Sampling distributions, simulation, and the linear model

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Review HW1

- Challenges?

Sampling and sampling distributions

The sampling model and population inference

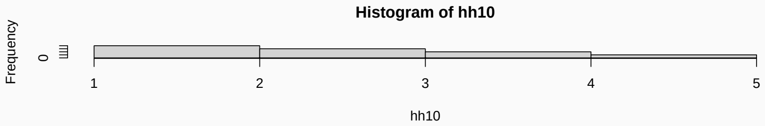
Under the **sampling model** we use a subset of the data to **infer** characteristics about the population.

I would like to know the average number of people living in a household in the United States.

Evaluating a sample

```r
draw_hh<-function(n){
  return(rpois(n, 1.53) + 1)
}

### sample 10 households
hh10<-draw_hh(10)
hist(hh10)
```



Infering population characteristics: mean

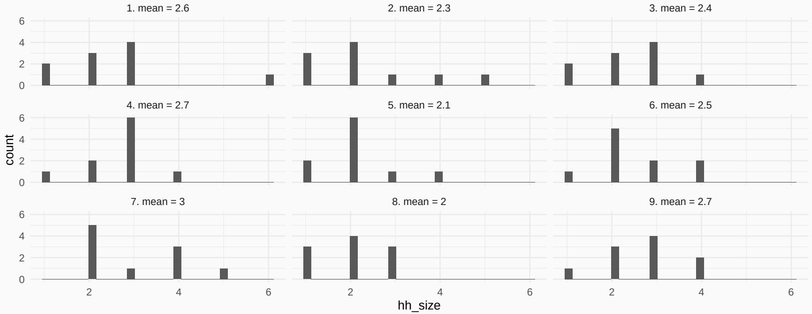
Let's assume this was a simple random sample (it was). We want to estimate μ, the population average household size. We've observed $\bar{hh}_{10}$, more commonly written as $\bar{x}$.

```
mean(hh10)
```

```
## [1] 2.8
```

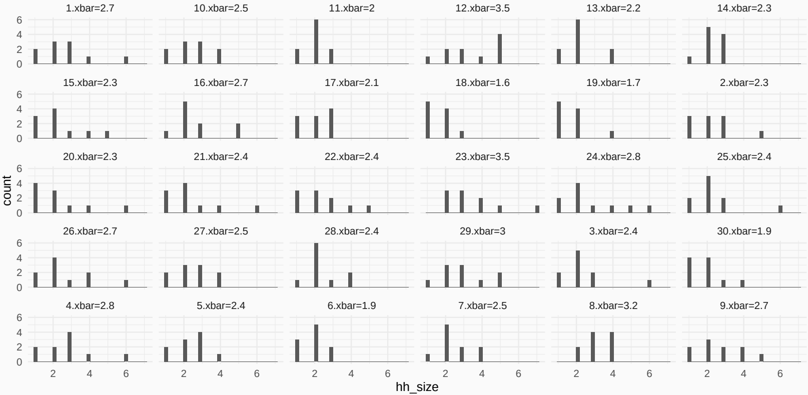
Describing uncertainty in our inference

We could have observed many possible samples of 10 households



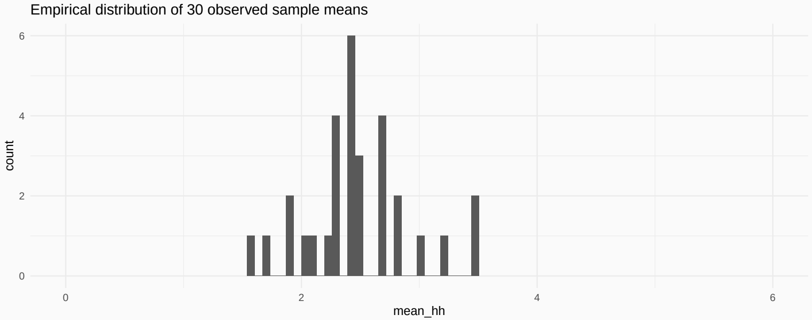
The approximate sampling distribution of hh_10

Each sample of 10 could draw any distribution of `hh_size`, here are 30 examples.



The sampling distribution of a parameter

Just as our sample has a theoretical sampling distribution, our estimate of the sample mean $\overline{x}$ has a sampling distribution.



Constructing a parameter estimate from a sampling distribution estimate

We can use the *central limit theorem* ($\bar{x} \sim N(\mu, \sigma)$ as $n \to \infty$) to estimate a sampling distribution for a parameter from our observed data.

We compute the sample mean ($\bar{x}$) and the *standard error* of the sample mean ($sd_x/\sqrt{n}$) to describe this distribution.

```
hh10 # the sample (x)
```

```
##  [1] 2 2 3 4 1 4 5 3 3 1
```

```
mean(hh10) # xbar
```

```
## [1] 2.8
```

```
sd(hh10) / sqrt(length(hh10)) # s_x
```

```
## [1] 0.4163332
```

Visualizing the sampling distribution of sample means

We can describe our uncertainty in the estimate of μ with the estimated sampling distribution for $\bar{x}$, or the possible values of the sample mean we *could have* observed based on these data.

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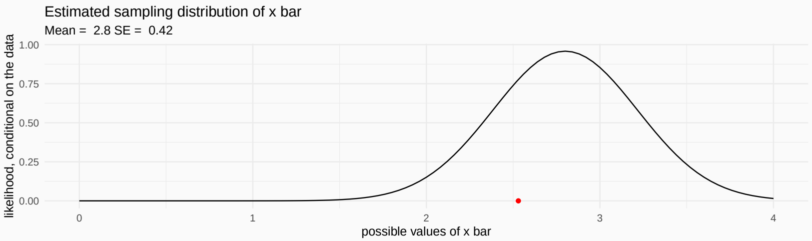

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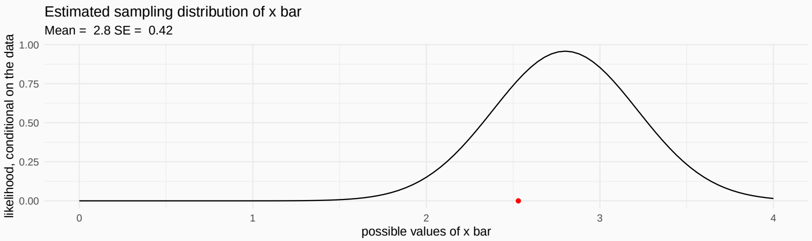
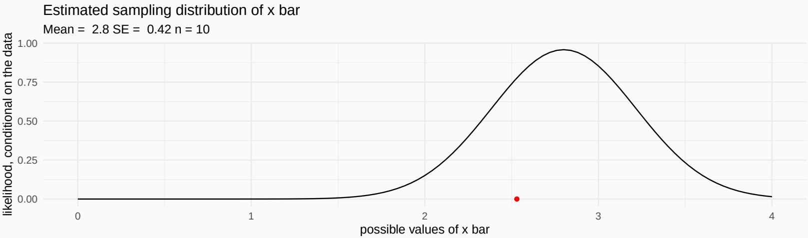


We use these estimates to describe our uncertainty in the value of the *population parameter* μ.

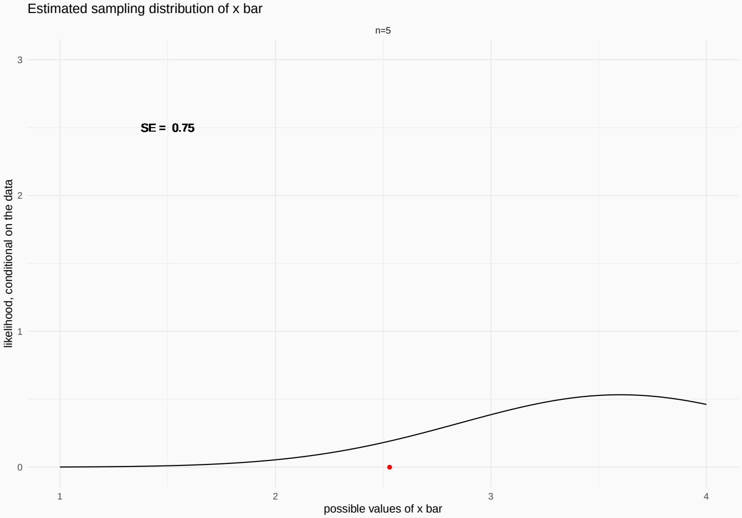
Question

Using this sampling distribution, compute a 95 percent confidence interval for μ.

Hint: you can use `pnorm(0.025, 0, 1)` and `pnorm(0.975, 0, 1)` to obtain critical values for z.



The sampling distribution of the mean



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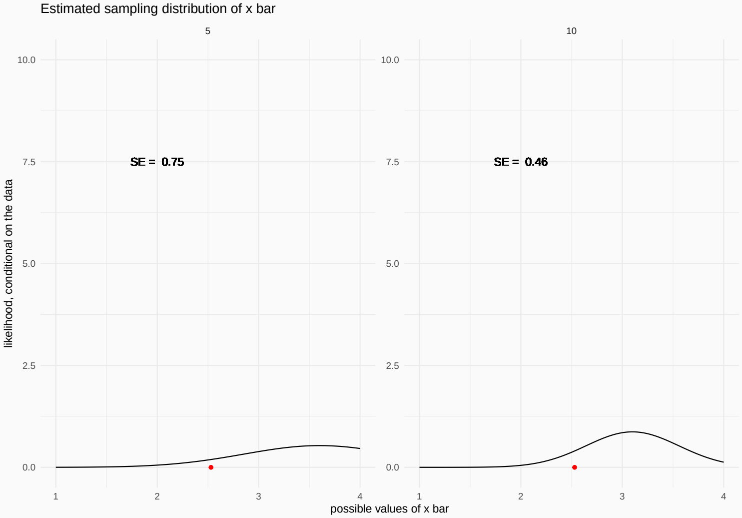

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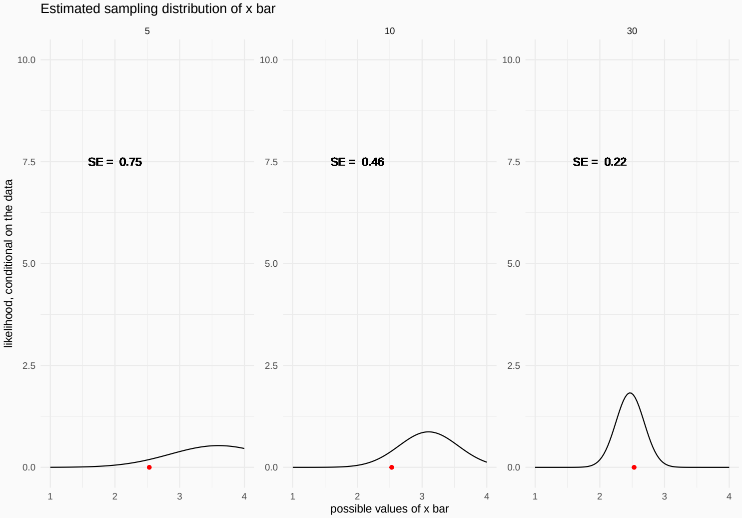

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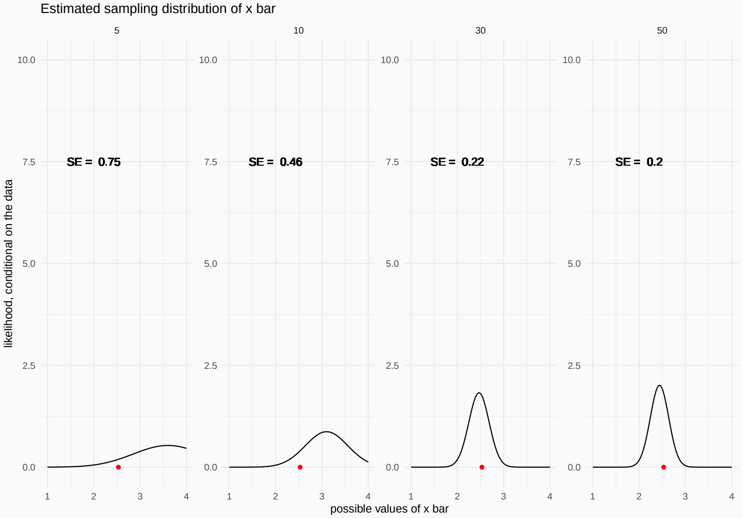

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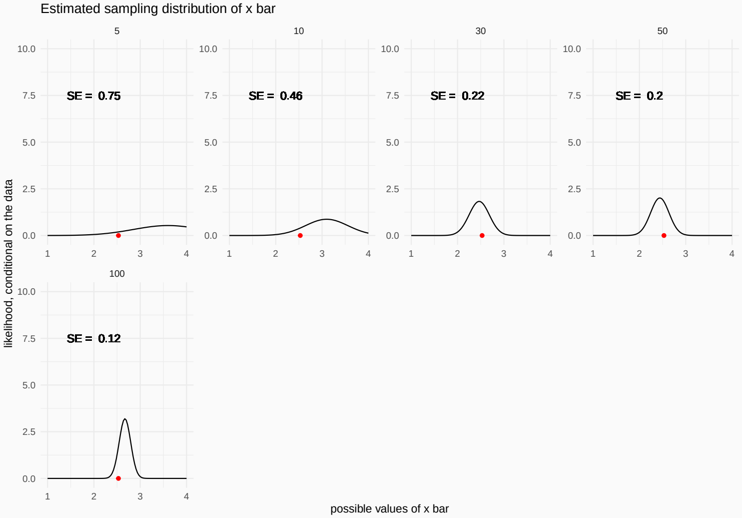

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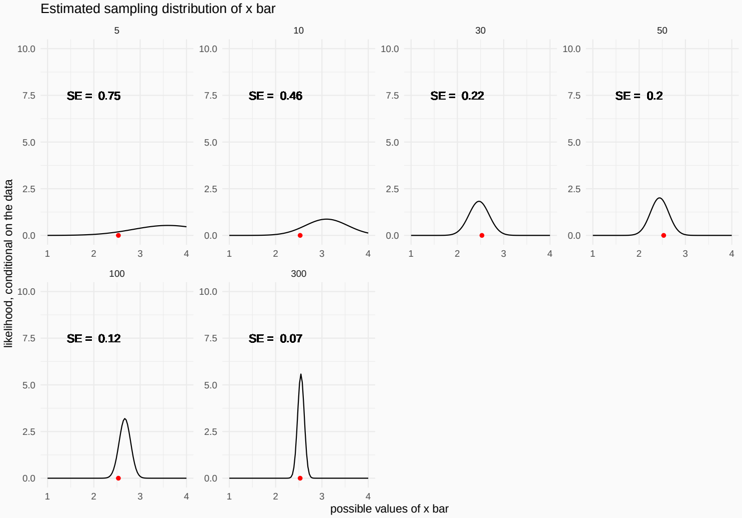

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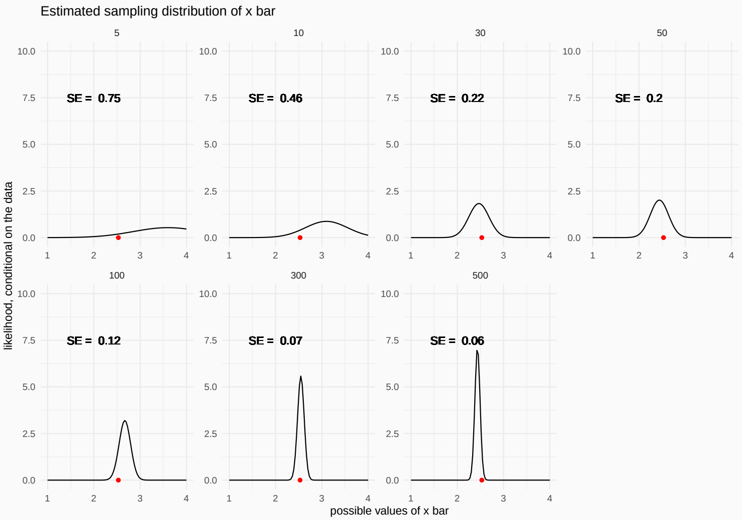

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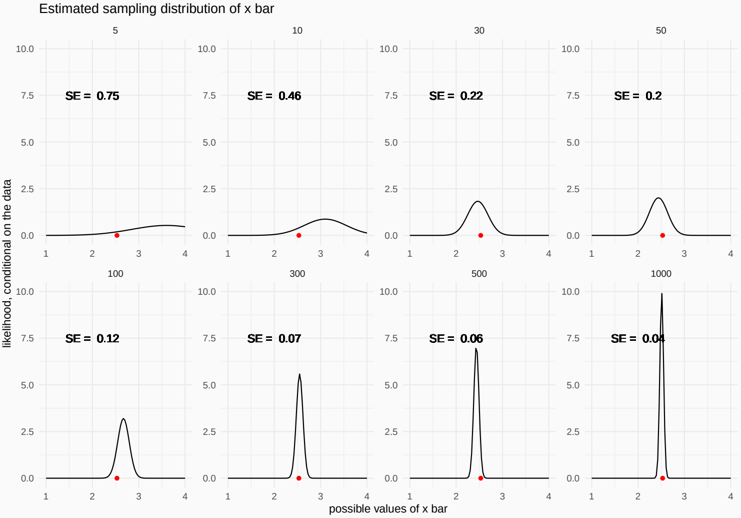

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2. What is the difference between $\bar{x}$ and μ?

Review

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Review

1. What is a parameter?
2. What is the difference between $\bar{x}$ and μ?
3. What is the difference between a sample and a sampling distribution?
4. Briefly explain the logic of a confidence interval through the logic of a sampling distribution

1. Let's draw 50 samples with 100 households sampled

```
samp_hh<-data.frame(sample_n = rep(1:50, each = 100))
temp<-draw_hh(100)
for(i in 2:50){
  temp<-c(temp,
          draw_hh(100))
}

samp_hh <- samp_hh %>%
  mutate(hh_size = temp)
```

Confidence intervals and sampling distributions

1. Let's draw 50 samples with 100 households sampled
2. Let's compute 95 percent confidence intervals for $\bar{x}$ for each sample

```
samp_ci<-samp_hh %>%
  group_by(sample_n) %>%
  summarise(xbarhat = mean(hh_size),
            se = sd(hh_size)/sqrt(100)) %>%
  mutate(ci_lwr = xbarhat - 1.96 * se,
         ci_upr = xbarhat + 1.96 * se)
```

Confidence intervals and sampling distributions

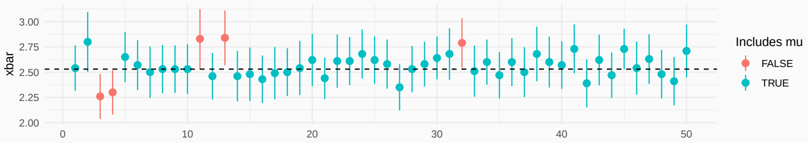
1. Let's draw 50 samples with 100 households sampled
2. Let's compute 95 percent confidence intervals for $\bar{x}$ for each sample
3. Let's add a binary variable indicating whether the interval includes μ (2.53)

```
samp_ci<- samp_ci %>%
  mutate(sig_test.95 = ci_lwr<2.53 & ci_upr>2.53)
```

Confidence intervals and sampling distributions

1. Let's draw 50 samples with 100 households sampled
2. Let's compute 95 percent confidence intervals for $\overline{x}$ for each sample
3. Let's add a binary variable indicating whether the interval includes μ (2.53)
4. Plot it!

```
ggplot(samp_ci,
       aes(ymin = ci_lwr, ymax = ci_upr,
           y = xbarhat, x = sample_n,
           color = sig_test.95)) +
  geom_pointrange() +
  geom_hline(yintercept = 2.53, lty = 2) +
  labs(x = "", y = "xbar", color = "Includes mu")
```



Break

Sampling distributions and regression parameters

We can apply the exact same logic to regression parameters. Let's use the `mpg` data to estimate the relationship between engine size (`displ`) and fuel efficiency (`hwy`).

```
glimpse(mpg)
```

```
## Rows: 234
## Columns: 11
## $ manufacturer <chr> "audi", "audi", "audi", "audi", "audi", "audi", "audi", "~
## $ model        <chr> "a4", "a4", "a4", "a4", "a4", "a4", "a4", "a4 quattro", "~
## $ displ        <dbl> 1.8, 1.8, 2.0, 2.0, 2.8, 2.8, 3.1, 1.8, 1.8, 2.0, 2.0, 2.~
## $ year         <int> 1999, 1999, 2008, 2008, 1999, 1999, 2008, 1999, 1999, 200~
## $ cyl          <int> 4, 4, 4, 4, 6, 6, 6, 4, 4, 4, 4, 6, 6, 6, 6, 6, 6, 8, 8, ~
## $ trans        <chr> "auto(l5)", "manual(m5)", "manual(m6)", "auto(av)", "auto~
## $ drv          <chr> "f", "f", "f", "f", "f", "f", "f", "4", "4", "4", "4", "4~
## $ cty          <int> 18, 21, 20, 21, 16, 18, 18, 18, 16, 20, 19, 15, 17, 17, 1~
## $ hwy          <int> 29, 29, 31, 30, 26, 26, 27, 26, 25, 28, 27, 25, 25, 25, 2~
## $ fl           <chr> "p", "p", "p", "p", "p", "p", "p", "p", "p", "p", "p", "p~
## $ class        <chr> "compact", "compact", "compact", "compact", "compact", "c~
```

Estimate the model

We model fuel efficiency as a linear function of engine size with the model

$$y \sim N(\mu, \sigma^2)$$

$$\mu = \beta_0 + \beta_1 x$$

```r
m0<-lm(hwy ~ displ, data = mpg)
```

What have we estimated?

```r
library(broom)
tidy(m0)
```

```
## # A tibble: 2 x 5
##   term        estimate std.error statistic   p.value
##   <chr>          <dbl>     <dbl>     <dbl>     <dbl>
## 1 (Intercept)    35.7      0.720      49.6 2.12e-125
## 2 displ          -3.53     0.195     -18.2 2.04e- 46
```

1. How does the `estimate` relate to the population mean?

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1. How does the `estimate` relate to the population mean?
2. What does the standard error tell us?

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1. How does the `estimate` relate to the population mean?
2. What does the standard error tell us?
3. What is `statistic`?
4. What about that p value?

Let’s interpret the model

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5. How certain are we in these findings? How precise are you willing to be?
6. What assumptions have we made?

Homework

1. complete Chapters 2, 5, 6, and 7 from STAT 545 (`https://stat545.com/r-basics.html`)
2. complete Introduction to R Markdown (`https://rmarkdown.rstudio.com/articles_intro.html`)
3. Write a brief RMarkdown report explaining how you are feeling about writing R and markdown code, and explaining areas where you feel you need support.