Programming Practical: rivers bathymetry estimation from surface measurements (altimetry-like). Part 1.

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1 The direct flow model and the inverse problem

We here consider a simplified flow model : the semi-linearized model which reads

$$\boxed{-\Lambda_{ref}(b)(x)\frac{\partial^2 H}{\partial x^2}(x)+\frac{\partial H}{\partial x}(x)=\frac{\partial b}{\partial x}(x) \quad \forall x\in[0,L]} \tag{1}$$

with Dirichlet boundary conditions, and

$$\Lambda_{ref}(b)\equiv\Lambda(H_{ref},b)=\frac{3}{10}\frac{(H_{ref}(x)-b(x))}{|\partial_x H_{ref}(x)|}$$

H_{ref} denotes a reference water surface elevation. In presence of observations, of course H_{ref} can be defined as : $H_{ref}(x)=H_{obs}(x)$.

For details, please refer to the dedicated document available on the Moodle page.

Recall that the considered inverse problem reads : *given some measurements $H^{obs}(x,t)$, estimate the river bathymetry $b(x)$.*

The objective function measures the discrepancy between the computed water surface elevation and the observation. It reads :

$$J^{obs}(H)=\sum_{m=1}^{M}\left(H(x_m)-H_m^{obs}\right)^2 \tag{2}$$

at x_m where observations are available.

The total cost function j reads :

$$\boxed{j_\alpha(b)=J_\alpha\left(H^b;b\right)=J^{obs}\left(H^b\right)+\alpha_{reg}J^{reg}(b)} \tag{3}$$

where H^b is the (unique) solution of the flow model, given b.

The principle of the VDA method is to solve the optimization problem :

$$\boxed{b^*_\alpha = arg \min_{b \in B} j_\alpha(b)} \quad (4)$$

Therefore providing the/an optimal bathymetry b^*.

2 Your tasks

2.1 Direct simulations analyses

Upload the Python code available on the course Moodle page. Consult the ReadMe file.

— Understand quickly the code (file main.py).
Run the direct model ($python main.py) with the provided data to get familiar with the flow model output(s).

— Improve your understanding of the direct model by performing it with different bathymetry values.
To do so, generate new test cases as follows :
- consider different bathymetry shapes $b(x)$ by changing frequencies and/or amplitudes,
- change the mean depth value.

Question : what flow or model properties seem to be the most sensitive with respect to the surface "signature" ?

2.2 Inverse problem : inferences of $b(x)$

— Consider the mathematical analysis on the (non-)uniqueness of the solution b^*. (See the supplementary material).

Questions : any chance to compute the target/true solution b^t ?
If few solutions possible, which one should you numerically obtain ?

— Perform the complete inverse problem resolution (VDA process).
Recover the "true" solution b^t as best as possible by playing with the few priors you can change, namely :
- the first guess value (in the main.py file),
- value(s) of α_{reg} (constant one or iteratively decreased) (in the callback routine / class vda.py file),
- the regularization term form (either 'grad' or 'b_b') (in the main.py file) and the background value b_b (in the generate case.py file).

Recall. After each inverse computations, and before analyzing the "physical" results, you must analyse the code numerical outputs : the stopping criteria, the plots cost function vs iterations, control variable vs iterations, and the gradient value vs iterations.

To go further

- Set up a case with quite low observations density, moreover uncertain. Perform the complete inverse problem resolution (VDA process) (potentially from different priors).
  Comment, analyse a few solutions your are able to obtain.

- Consider the case where one (1) in-situ measurement of h is given : b^* is known at one (1) location x_0.