

Résolution de systèmes linéaires issus d'EDP

Partie Manquante

Théorème

Let $\rho(S) = \max(\lambda(S))$ be the *spectral radius* of the matrix S . The iteration associated to the matrix S converges for all initial guess $x^{(0)}$ if and only if $\rho(S) < 1$.

Remarque : Suppose that the matrix S is symmetric and positive definite. We have

$$\frac{\|e^{(m)}\|}{\|e^{(0)}\|} \leq \|S^m\| \leq \rho(S)^m$$

Remarque : How many iterations are needed to guarantee the reduction of the error by a factor of 10^{-p} ?

$$m \geq \frac{\log(10^{-p})}{\log(|\rho(S)|)}$$

Définition - Convergence factor

The *convergence factor* is defined as the worst factor of reduction of the error in one step : $\rho(S)$.

Définition - Convergence rate

The *convergence rate* highlights the speed of convergence of the method : $-\log_{10}(\rho(S))$. The convergence becomes faster, the higher the convergence rate is.