Intermediate statistics: introduction

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Introductions: What are you planning to do with statistical models?

Before we begin

Remember: All models are wrong, some are useful.

What we will cover

- How to explore, visualize, and model diverse kinds of data with an emphasis on generalized linear models

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- How to explore, visualize, and model diverse kinds of data with an emphasis on generalized linear models
- How to program in R
- Developing a workflow for producing replicable quantitative social science
- Some advanced topics that are relevant for the kinds of data we're dealing with in the course

My general approach to data analysis

1. Explore and visualize data

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2. Fit models

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My general approach to data analysis

1. Explore and visualize data
2. Fit models
3. Assess model fit
4. Interpret and describe results through simulation

Review the syllabus

https://f-edwards.github.io/intermediate_stats/

1. Install R

cran.r-project.org

2. Install RStudio

posit.co/download/rstudio-desktop/

3. File structure and project organization basics
4. R console basics

Break

The Generalized Linear Model

We know we can model data as:

$$y = \beta_0 + \beta_1 x_1 \cdots \beta_n x_n + \varepsilon$$

The linear model

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Or, more succinctly:

$$y = \mathsf{X}\beta + \varepsilon$$

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Or, more succinctly:

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Where the likelihood for the outcome conditional on the data takes the form:

$$Y|X \sim Normal(\mu, \sigma^2)$$

The linear model:

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Can be written as a more general formulation for a likelihood function f

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$$Y|X \sim f(\mu, \sigma^2)$$

Now we can extend the (very) useful linear model to data with discrete outcomes

An expected value $E(Y|X) = \mu$

A linear predictor:

$$\mathrm{X}\beta$$

Generalizing the linear model

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A link function g

$$g(\mu) = \mathsf{X}\beta$$

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An expected value $E(Y|X) = \mu$

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A link function g

$$g(\mu) = \mathsf{X}\beta$$

$$\mu = g^{-1}(\mathsf{X}\beta)$$

From OLS to GLM

OLS:

$$Y|X \sim Normal(\mu, \sigma^2)$$

GLM, for a likelihood function f with parameters θ:

$$Y|X \sim f(\theta)$$

Models we'll consider this semester

- Binary data: logistic models

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- Binary data: logistic models
- Categorical data: Multinomial models
- Count data: Poisson and negative binomial models

Returning to the linear model

What do we know about the linear regression model?

$$y = \mathsf{X}\beta + \varepsilon$$

$$\varepsilon \sim Normal(0, \sigma^2)$$

OR

$$\mu = \mathsf{X}\beta$$

$$y \sim Normal(\mu, \sigma^2)$$

Let’s analyze some data?

Two ways to access course data

- All data is accessible through the the course website (see the data link, or data folder on the GitHub page)

Read in

```r
library(tidyverse)
### data available in intermediate_stats/data/revenue_dat.csv
cj_budgets <- read_csv("http://tinyurl.com/revenuedata1")
```

About the data

It documents police involved deaths, demographics, and local government budgets at the county-level for two time periods, 2007-11 and 2012-16. Sources include Fatal Encounters, American Community Survey 5-year data, Annual Survey of State and Local Government Finance, and Uniform Crime Reports.

Evaluate the structure of the data

```r
head(cj_budgets)
```

```
## # A tibble: 6 x 26
##   year_range fips_st fips_cnty deaths  exp_tot exp_correction exp_police rev_tot
##   <chr>      <chr>   <chr>      <dbl>    <dbl>          <dbl>      <dbl>   <dbl>
## 1 2007-2011  01      001            3 49742600        2101800    9306200  5.65e7
## 2 2007-2011  01      005            1 28588200        1037880    5537840  3.37e7
## 3 2007-2011  01      007            0 13036120          80600    2421720  1.36e7
## 4 2007-2011  01      009            0 36644480        1703760    6853480  3.33e7
## 5 2007-2011  01      011            0 10940520              0    2285320  1.18e7
## 6 2007-2011  01      013            1 30533760         487320    4067200  2.50e7
## # i 18 more variables: rev_gen_ownsource <dbl>, rev_int_gov <dbl>,
## #   rev_prop_tax <dbl>, rev_tax <dbl>, pop_tot <dbl>, pop_wht <dbl>,
## #   pop_blk <dbl>, pop_lat <dbl>, pop_pct_pov <dbl>, pop_pct_deep_pov <dbl>,
## #   pop_med_income <dbl>, pop_pc_income <dbl>, violent.yr <dbl>,
## #   property.yr <dbl>, murder.yr <dbl>, ft_sworn <dbl>, cbsa <dbl>,
## #   metroname <chr>
```

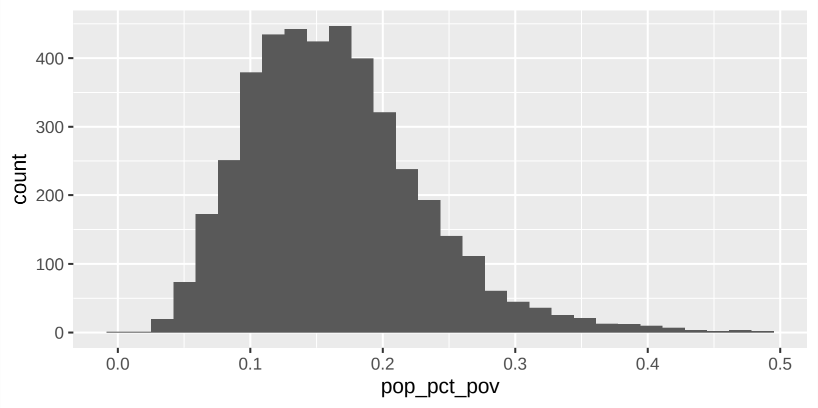
Descriptives

```
summary(cj_budgets)
```

```
##   year_range          fips_st            fips_cnty              de
##  Length:4286        Length:4286        Length:4286        Min.  
##  Class :character   Class :character   Class :character   1st Qu
##  Mode  :character   Mode  :character   Mode  :character   Median
##                                                           Mean  
##                                                           3rd Qu
##                                                           Max.  
##
##     exp_tot          exp_correction       exp_police
##  Min.   :2.531e+05   Min.   :0.000e+00   Min.   :1.221e+04
##  1st Qu.:1.965e+07   1st Qu.:1.996e+05   1st Qu.:1.679e+06
##  Median :4.972e+07   Median :1.280e+06   Median :4.257e+06
##  Mean   :3.614e+08   Mean   :9.856e+06   Mean   :3.005e+07
##  3rd Qu.:1.539e+08   3rd Qu.:4.076e+06   3rd Qu.:1.241e+07
##  Max.   :1.177e+11   Max.   :1.747e+09   Max.   :5.623e+09
```

Visualize the distribution of deep poverty across counties with ggplot

```r
ggplot(cj_budgets, aes(x = pop_pct_pov)) + geom_histogram()
```

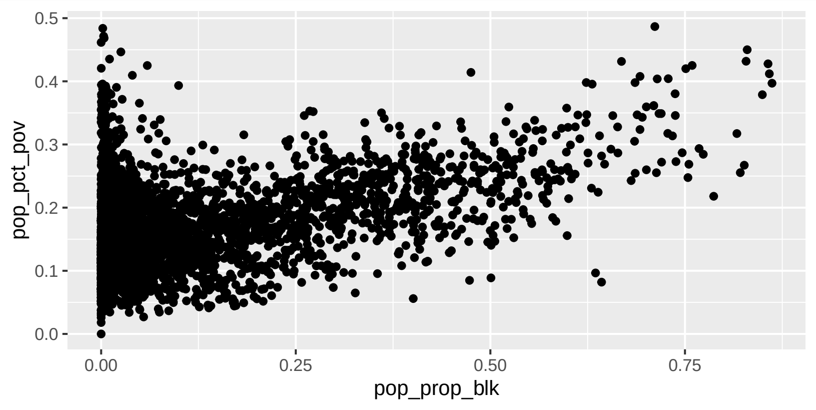


Create a new variable using mutate()

```r
cj_budgets <- cj_budgets %>%
    mutate(pop_prop_blk = pop_blk/pop_tot)
```

Visualize a bivariate relationship with ggplot

```
ggplot(cj_budgets, aes(y = pop_pct_pov, x = pop_prop_blk)) + geom_point()
```



Fitting a linear model with lm()

```r
model_1 <- lm(pop_pct_pov ~ pop_prop_blk, data = cj_budgets)
```

Display the model fit

```r
summary(model_1)
```

```
##
## Call:
## lm(formula = pop_pct_pov ~ pop_prop_blk, data = cj_budgets)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.18870 -0.04002 -0.00586  0.03362  0.33849
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.144985   0.001090  133.05   <2e-16 ***
## pop_prop_blk 0.195611   0.006007   32.56   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.05978 on 4284 degrees of freedom
## Multiple R-squared:  0.1984, Adjusted R-squared:  0.1982
## F-statistic:  1060 on 1 and 4284 DF,  p-value: < 2.2e-16
```

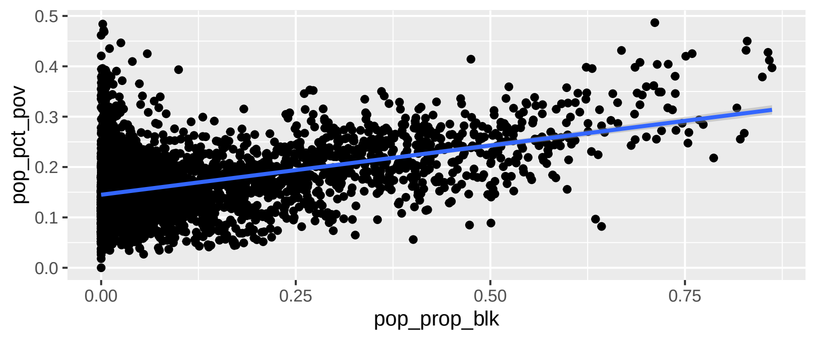
Display the model fit (nicer)

```r
library(broom)
tidy(model_1)
```

```
## # A tibble: 2 x 5
##   term        estimate std.error statistic   p.value
##   <chr>          <dbl>     <dbl>     <dbl>     <dbl>
## 1 (Intercept)    0.145   0.00109     133.  0
## 2 pop_prop_blk   0.196   0.00601      32.6 4.85e-208
```

Visualize the model fit

```
ggplot(cj_budgets, aes(y = pop_pct_pov, x = pop_prop_blk)) + geom_point() + geom_smooth(method = "lm",
    formula = y ~ x)
```



HW 1 guidelines

HW tips

- Work together!
- Google it: StackOverflow will become your best friend
- Accept that this is hard and you will probably struggle with it