Logistic regression, 1

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Logistic regression

Read in the data for today

```
admissions <- read.csv("https://stats.idre.ucla.edu/stat/data/binary.csv")
head(admissions)
```

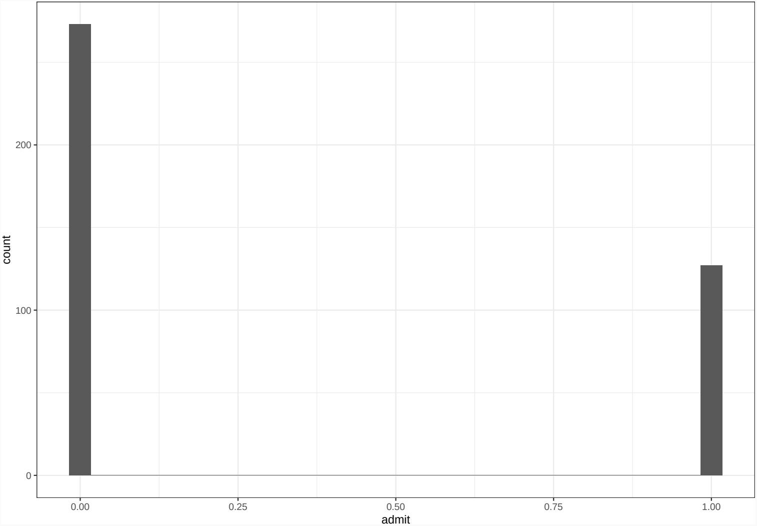
```
##   admit gre  gpa rank
## 1     0 380 3.61    3
## 2     1 660 3.67    3
## 3     1 800 4.00    1
## 4     1 640 3.19    4
## 5     0 520 2.93    4
## 6     1 760 3.00    2
```

```
nrow(admissions)
```

```
## [1] 400
```

Evaluate distribution of binary admission variable

```
ggplot(admissions, aes(x = admit)) + geom_histogram()
```



If y is an i.i.d. Bernoulli variable with probability p:

$$y \sim Bernoulli(p)$$

$$\Pr(y = 1) = p = 1 - \Pr(y = 0)$$

$$E(y) = \bar{y} = p$$

$$Var(y) = p(1 - p)$$

Summary of admit: What can we say about the probability of admission?

```r
mean(admissions$admit)
```

```
## [1] 0.3175
```

```r
sum(admissions$admit==1)/nrow(admissions)
```

```
## [1] 0.3175
```

```r
var(admissions$admit)
```

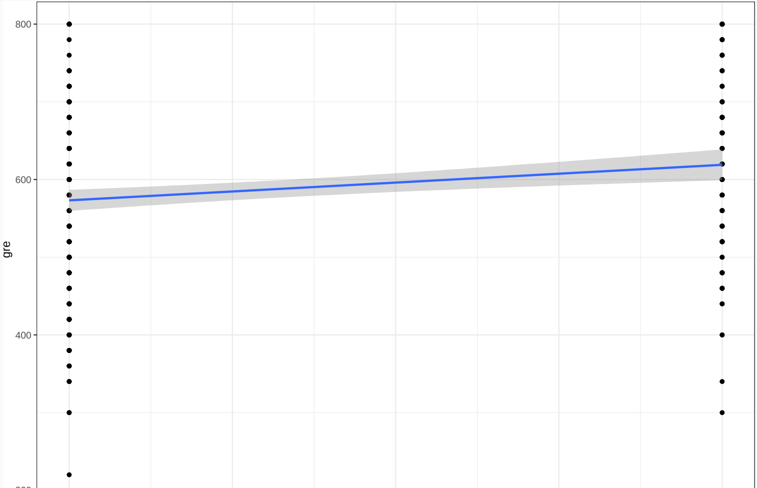
```
## [1] 0.2172368
```

```r
mean(admissions$admit) * (1 - mean(admissions$admit))
```

```
## [1] 0.2166937
```

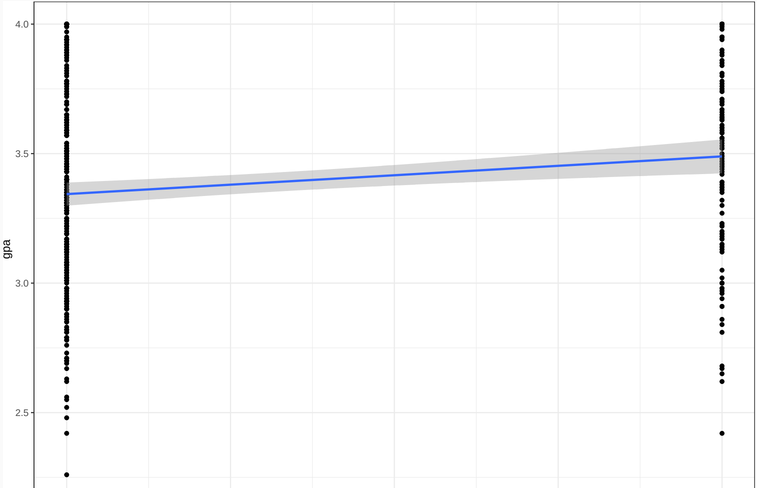
How does GRE relate to admission?

```
ggplot(admissions,
       aes(x = admit, y = gre)) + geom_point() +
  geom_smooth(method = "lm")
```



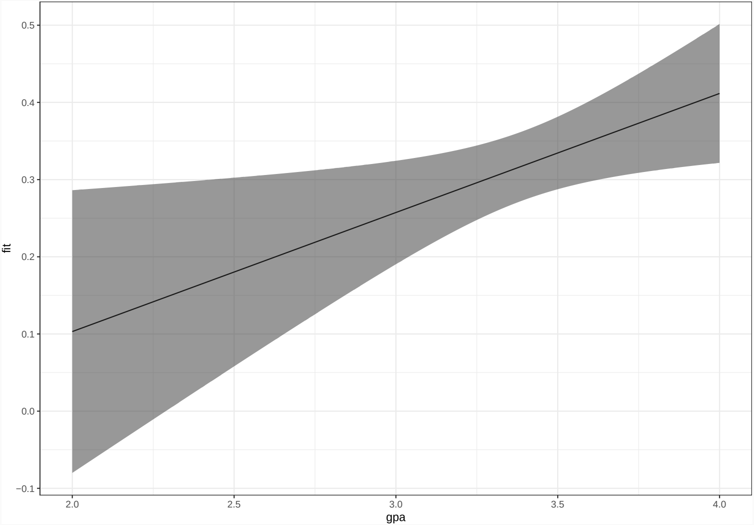
GPA?

```
ggplot(admissions,
       aes(x = admit, y = gpa)) + geom_point() +
  geom_smooth(method = "lm")
```



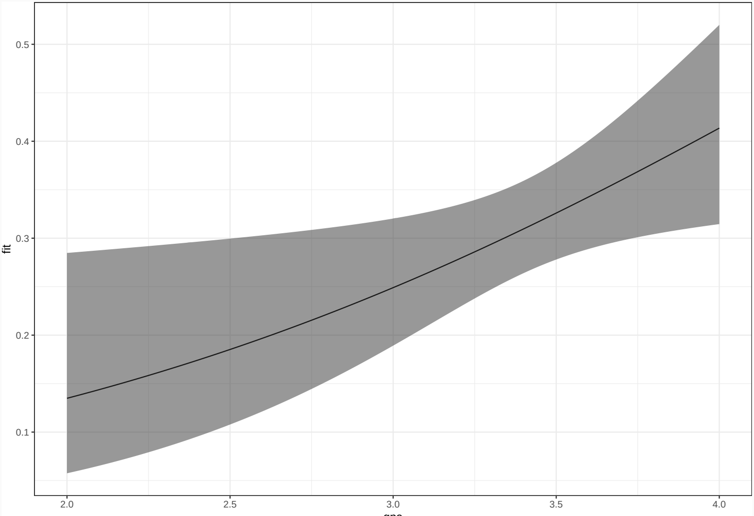
Can we fit a model to predict admission?

```
m1<-lm(admit ~ gre + gpa,
       data = admissions)
```



Let’s try a different approach

```
m2<-glm(admit ~ gre + gpa,
        data = admissions,
       family = "binomial")
```



A generalized linear model

Our linear probability model was:

$$Pr(admit = 1) = \beta_0 + \beta_1 GRE + \beta_2 GPA + \beta_3 Rank + \varepsilon$$

Our logistic regression model takes the form:

$$logit(Pr(admit = 1)) = \beta_0 + \beta_1 GRE + \beta_2 GPA + \beta_3 Rank$$

The logit function is our link between the linear predictor term $X\beta$ and the outcome *admit*.

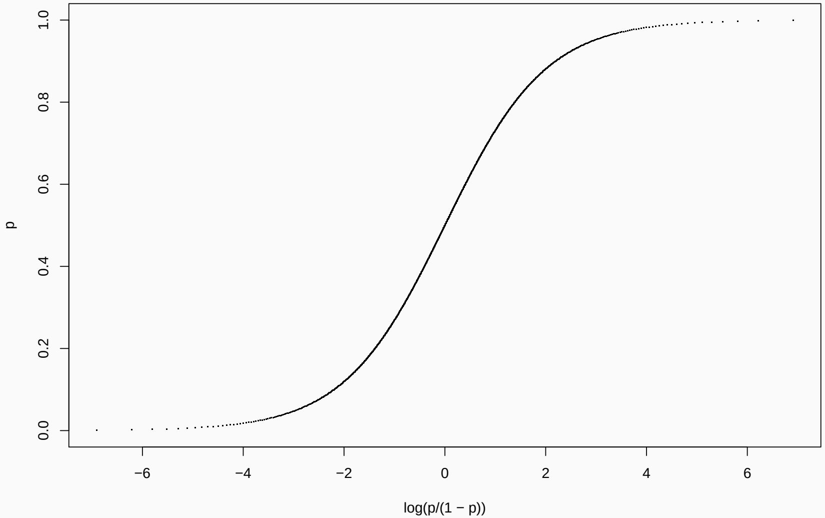
The logit function

The logit function transforms a probability value on $[0, 1]$ to a continuous distribution

$$logit(p) = log\frac{p}{1-p}$$

The logit function

```
p<-seq(0,1,0.001)
plot(log(p/(1-p)), pch = ".", p)
```



Logistic regression is a GLM with a logit link

A generalized linear model with link function g takes the form:

$$g(y) = X\beta$$

For OLS, the link function is the identity function $g(y) = y$

For logistic regression, the link function is the logit function

$$logit(y) = X\beta$$

$$y = logit^{-1}(X\beta)$$

Defining logit and its inverse

$$logit(p) = log\frac{p}{1-p}$$

$$logit^{-1}(x) = \frac{exp(x)}{exp(x)+1}$$

We can use these functions to transform values back and forth from our logit-linear scale and the probability scale.

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Challenge: create two functions in R. logit() and inv_logit() that will compute the logit for any p and the inverse logit for any x.

Defining logit and its inverse

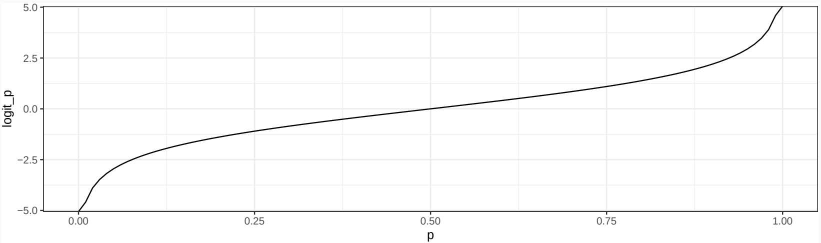
```r
logit<-function(p){
  log(p/(1-p))
  }

inv_logit<-function(x){
  exp(x) / (exp(x) + 1)
}
```

Challenge: practice with these functions

1. Create a tibble with values of p ranging from 0 to 1
2. Use mutate to add a variable called logit_p that is equal to $\text{logit}(p)$
3. Use ggplot to plot p and logit p

Solution

```
#1.
new_dat<-tibble(p = seq(0, 1, by = 0.01))
#2.
new_dat<-new_dat %>%
  mutate(logit_p = logit(p))
#3.
ggplot(new_dat,
       aes(x = p, y = logit_p)) +
  geom_line()
```

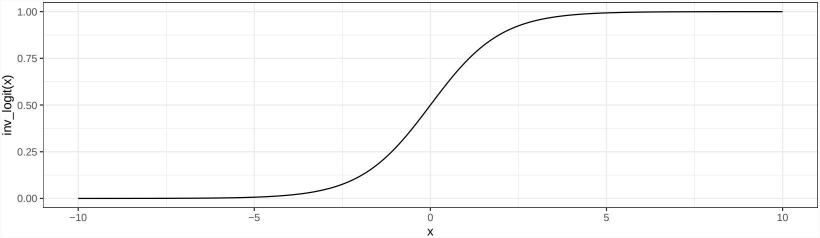


Challenge: practice with these functions

1. Create a tibble with values of x ranging from -10 to 10
2. Use mutate to add a variable called inv_logit_x that is equal to $\mathrm{logit}^{-1}(x)$
3. Use ggplot to plot x and inv_logit_x

Solution

```
#1.
new_dat<-tibble(x = seq(-10, 10, by = 0.1))
#2.
new_dat<-new_dat %>%
  mutate(inv_logit_x = inv_logit(x))
#3.
ggplot(new_dat,
       aes(x = x, y = inv_logit(x))) +
  geom_line()
```



1. Select a model that works for binary outcomes
2. Preserve the linear structure for predictors
3. Map unbounded $(-\infty, \infty)$ linear predictors onto probability $(0, 1)$
4. Map expected probability into binary outcomes

Running logistic models in R: the glm() function

```
m1<-glm(admit ~ gpa,
        data = admissions,
        family = "binomial")

m1_b<-stan_glm(admit ~ gpa,
               data = admissions,
               family = "binomial")
```

What do these models mean?

```r
coef(m1_b)
```

```
## (Intercept)         gpa
##   -4.367779    1.056530
```

This is the direct interpretation in terms of `admit`

$$Pr(admit_i = 1 | gpa) = \mathrm{logit}^{-1}(\beta_0 + \beta_1 gpa_i)$$

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Common interpretations

Because $\mathrm{logit}(p_i) = \log(\frac{p}{1-p}) = \beta_0 + \beta_1 x_i$

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- Odds ratio: e^{β_1}

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Because $\mathrm{logit}(p_i) = \log(\frac{p}{1-p}) = \beta_0 + \beta_1 x_i$

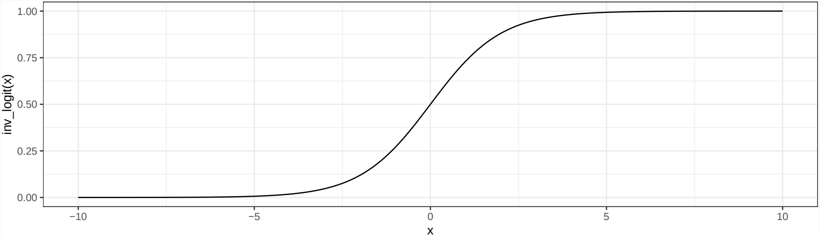
We can interpret β_1 several ways

- Log odds: β_1
- Odds ratio: e^{β_1}
- Probability: $logit^{-1}(x) = \frac{exp(X\beta)}{exp(X\beta)+1}$

The challenge of interpretation

The expected change in probability for *p* for a 1 unit change in *x* (or slope) is not constant!

```
ggplot(new_dat,
       aes(x = x, y = inv_logit(x))) +
  geom_line()
```



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2. For a student with a 3.0 GPA

The challenge of interpretation

1. Compute the expected probability of admission for a student with a 2.0 GPA
2. For a student with a 3.0 GPA
3. Assume the 2.0 and 3.0 student each bumped their GPA up by 0.5. How much does their expected probability of admission change?

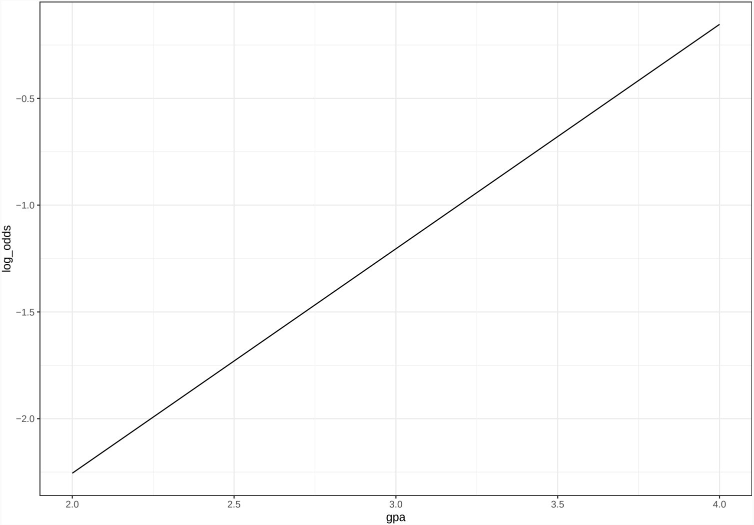
The challenge of interpretation

```r
new_dat<-data.frame(gpa = seq(2, 4, by= 0.25))
new_dat<-new_dat %>%
  mutate(log_odds = predict(m1, new_dat),
         probability = predict(m1, new_dat, type = "response"))
new_dat
```

```
##    gpa   log_odds probability
## 1 2.00 -2.2553699  0.09488728
## 2 2.25 -1.9925927  0.11998284
## 3 2.50 -1.7298155  0.15061118
## 4 2.75 -1.4670383  0.18739319
## 5 3.00 -1.2042611  0.23071805
## 6 3.25 -0.9414839  0.28060069
## 7 3.50 -0.6787068  0.33655000
## 8 3.75 -0.4159296  0.39749117
## 9 4.00 -0.1531524  0.46178656
```

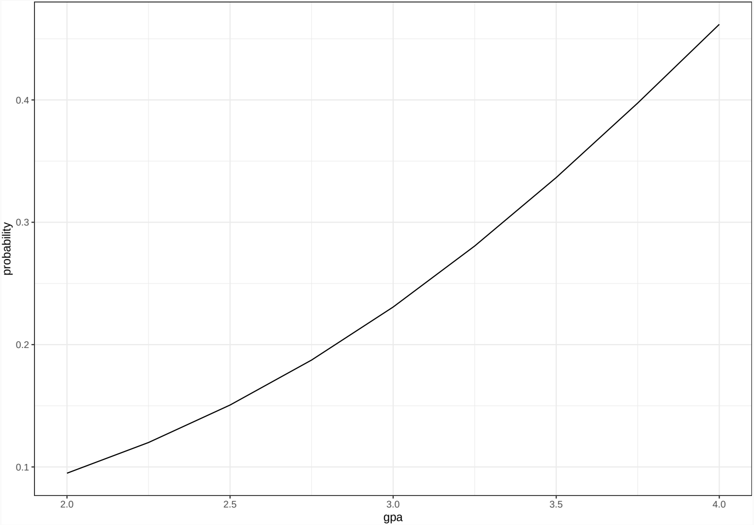
The challenge of interpretation: log odds scale

```
ggplot(new_dat, aes(x = gpa, y = log_odds)) +
  geom_line()
```



The challenge of interpretation: probability scale

```
ggplot(new_dat, aes(x = gpa, y = probability)) +
  geom_line()
```



Break

Lab: Let's fit a more complex model

What else might predict admission?

```
head(admissions)
```

```
##   admit gre  gpa rank
## 1     0 380 3.61    3
## 2     1 660 3.67    3
## 3     1 800 4.00    1
## 4     1 640 3.19    4
## 5     0 520 2.93    4
## 6     1 760 3.00    2
```

1. Write out a model
2. Fit the model
3. Think about revising the model
4. Compare model fits
5. Interpret the model

Who was most (and least) likely to die on the Titanic? Use `~/hw/data/titanic.csv` for this one.

1. Write out a model
2. Fit the model
3. Think about revising the model
4. Compare model fits
5. Interpret the model