Intermediate statistics: introduction

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Introductions: What are you planning to do with statistical models?

Before we begin

Remember: All models are wrong, some are useful.

What we will cover

- How to explore, visualize, and model diverse kinds of data with an emphasis on generalized linear models

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- How to explore, visualize, and model diverse kinds of data with an emphasis on generalized linear models
- How to program in R
- Developing a workflow for producing replicable quantitative social science
- Some advanced topics that are relevant for the kinds of data we're dealing with in the course, subject to class interest

Review the syllabus

https://f-edwards.github.io/intermediate_stats/

My general approach to data analysis

1. Explore and visualize data

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1. Explore and visualize data
2. Fit models

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3. Assess model fit

My general approach to data analysis

1. Explore and visualize data
2. Fit models
3. Assess model fit
4. Interpret and describe results through simulation

The Generalized Linear Model

The linear model

We know we can model data as:

$$y = \beta_0 + \beta_1 x_1 \cdots \beta_n x_n + \varepsilon$$

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Or, more succinctly:

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Where the likelihood for the outcome conditional on the data takes the form:

$$Y|X \sim Normal(\mu, \sigma^2)$$

Generalzing the linear model

The linear model:

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Can be written as a more general formulation for a likelihood function f

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Now we can extend the (very) useful linear model to data with discrete outcomes

Generalizing the linear model

A linear predictor η:

$$\eta = \mathsf{X}\beta$$

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A link function g

$$g(E(Y|X)) = \eta$$

A mean expectation $E(Y|X) = \mu$

$$\mu = g^{-1}(\eta)$$

From OLS to GLM

OLS:

$$Y|X \sim Normal(\mu, \sigma^2)$$

GLM, for a likelihood function f with parameters θ:

$$Y|X \sim f(\theta)$$

Diverse likelihood functions

- Binary data: linear probability (Normal/Gaussian) and logistic models

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Diverse likelihood functions

- Binary data: linear probability (Normal/Gaussian) and logistic models
- Categorical data: Multinomial model
- Count data: Poisson and negative binomial models
- Positive continuous data: Gamma model

Getting started: software

Required installations

All software we are using is free and open source.

Install R:

`https://cran.r-project.org/`

Install RStudio:

`https://www.rstudio.com/products/rstudio/download/`

Recommended software: Git and GitHub

Git and GitHub are powerful tools for backing up and sharing your research.

All course materials, source code, and most of my research are hosted on GitHub (`https://github.com/f-edwards`).

Install Git:

`https://git-scm.com/`

Set up a GitHub account:

`https://github.com/`

Using GitHub for social science:

`https://happygitwithr.com/`

Recommended software: LaTeX

LaTeXis a powerful typesetting tool that works well with RMarkdown. It makes very attractive academic papers and slides.

Install it from the console

```
install.packages("tinytex")
```

```
tinytex::install_tinytex()
```

Break

Returning to the linear model

What do we know about the linear regression model?

$$y = \mathbf{X}\beta + \varepsilon$$

$$\varepsilon \sim Normal(0, \sigma^2)$$

Review

1. What forms can y take?

Review

1. What forms can y take?
2. What assumptions does the linear regression model require?

Assumptions of linear regression model

1. Validity of data relative to the research question

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Let’s analyze some data?

Two ways to access course data

- All data is accessible through the the course website (see the data link, or data folder on the GitHub page)
- *Recommended approach:* In a terminal (terminal.app on mac, Git Bash on windows):

```
git clone
https://github.com/f-edwards/intermediate_stats.git
```

Before beginning your work each session, pull updates I've pushed to the repo with:

```
git pull
```

Now you have an intermediate_stats folder with all code, slides, and data. Data is in intermediate_stats/data

Read in

```
#library(tidyverse)
### directly from the web
cj_budgets<-read_csv("./hw/data/revenue_dat.csv")
### from a project directory root
#cj_budgets<-read_csv("./hw/data/revenue_dat.csv")
```

About the data

Data are for an ongoing research project I'm working on. It's real, and can be a bit messy!

It documents police involved deaths, demographics, and local government budgets at the county-level for two time periods, 2007-11 and 2012-16. Datasets used include Fatal Encounters, American Community Survey, Annual Survey of State and Local Government Finance, and Uniform Crime Reports.

Full code for the project is up at:

`https://github.com/f-edwards/police-mort-revenue`

`merge.r` contains the code to make this merged file from a variety of source files (available if you want the raw data).

Evaluate the structure of the data

```
head(cj_budgets)
```

```
## # A tibble: 6 x 33
##   year_range fips_st fips_cnty deaths exp_tot exp_correction exp_police
##   <chr>      <chr>   <chr>      <dbl>   <dbl>          <dbl>      <dbl>
## 1 2007-2011  01      001            3  4.97e7       2101800     9306200
## 2 2007-2011  01      005            1  2.86e7       1037880.    5537840
## 3 2007-2011  01      007            0  1.30e7         80600     2421720
## 4 2007-2011  01      009            0  3.66e7       1703760     6853480
## 5 2007-2011  01      011            0  1.09e7             0     2285320
## 6 2007-2011  01      013            1  3.05e7        487320     4067200
## # ... with 26 more variables: exp_welfare <dbl>, rev_tot <dbl>,
## #   rev_fines <dbl>, rev_gen_ownsource <dbl>, rev_int_gov <dbl>,
## #   rev_prop_tax <dbl>, rev_tax <dbl>, pop_tot <dbl>, pop_pct_men_15_34 <dbl>,
## #   pop_wht <dbl>, pop_blk <dbl>, pop_ami <dbl>, pop_api <dbl>, pop_lat <dbl>,
## #   pop_pct_pov <dbl>, pop_pct_deep_pov <dbl>, pop_med_income <dbl>,
## #   pop_pc_income <dbl>, violent.yr <dbl>, property.yr <dbl>, murder.yr <dbl>,
## #   ft_sworn <dbl>, cbsa <dbl>, metroname <chr>, dissim_bw <dbl>,
## #   dissim_wl <dbl>
```

Evaluate the structure of the data

```
nrow(cj_budgets)
```

```
## [1] 4286
```

```
table(cj_budgets$year_range)
```

```
##
## 2007-2011 2012-2016
##      2308      1978
```

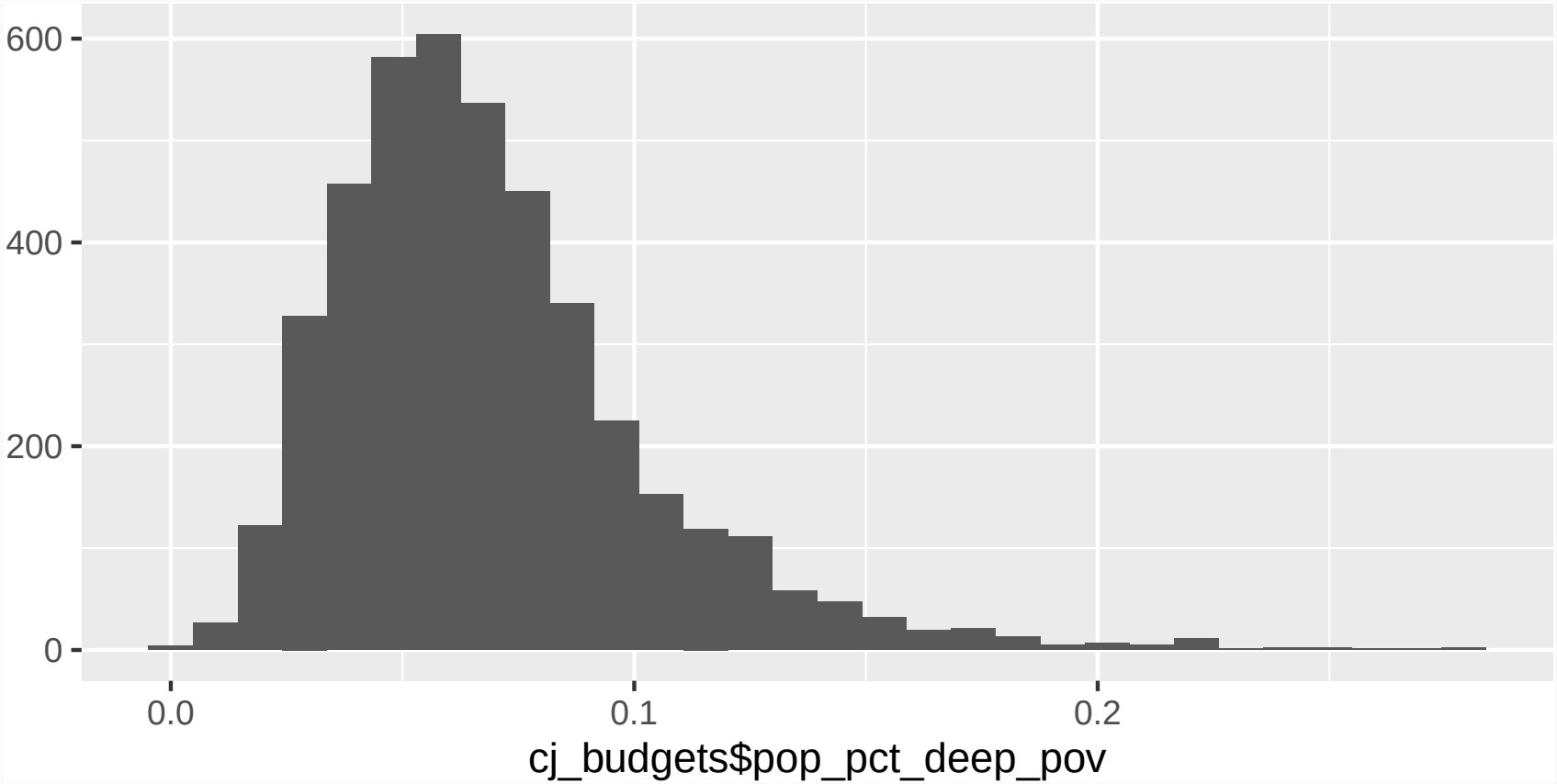
Descriptives

```
summary(cj_budgets$pop_pct_deep_pov)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.00000 0.04553 0.06285 0.06884 0.08442 0.27901
```

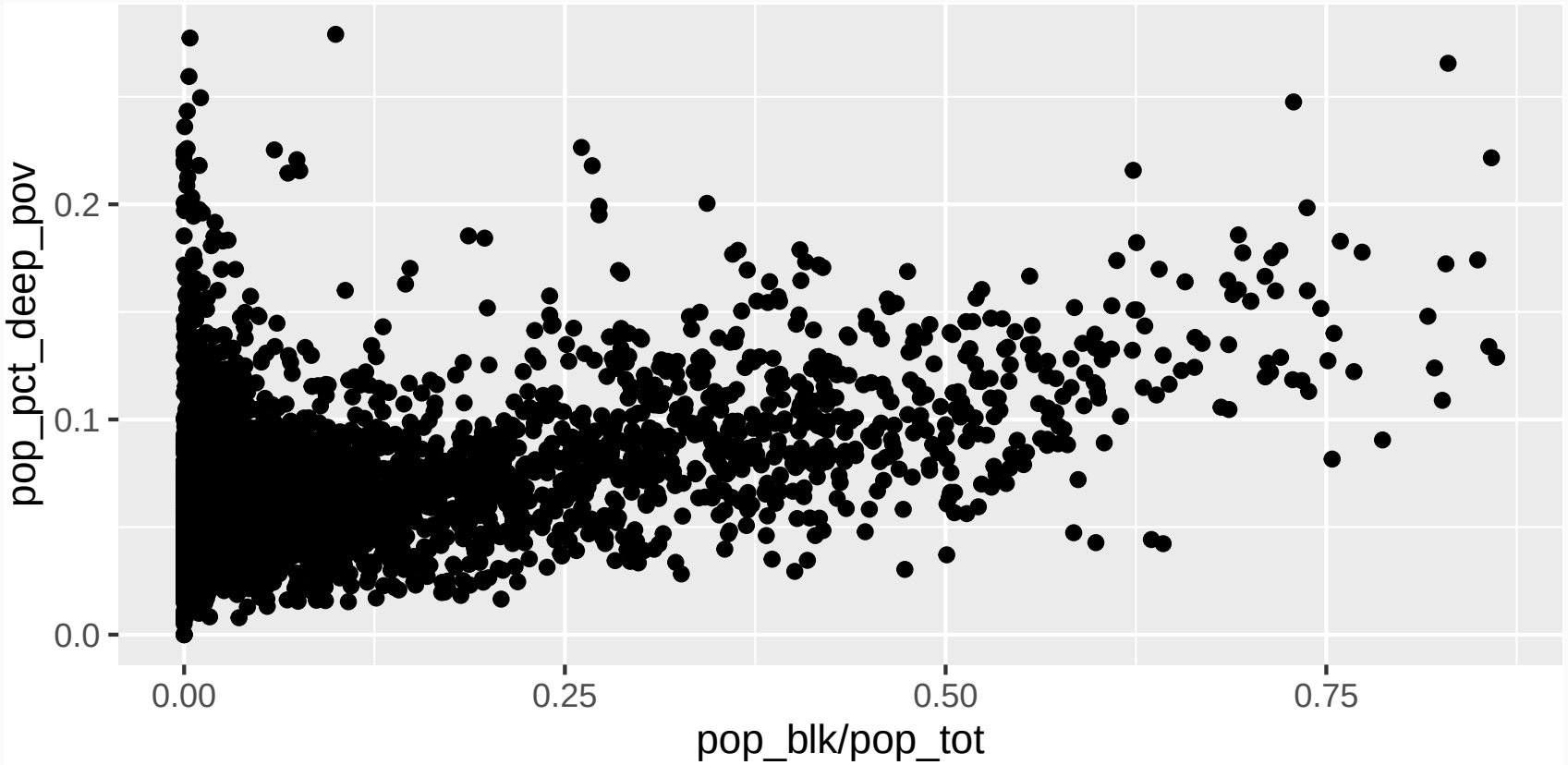
Visualize the distribution of a variable

```
qplot(cj_budgets$pop_pct_deep_pov)
```



Visualize a bivariate relationship

```
qplot(pop_blk/pop_tot,
      pop_pct_deep_pov,
      data = cj_budgets)
```



Fitting a linear model

```
model_1<-lm(pop_pct_deep_pov ~
              I(pop_blk/pop_tot),
            data =cj_budgets)
```

Display the model fit

```
summary(model_1)
```

```
##
## Call:
## lm(formula = pop_pct_deep_pov ~ I(pop_blk/pop_tot), data = cj_budgets)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.079709 -0.019343 -0.004579  0.013753  0.217773
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)        0.0591712  0.0005603  105.61   <2e-16 ***
## I(pop_blk/pop_tot) 0.0977188  0.0030884   31.64   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.03074 on 4284 degrees of freedom
## Multiple R-squared:  0.1894, Adjusted R-squared:  0.1892
## F-statistic:  1001 on 1 and 4284 DF,  p-value: < 2.2e-16
```

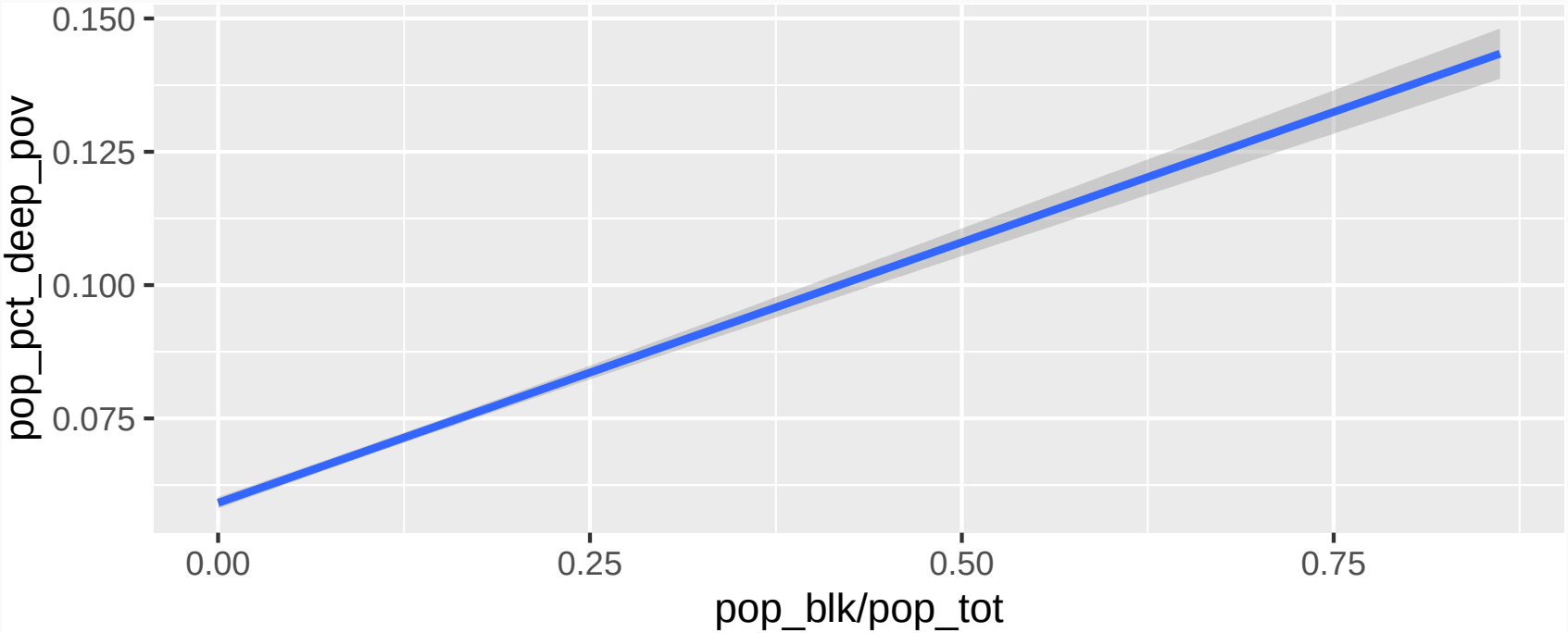
Display the model fit (nicer)

```
library(broom)
tidy(model_1)
```

```
## # A tibble: 2 x 5
##   term               estimate std.error statistic   p.value
##   <chr>                 <dbl>     <dbl>     <dbl>     <dbl>
## 1 (Intercept)          0.0592  0.000560     106.  0.
## 2 I(pop_blk/pop_tot)   0.0977  0.00309       31.6 1.22e-197
```

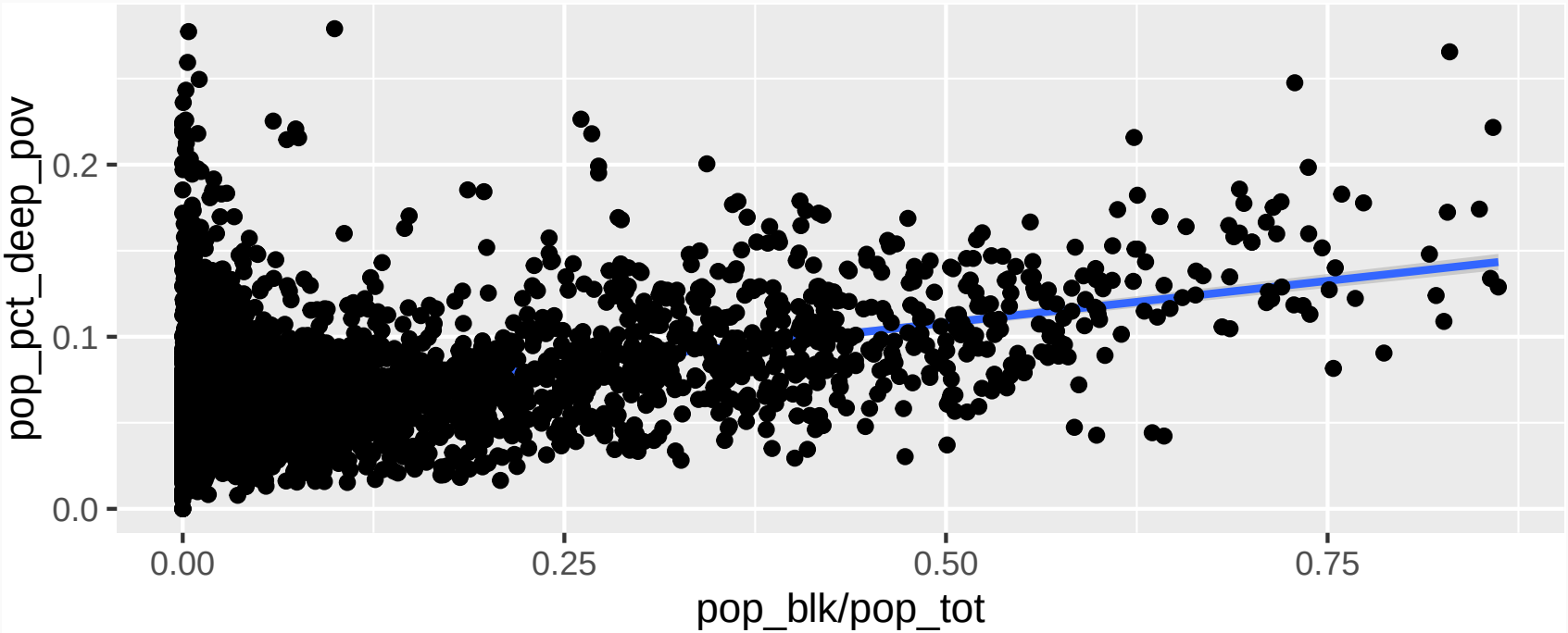
Visualize the model fit

```
library(ggplot2)
ggplot(cj_budgets,
       aes(x=pop_blk/pop_tot, y=pop_pct_deep_pov))+
  geom_smooth(method = "lm",
              formula = y~x)
```



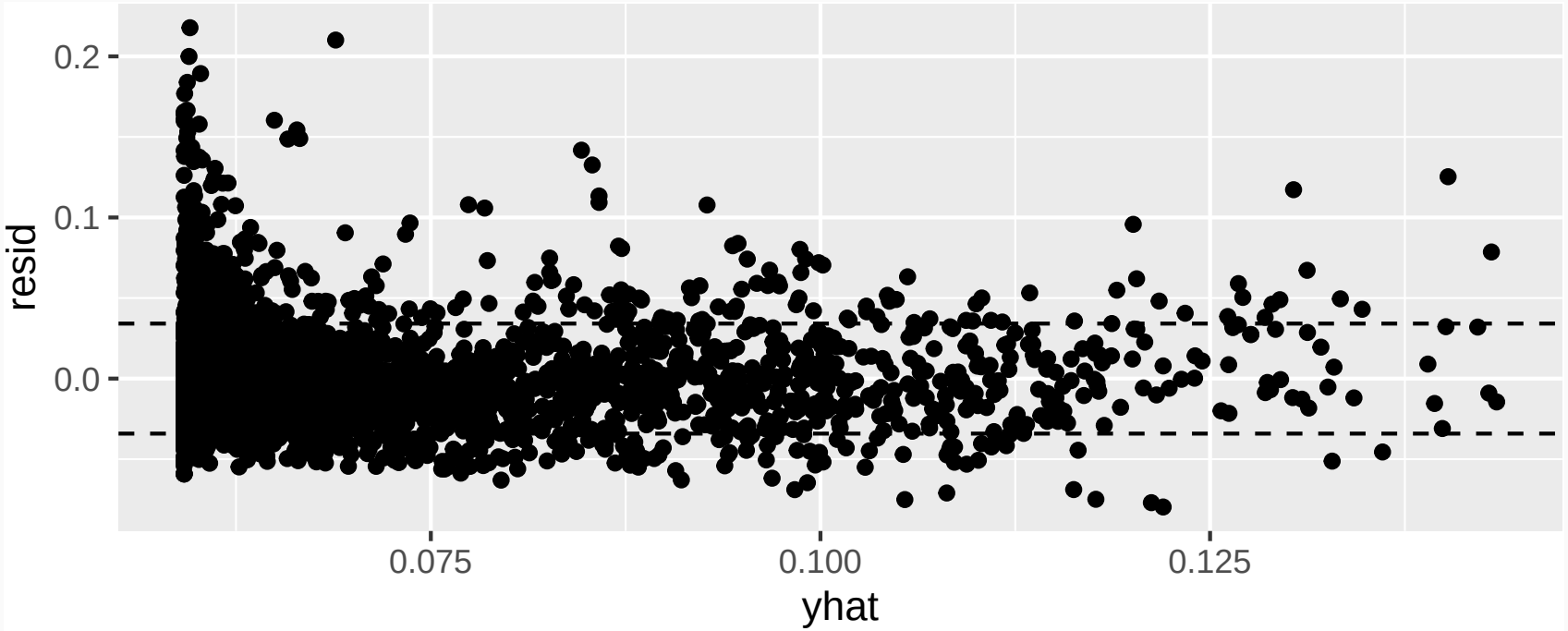
Visualize the model fit (against the data)

```
library(ggplot2)
ggplot(cj_budgets,
       aes(x=pop_blk/pop_tot, y=pop_pct_deep_pov))+
  geom_smooth(method = "lm",
              formula = y~x) +
  geom_point()
```



Residuals vs fitted

```
sd_outcome<-sd(cj_budgets$pop_pct_deep_pov)
plot_dat<-data.frame(resid = model_1$residuals, yhat = model_1$fitted.values)
ggplot(plot_dat, aes(y = resid, x = yhat)) +
  geom_point() +
  geom_hline(yintercept = sd_outcome, lty=2) + geom_hline(yintercept = -sd_outcome, lty=2)
```



Can we fit a better model?

Homework programming prep

Needed concepts / syntax

HW 1 asks you to apply some basic programming, data wrangling, and data visualization to common linear regression challenges.

- Reading data
- Loops
- Lists
- Matrix and data frame indexing
- dplyr::mutate
- ggplot2
- RMarkdown

HW tips

- Work together!
- Check the Wickham text from the syllabus or other online R courses
- Google it: StackOverflow will become your best friend
- Accept that this is hard and you will probably struggle with it