Y = i^2] = \underbrace{P[X = 2+i]}_{\frac{\lambda^{2+i}}{} e^{-\lambda}} + P[X = 2-i]$$

$(2+i)!$

$$P[X=2-i] = \begin{cases} 0 & \text{si } 2-i<0 \iff \boxed{i>2} \\ P(X=2) & \text{si } i=0 \text{ (déjà fait)} \\ \underbrace{P(X=1)}_{\lambda e^{-\lambda}} & \text{si } i=1 \\ \underbrace{P(X=0)}_{e^{-\lambda}} & \text{si } i=2 \end{cases}$$

Résumé

$$P[Y=0] = \frac{\lambda^2}{2} e^{-\lambda}$$

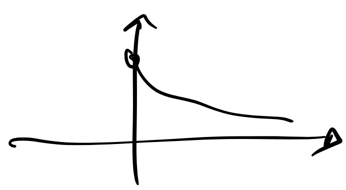
$$P(Y=1) = \frac{\lambda^3}{6} e^{-\lambda} + \lambda e^{-\lambda}$$

$$P[Y=2^2] = \frac{\lambda^4}{24} e^{-\lambda} + e^{-\lambda}$$

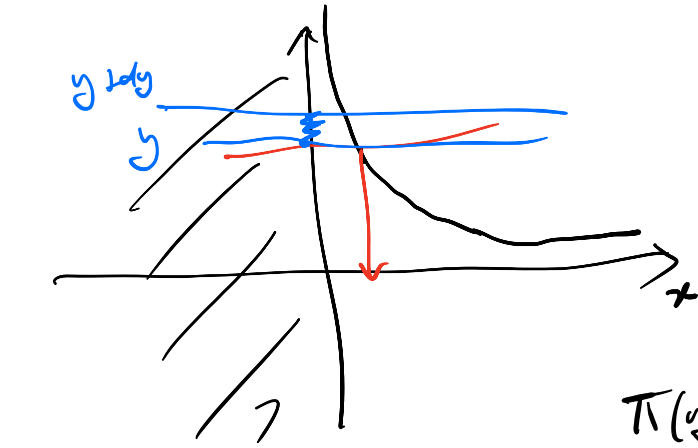
$$\underset{i>2}{P(Y=i^2)} = \frac{\lambda^{2+i}}{(2+i)!} e^{-\lambda}$$

$X \sim \mathcal{E}(1)$

$$p(x) = \begin{cases} e^{-x} & x>0 \\ 0 & x<0 \end{cases}$$



loi de $Y = \frac{1}{X}$



$y = \frac{1}{x} \iff x = \frac{1}{y}$

Soit $\pi(y)$ la densité de Y

$$\pi(y) = e^{-\frac{1}{y}} \underbrace{\left|-\frac{1}{y^2}\right|}_{\text{Jacobien de la transformation}} \qquad \boxed{y \in ?}$$

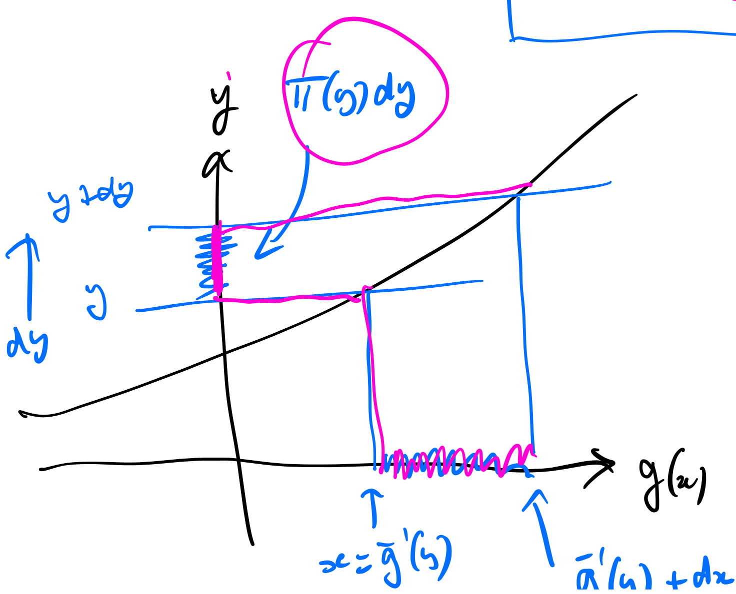
$\pi(y)\,dy = P[Y \in]y, y+dy[]$

$x > 0 \iff \frac{1}{y} > 0 \iff y > 0$

Conclusion
$$\pi(y) = \begin{cases} \frac{1}{y^2} e^{-\frac{1}{y}} & y > 0 \\ 0 & y \leq 0 \end{cases}$$

$$\boxed{\pi(y) = \frac{1}{y^2} e^{-\frac{1}{y}} \, \underline{I_{\mathbb{R}^{+*}}(y)}}$$

$$\begin{cases} 1 & \text{si } y \in \mathbb{R}^{+*} \\ 0 & \text{sinon} \end{cases}$$



$\pi(y)\,dy = p[g^{-1}(y)]\,dx$

$$\boxed{\pi(y) = p[g^{-1}(y)] \left|\frac{dx}{dy}\right|}$$

dx

Preuve

$$P[Y \in \Delta] = \boxed{\int_{\Delta} q(y)\,dy}$$

$$P[Y \in \Delta] = P[Y = g(X) \in \Delta]$$

$$= P[X \in g^{-1}(\Delta)]$$

$$= \int_{g^{-1}(\Delta)} p(x)\,dx$$

$\forall \Delta$

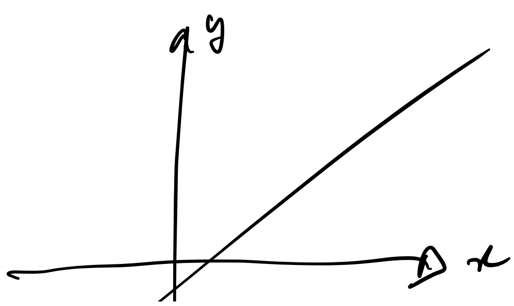
$$\boxed{= \int_{\Delta} p\left[g^{-1}(y)\right] \left|\frac{dx}{dy}\right| dy}$$

$x = g^{-1}(y)$

$(y = g(x))$

$X \sim N(m, \sigma^2)$ $\qquad p(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\dfrac{(x-m)^2}{2\sigma^2}\right]$ $x \in \mathbb{R}$

Loi de $Y = aX + b$ $\quad a \neq 0$



$Y = aX + b \iff X = \dfrac{Y - b}{a}$

On a une bijection de $\mathbb{R}$ dans $\mathbb{R}$

Densité de Y

$$\pi(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{1}{2\sigma^2}\left[\frac{y-b}{a} - m \right]^2 \right\} \underbrace{\left| \frac{1}{a} \right|}_{\frac{dx}{dy}}$$

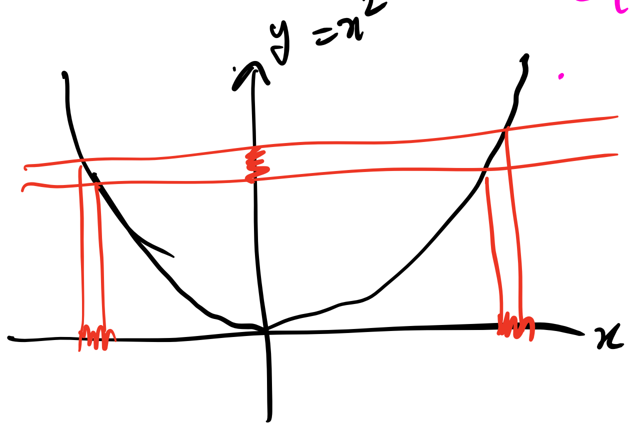
$$= \frac{1}{\sqrt{2\pi a^2\sigma^2}} \exp\left\{ -\frac{1}{2}\left[\frac{(y-(am+b))^2}{\sigma^2 a^2} \right] \right\} \quad y \in \mathbb{R}$$

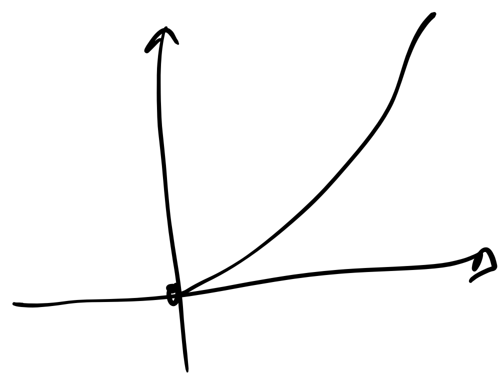
$$\boxed{p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp -\frac{(x-m)^2}{2\sigma^2} \quad x \sim \mathcal{N}(m, \sigma^2)}$$

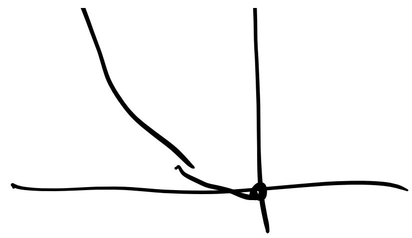
$$\boxed{Y \sim \mathcal{N}(am+b,\ a^2\sigma^2)}$$

$\operatorname{Var} Y = \operatorname{Var}(ax+b) = a^2\sigma^2$

$E(Y) = E[ax+b] = am+b$

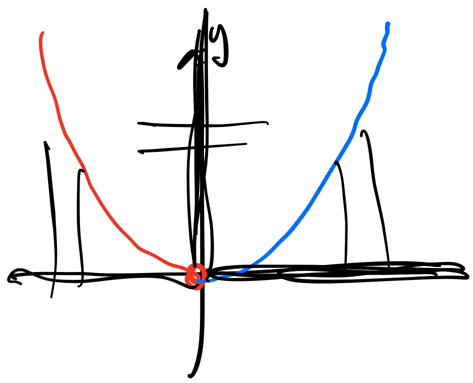






$X \sim \mathcal{N}(0,1)$ $\quad p(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ $\quad x \in \mathbb{R}$

$Y = X^2$ — Loi de Y ?



Bijection 1

$x \longrightarrow y = x^2 \iff x = \sqrt{y}$

$\mathbb{R}^+ \longrightarrow \mathbb{R}^+$

$$\pi_1(y) = \frac{1}{\sqrt{2\pi}} e^{-y/2} \left| \frac{1}{2\sqrt{y}} \right| \quad y > 0$$

$$= \frac{1}{2\sqrt{2\pi y}} e^{-y/2} \quad y > 0$$

Bijection 2

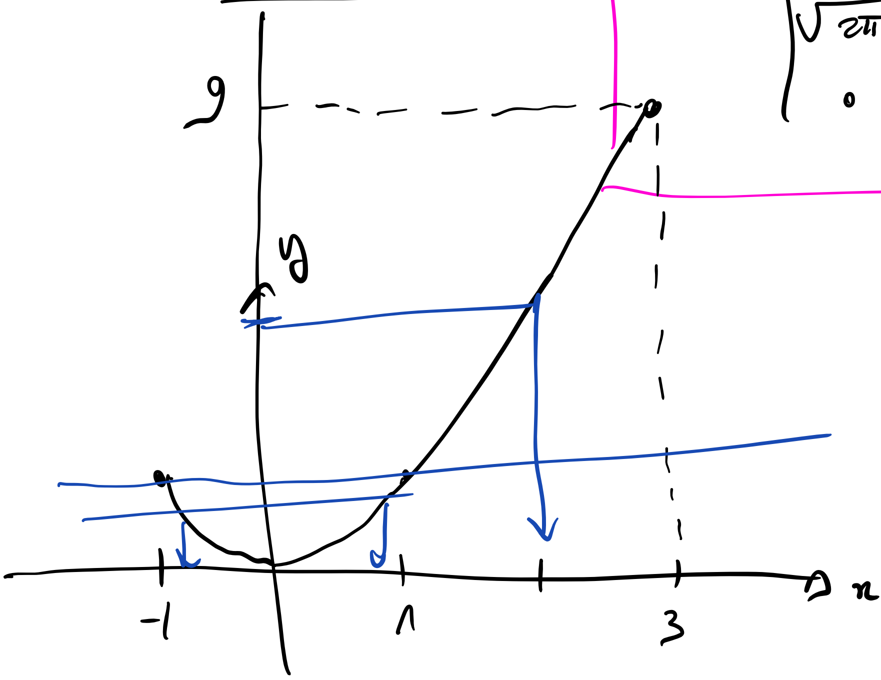
$x \longrightarrow y = x^2 \iff x = -\sqrt{y}$

$\mathbb{R}^- \longrightarrow \mathbb{R}^+$

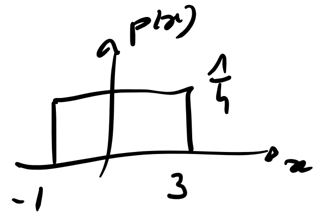
$$\pi_2(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y}{2}} \left| -\frac{1}{2\sqrt{y}} \right| \quad y > 0$$

$$= \frac{1}{2\sqrt{2\pi y}} e^{-y/2} \quad y > 0$$

Densité de Y



$$\Pi(y) = \begin{cases} \dfrac{1}{\sqrt{2\pi y}} e^{-y/2} & y > 0 \\ 0 & y \leq 0 \end{cases}$$

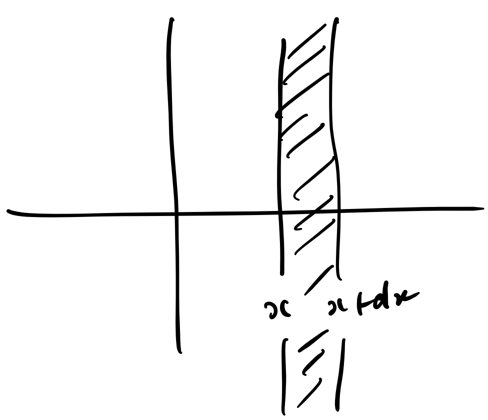


$Y = X^2$

Cours du 20/09/2022

$P[X \in]x, x+dx[] \approx \boxed{p(x)\,dx}$

$P[X \in]x, x+dx[] = P[X \in]x, x+dx[, Y \in \mathbb{R}]$



$$= \int_x^{x+dx} \int_{\mathbb{R}} \underbrace{p(u,v)}_{\text{densité du couple } (X,Y)} du\,dv$$

$$= \int_x^{x+dx} \underbrace{\left[\int_{\mathbb{R}} p(u,v)\,dv \right]}_{\approx \int_{\mathbb{R}} p(x,v)\,dv} du$$

$$\approx \left(\int_{\mathbb{R}} p(x,v)\,dv \right) \int^{x+dx} du$$

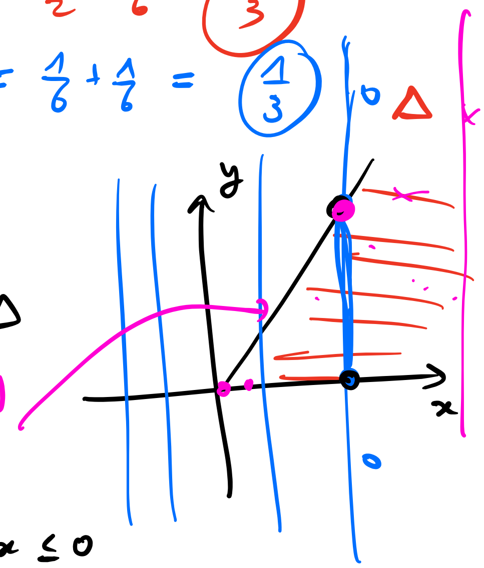
$$\underbrace{\qquad\qquad}_{dx}$$

Ex1

$Y \backslash X$	0	1
0	$\frac{1}{2}$	$\frac{1}{6}$
1	$\frac{1}{6}$	$\frac{1}{6}$

$$P(X=0) = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$$

$$P(X=1) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$



Ex2 $\quad p(x,y) = \theta^2 e^{-\theta x} \qquad (x,y) \in \Delta$

loi de X

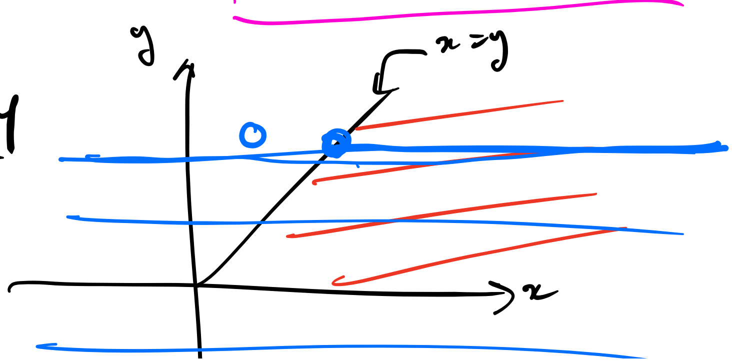
X est à valeurs dans $]0, +\infty[$

$$p(x,\cdot) = \int_{\mathbb{R}} p(x,y)\,dy = \begin{cases} 0 & x \le 0 \\ \underbrace{\int_0^x \theta^2 e^{-\theta x}\,dy}_{\theta^2 x e^{-\theta x}} & x > 0 \end{cases}$$

$$\boxed{p(x,\cdot) = \theta^2 x e^{-\theta x}\, I_{\mathbb{R}^{+*}}(x)}$$

donc $\boxed{X \sim \Gamma(\theta, 2)}$

loi de Y



Y est à valeurs dans $]0,+\infty[$

La densité de Y est $p(\cdot,y) = \int_{\mathbb{R}} p(x,y)\,dx$

$$= \begin{cases} 0 & y \le 0 \\ \underbrace{\int_y^{+\infty} \theta^2 e^{-\theta x}\,dx}_{\left[-\theta e^{-\theta x}\right]_y^{+\infty}} & y > 0 \end{cases}$$

$$\boxed{p(\cdot,y) = \theta e^{-\theta y} \underline{\mathbb{1}}_{]0,+\infty[}(y)}$$

$$\boxed{Y \sim \Gamma(\theta, 1)}$$

$$p(x|y)\,dx \approx \underbrace{P[X \in]x,x+dx[\mid Y = y]}_{\frac{P[X\in]x,x+dx[,\ Y=y]}{\underbrace{P(Y=y)}}} = \frac{0}{0}$$

$$p(x|y)\,dx = \lim_{dy \to 0} P[X \in]x,x+dx[\mid Y \in]y,y+dy[]$$

$\Delta = \{x_1, x_2\}$

$\Delta' = \{y_3\}$

$$P[X\in\Delta, Y\in\Delta'] = P[(X=x_1 \text{ ou } X=x_2) \text{ et } Y=y_3]$$

$$= P[\text{"}X=x_1, Y=y_3\text{" ou "}X=x_2, Y=y_3\text{"}]$$

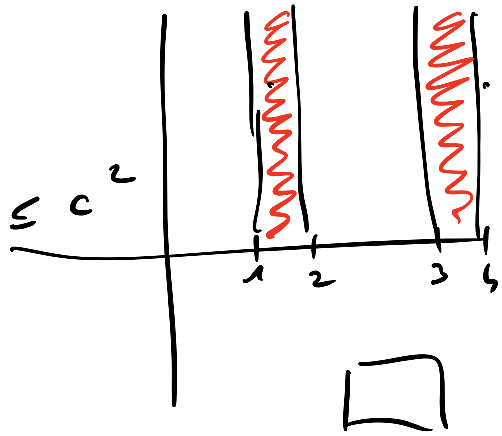
$$= P[X=x_1]\times P[Y=y_3] + P[X=x_2]\,P[Y=y_3]$$

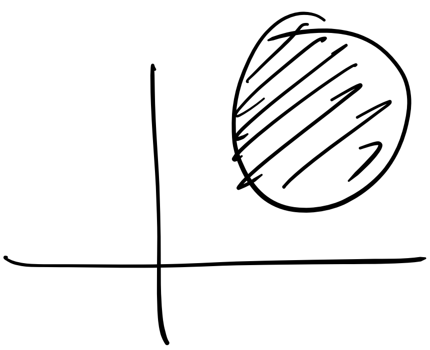
$$= \Big(\underbrace{P[X=x_1]+P[X=x_2]}_{P(X\in\Delta)}\Big)\underbrace{P(Y=y_3)}_{P(Y\in\Delta')}$$

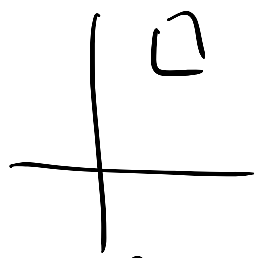
$X\in]1,2[\cup]3,4[$

$Y\in]0,+\infty[$

$(x-a)^2+(y-b)^2 \le c^2$







en 1D

$$E(X) \to \sum x_i\, P[X=x_i]$$

$$E(X) \to \int x\, p(x)\,dx$$

$$E[XY] \to \sum_i\sum_j x_i y_j\, p_{ij}$$

$$E[XY] \to \iint xy\, p(x,y)\,dx\,dy$$

$p(x,y)$

$p(x,\cdot)$

$p(\cdot,y)$

$\sum_{i \in I} \int x_i\, y\, p_i(y)\, dy$

$$\boxed{E[X] = \int x\, p(x,\cdot)\, dx}$$ version 1D

$$E(X) = \iint x\, p(x,y)\, dx\, dy$$ version 2D

$$= \int x \Big[\underbrace{\int p(x,y)\, dy}_{p(x,\cdot)} \Big]\, dx$$

$$E[\alpha(X)\beta(Y)] = \iint \alpha(x)\beta(y)\, p(x,y)\, dx\, dy$$

$$= \iint \alpha(x)\beta(y)\, p(x,\cdot)\, p(\cdot,y)\, dx\, dy$$

X et Y ind $\Leftrightarrow p(x,y) = p(x,\cdot) \times p(\cdot,y)\ \forall x\ \forall y$

$$= \underbrace{\Big[\int \alpha(x)\, p(x,\cdot)\, dx\Big]}_{E(\alpha(X))} \times \underbrace{\Big[\int \beta(y)\, p(\cdot,y)\, dy\Big]}_{E[\beta(Y)]}$$

<u>En particulier</u> X et Y ind $\Rightarrow E[XY] = E(X)E(Y)$ $\cancel{\Leftarrow}$

$$\mu_{11} = E[(X-E(X))(Y-E(Y))] \qquad \text{si } X=Y \quad Var(X)$$

$= \text{"covariance" du couple } (X,Y)$

$$\left(\text{Rappel} \quad Var X = E[(X-E(X))^2] = E(X^2) - E^2(X)\right)$$

$$\mu_{11} = E[\,XY \overset{a}{-XE(Y)} \overset{b}{-YE(X)} + \overbrace{E(X)E(Y)}^{c}\,]$$

$$= E(XY) - E(Y)E(X) - E(X)E(Y) + E(X)E(Y)$$

$$\boxed{cov(X,Y) = E(XY) - E(X)E(Y)}$$

$$\boxed{\langle X, Y \rangle = E(XY)} \quad \text{définit un produit scalaire}$$

CAUCHY SCHWARTZ

$$\boxed{\langle \underset{X-E(X)}{X}, \underset{Y-E(Y)}{Y} \rangle^2 \leq \|X\|^2 \; \|Y\|^2}$$

$$(\langle X-E(X), Y-E(Y) \rangle)^2 = \underbrace{E[(X-E(X))(Y-E(Y))]^2}_{cov^2(X,Y)}$$

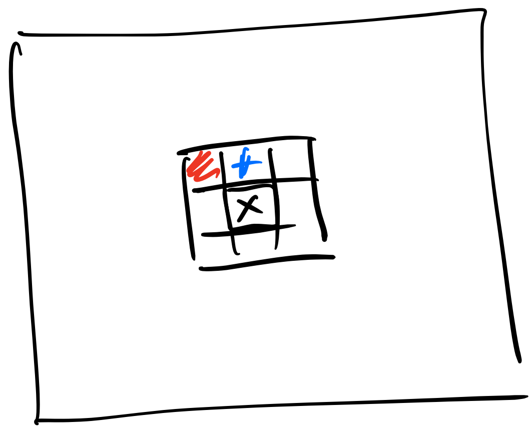
$$\|X\|^2 = \langle X, X \rangle =$$

$$\|X-E(X)\|^2 = \langle X-E(X), X-E(X) \rangle$$

$$= E[(X-E(X))^2] = Var X$$

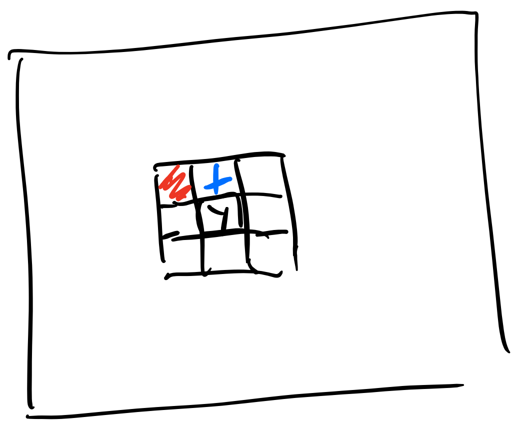
$$\text{cov}^2(X,Y) \le \text{Var}X \; \text{Var}Y$$

$$\boxed{r^2(X,Y) = \frac{\text{cov}^2(X,Y)}{\text{Var}X \; \text{Var}Y} \le 1}$$



$$E[X] \approx \frac{1}{9}\sum_{i=1}^{9} x_i$$

$$E(XY) \approx \frac{1}{9}\sum_{i=1}^{9} x_i y_i$$



$$E(Y) \approx \frac{1}{9}\sum y_i$$

$$E[\alpha(X,Y)] = \iint \alpha(x,y) \underbrace{p(x,y)}_{p(y|x)\,p(x,\cdot)} \, dx\, dy$$

$$p(y|x) = \frac{p(y,x)}{p(x,\cdot)} \qquad = \int p(x,\cdot) \underbrace{\left[\int \alpha(x,y)\, p(y|x)\, dy\right]}_{E[\alpha(X,Y) \mid X]} dx$$

$$= E_X\left[E_Y^Y \left[\alpha(X,Y) \mid X \right] \right]$$

$\boxed{\text{TD du } 20/09/2022}$

$$f(x) = \begin{cases} k x^a & x \in]0,1[\\ 0 & \text{sinon} \end{cases}$$

1) $\displaystyle\int_{\mathbb{R}} f(x)\,dx = 1 \iff \int_0^1 k x^a\,dx = k\left[\frac{x^{a+1}}{a+1}\right]_0^1 = \frac{k}{a+1} = 1$

$$\boxed{k = a+1}$$

2) $\displaystyle E[X^n] = \int x^n f(x)\,dx = \int_0^1 x^n (a+1) x^a\,dx$

$$= (a+1)\left[\frac{x^{a+n+1}}{a+n+1}\right]_0^1$$

$$= \boxed{\frac{a+1}{a+n+1}}$$

Moyenne $\quad E(X) = \boxed{\dfrac{a+1}{a+2}}$

$n=1$

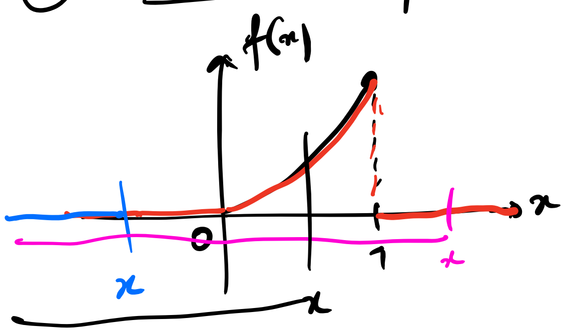
Variance $\quad E(X^2) - E^2(X) = \dfrac{a+1}{a+3} - \left(\dfrac{a+1}{a+2}\right)^2$

$n=2$

$$= \frac{(a+1)\left[(a+2)^2 - (a+1)(a+3)\right]}{(a+3)(a+2)^2}$$

$$= \boxed{\frac{(a+1)}{(a+3)(a+2)^2}}$$

3) Fonction de répartition



$$F(x) = P(X < x) = \int_{-\infty}^{x} f(u)\,du$$

$$= \begin{cases} 0 & x < 0 \\ 1 & x > 1 \\ \underbrace{\int_0^x (a+1)u^a\,du}_{[u^{a+1}]_0^x} & x \in [0,1] \end{cases}$$

//

$$\boxed{F(x) = \begin{cases} 0 & x \leq 0 \\ x^{a+1} & x \in]0,1[\\ 1 & x \geq 1 \end{cases}}$$

$$f(x) = \begin{cases} (a+1)x^a & x \in]0,1[\\ 0 & \text{sinon} \end{cases}$$

4) $\boxed{\begin{array}{l} Y = -\ln X \\ \text{Loi de } Y\,? \end{array}}$

$X = e^{-Y}$

$$\left|\frac{dx}{dy}\right| \qquad \left|\frac{dy}{dx}\right|$$

$$\uparrow \qquad\qquad \uparrow$$

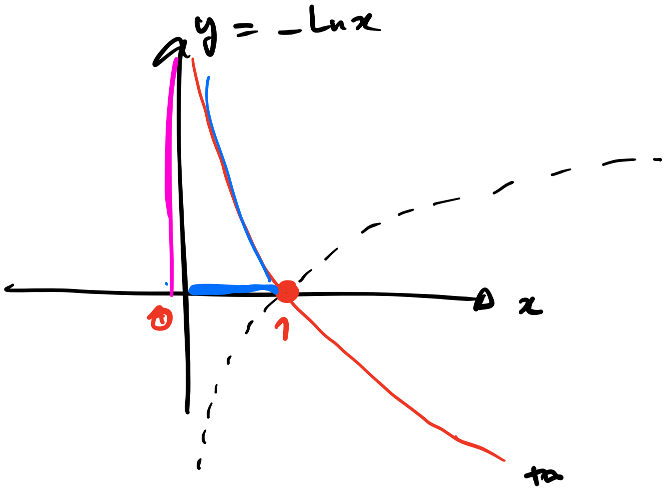
$$-e^{-y} \qquad -\frac{1}{x}$$

$$\pi(y) = (a+1)\,e^{-ay}\,\left|-e^{-y}\right|$$

$$\boxed{\pi(y) = (a+1)\,e^{-(a+1)y}}$$

$y \in ?$

$y \in]0, +\infty[$



Fonction gamma

$$\int_0^{+\infty} u^{n-1} e^{-u}\, du = \Gamma(n)$$
$$= (n-1)!$$

Calcul de $E[y^n]$ $= \displaystyle\int_0^{+\infty} y^n (a+1)\, e^{-\overset{u}{(a+1)y}}\, dy$

$$= \int_0^{+\infty} \left(\frac{u}{a+1}\right)^n \cancel{(a+1)}\, e^{-u}\, \frac{du}{\cancel{(a+1)}}$$

$u = (a+1)y$

$y = \dfrac{u}{a+1}$

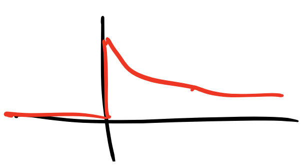
$$= \frac{1}{(a+1)^n} \underbrace{\int_0^{+\infty} u^n e^{-u}\, du}_{\Gamma(n+1) = n!}$$

$$= \boxed{\frac{n!}{(a+1)^n}}$$

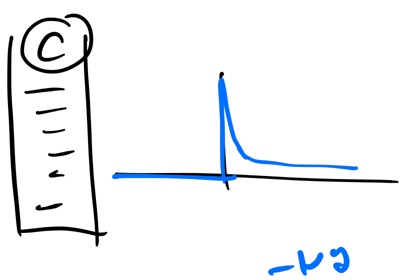
$\boxed{\text{Loi de } Z = \left|X - \tfrac{1}{2}\right|}$

| A | | B |

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$



C



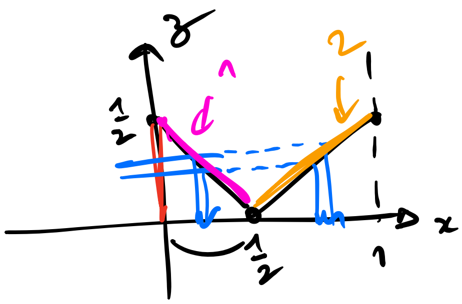
$E(X) = \frac{1}{\lambda}$

$f(y) = \begin{cases} \mu e & y > 0 \\ 0 & \text{sinon} \end{cases}$

$E(Y) = \frac{1}{\mu}$

$$\boxed{T = \mathrm{Inf}(\underset{X}{T_A}, \underset{Y}{T_B}) \ ?}$$

$Z = |X - \frac{1}{2}|$



- Z est à valeurs dans $]0, \frac{1}{2}[$
- $\pi(z) = \pi_1(z) + \pi_2(z)$

__Bijection 1__ $]0, \frac{1}{2}[\to]0, \frac{1}{2}[$

$x \mapsto z = |x - \frac{1}{2}|$

$z = \frac{1}{2} - x$

ou

$\boxed{x = \frac{1}{2} - z}$

$$\pi_1(z) = (a+1)\left(\frac{1}{2} - z\right)^a \underbrace{\left|\frac{dx}{dz}\right|}_{-1} = (a+1)\left(\frac{1}{2} - z\right)$$

__Bijection 2__ $]\frac{1}{2}, 1[\to]0, \frac{1}{2}[$

$x \mapsto z = |x - \frac{1}{2}| = x - \frac{1}{2} \iff \boxed{x = z + \frac{1}{2}}$

$$\pi_2(z) = (a+1)\left(\frac{1}{2} + z\right)^a |1| = (a+1)\left(\frac{1}{2} + z\right)^a$$

__Conclusion__

$$\boxed{\pi(z) = \begin{cases} (a+1)\left[\left(\frac{1}{2} + z\right)^a + \left(\frac{1}{2} - z\right)^a\right] & z \in]0, \frac{1}{2}[\\ 0 & \text{sinon} \end{cases}}$$

$X \sim \mathcal{E}(\lambda)$ $\quad p(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & x \leq 0 \end{cases}$

$Y \sim \mathcal{E}(\mu)$ $\quad p(y) = \begin{cases} \mu e^{-\mu y} & y > 0 \\ 0 & \text{sinon} \end{cases}$

$\mathrm{Inf}(X,Y)$

$\downarrow$

$E\left[e^{itT}\right]$

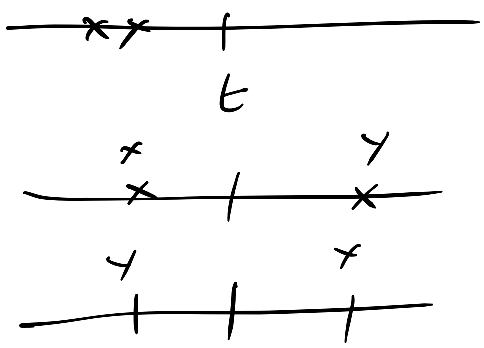
Loi de $T = \mathrm{Inf}(X,Y)$?

$$P[T < t] = P[\mathrm{Inf}(X,Y) < t]$$
$$= 1 - P[\mathrm{Inf}(X,Y) \geq t]$$

$X < t$ $\quad Y < t$

$X < t$ $\quad Y > t$

$Y < t$ $\quad X > t$



$$= 1 - \underbrace{P[X \geq t, Y \geq t]}_{P(X \geq t)\, P(Y \geq t)}$$

$$\underbrace{\int_t^{+\infty} \lambda e^{-\lambda x}\, dx}_{\left[-e^{-\lambda x}\right]_t^{+\infty} = e^{-\lambda t}} \qquad \int_t^{+\infty} \mu e^{-\mu y}\, dy$$

$$P(T < t) = \begin{cases} 1 - e^{-\lambda t} e^{-\mu t} = 1 - e^{-(\lambda+\mu)t} & t > 0 \\ 0 & t \leq 0 \end{cases}$$

Densité

$$\boxed{\pi(t) = \begin{cases} (\lambda+\mu)\, e^{-(\lambda+\mu)t} & t > 0 \\ 0 & t \leq 0 \end{cases}}$$

$$\boxed{\mathrm{Inf}(X,Y) \sim \mathcal{E}(\lambda+\mu)}$$

$$E[\quad] = \frac{1}{\lambda+\mu}$$