Testing for homogeneity of a Poisson process

Keywords: Poisson process, Statistical test

Context of the project

Inhomogeneous Poisson processes provide models for a variety of physical phenomena. For instance, if at each failure a system is repaired to its condition at the time of failure and placed in service again, then the failures are often modeled by an inhomogeneous Poisson process, provided the repair times can be neglected. In some of these situations, it may be reasonable to assume that the intensity, $\lambda(\cdot)$, is nondecreasing, so tests of

$$\mathcal{H}_0 : \text{"}\lambda(\cdot) \text{ is constant"} \text{ versus } \mathcal{H}_1 : \text{"}\lambda(\cdot) \text{ is increasing"}$$

are of interest. The results of such tests could indicate whether the simple homogeneous Poisson process (HPP) may be adequate or whether a more general NHPP model is required.

Several tests are reviewed in [1], based on different characteristics of homogeneous Poisson processes, such as i.i.d. exponential interarrival times (see [2]) or the conditional distribution of the arrival times given the number of events.

Work to be performed

The work asked to the students goes in two different directions.

Theoretical results

The students to understand the methods described in [1] and justify some constructions.

Numerical results

The second types of results is to perform numerical simulations for these tests in order to validate them on simulated data (under both the null hypothesis and the alternative).

Finally, an application of these homogeneity tests to the large fire insurance claims in Denmark from 1980 to 1990 should be performed. The dates of each observation are contained in the `danish` dataset from the `evir` R-package.

The numerical simulations, will be presented with R Markdown.

References

[1] Lee J Bain, Max Engelhardt, and FT Wright. Tests for an increasing trend in the intensity of a poisson process: A power study. *Journal of the American Statistical Association*, 80(390):419–422, 1985.

[2] Hubert W Lilliefors. On the kolmogorov-smirnov test for the exponential distribution with mean unknown. *Journal of the American Statistical Association*, 64(325):387–389, 1969.