TD: Basic knowledge

1 Matrix Calculus

Question 1 : Let A and X be two real-valued matrices, computes

$$\frac{\partial \mathrm{Tr}(XA)}{\partial X}$$

Answer :

$$\begin{aligned}
\frac{\partial \mathrm{Tr}(XA)}{\partial X} &= \left(\frac{\partial \mathrm{Tr}(XA)}{\partial X_{ij}}\right)_{1\leq i,j\leq d} \\
&= \left(\frac{\partial}{\partial X_{ij}} \sum_k [XA]_{kk}\right)_{1\leq i,j\leq d} \\
&= \left(\frac{\partial}{\partial X_{ij}} \sum_k \sum_l X_{kl}A_{lk}\right)_{1\leq i,j\leq d} \\
&= \left(\frac{\partial}{\partial X_{ij}} \sum_k \sum_l X_{kl}A_{lk}\right)_{1\leq i,j\leq d} \\
&= (A_{ji})_{1\leq i,j\leq d} \\
&= A^T
\end{aligned}$$

Question 2 : Let X be an invertible real-valued matrix, computes

$$\frac{\partial \det(X)}{\partial X}$$

$$
\begin{aligned}
\frac{\partial \det(X)}{\partial X} &= \left(\frac{\partial \det(X)}{\partial X_{ij}}\right)_{1\leq i,j\leq d} \\
&= \left(\frac{\partial}{\partial X_{ij}}\det(X)\right)_{1\leq i,j\leq d} \\
&= \left(\det(X)\mathrm{tr}\left(X^{-1}\frac{\partial X}{\partial X_{ij}}\right)\right)_{1\leq i,j\leq d} \\
&= \left(\det(X)\mathrm{tr}\left(X^{-1}E_{ij}\right)\right)_{1\leq i,j\leq d} \\
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\end{aligned}
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