Introducing..... Linear Regression!

Frank Edwards

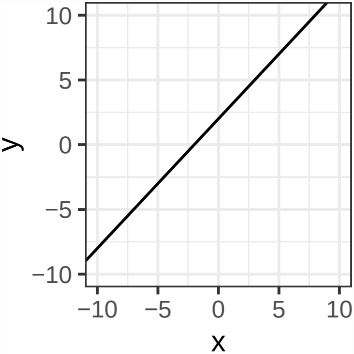
Lines

We can define a line as:

$$y = mx + b$$

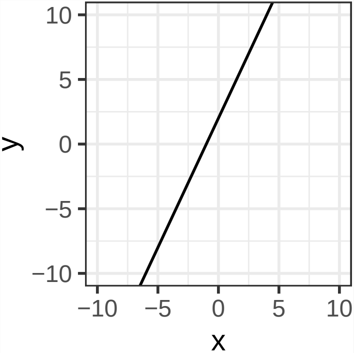
Where m is the slope and b is the y-intercept.

$y = x + 2$



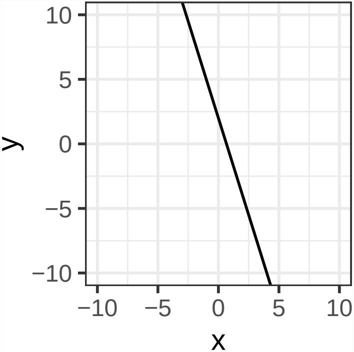
Lines: slopes

$y = 2x + 2$

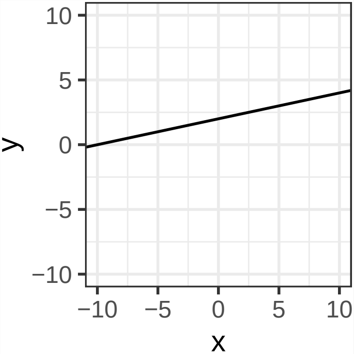


Lines: slopes

$y = -2x + 2$

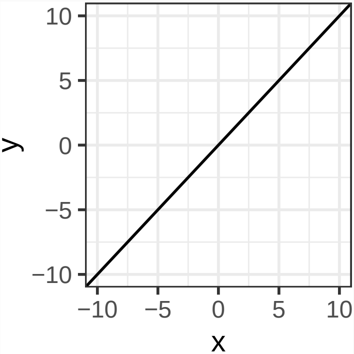


$y = 0.2x + 2$



Lines: intercepts

$y = x + 0$



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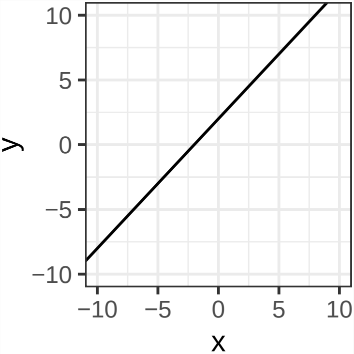
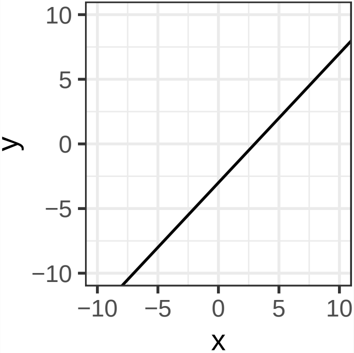


$y = x - 3$



The linear regression model

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What does m describe?

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What does b describe

- The location of y when x = 0

The linear regression model: expected value

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What does β_0 describe?

What does β_1 describe?

The error term in linear regression

We can describe the relationship between a predictor variable X and an outcome variable Y with the line:

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

Where β_0 is the y-intercept of the line, β_1 is the slope of the line, and ε is the error between the fitted line and the coordinates (X, Y)

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ε: The distance between the line $y = \beta_0 + \beta_1 X$ and the actual observed values of y.

The line $y = \beta_0 + \beta_1 X$ provides an expected value for y based on the values of x.

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- When $x = 2$?

The linear regression model and prediction

We put a $\hat{hat}$ on variables to indicate that they are estimated from the data, or predicted.

A regression line predicts values Y, $\hat{Y}$ with the equation:

$\hat{Y} = \beta_0 + \beta_1 X$

and the residual, or prediction error is the difference between the observed and predicted values of Y

$\varepsilon = Y_{obs} - \hat{Y}$

Understanding the regression line for real data

```
## # A tibble: 10 x 2
##         x      y
##     <dbl>  <dbl>
##  1 -0.311 -0.237
##  2  0.932  0.883
##  3  1.04   1.62
##  4  3.55   3.41
##  5  0.507  0.254
##  6  0.655  1.11
##  7  1.39   1.29
##  8  0.868  0.477
##  9  0.859  0.561
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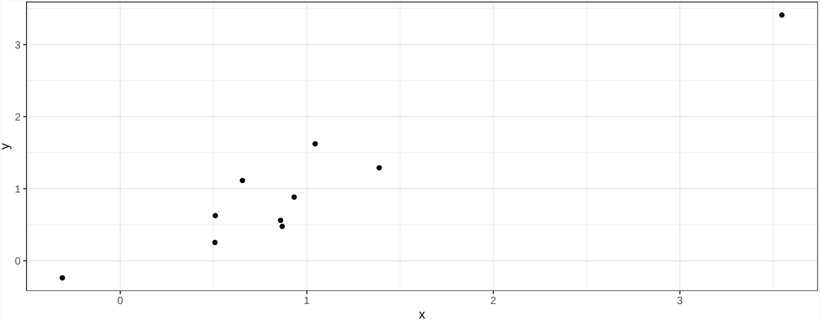
- Estimate $\hat{Y}$. Recall that $\hat{Y} = \beta_0 + \beta_1 X$
- Estimate ε. Recall that $\varepsilon = Y - \hat{Y}$

Understanding the regression line

$\beta_0 = 0.05$, $\beta_1 = 0.95$



Understanding the regression line: adding the fit

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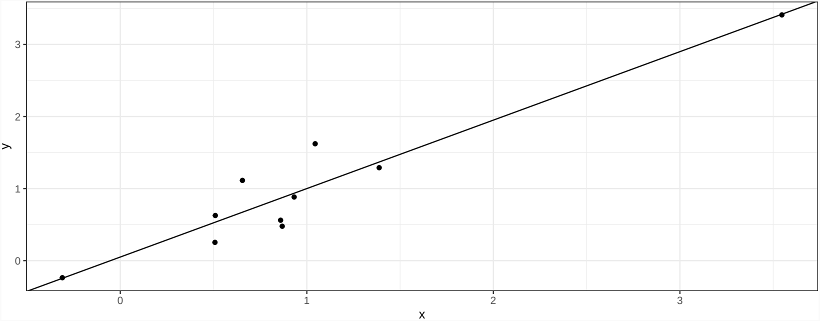

Understanding the regression line: adding $\hat{y}$

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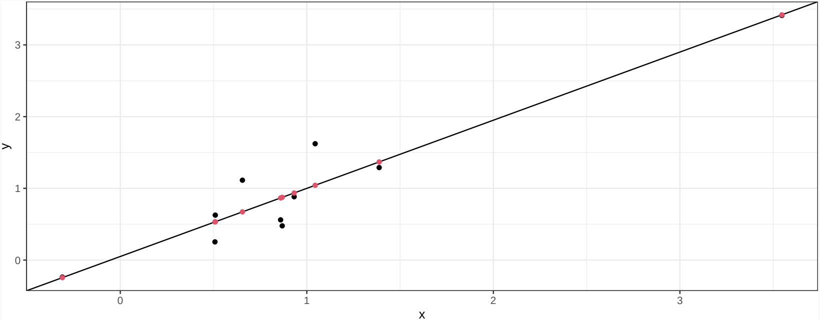

Understanding the regression line: adding ε

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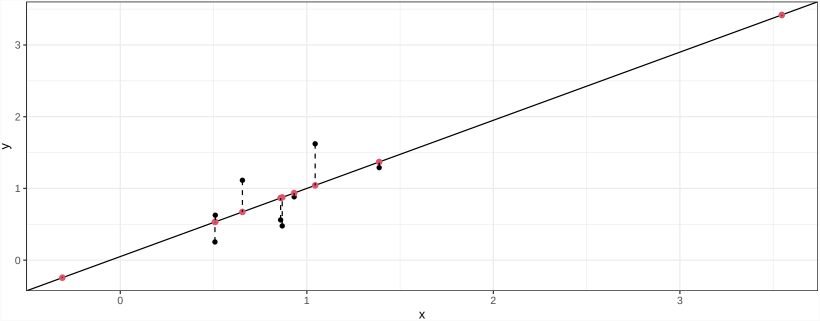

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In other words, we solve for the values of β_0 and β_1 that result in the smallest possible value for SSR

$$\mathrm{SSR} = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 X)^2$$

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Estimating a regression model in R, the basics

Let's use **iris** to estimate this regression model:

$$\text{Sepal.Length} = \beta_0 + \beta_1 \text{Petal.Length} + \varepsilon$$

```
my_cool_model<-lm(Sepal.Length ~ Petal.Length,
                  data = iris)

my_cool_model
```

```
##
## Call:
## lm(formula = Sepal.Length ~ Petal.Length, data = iris)
##
## Coefficients:
##  (Intercept)  Petal.Length
##       4.3066        0.4089
```

What does this model tell us?

- $\beta_0 = 4.3$

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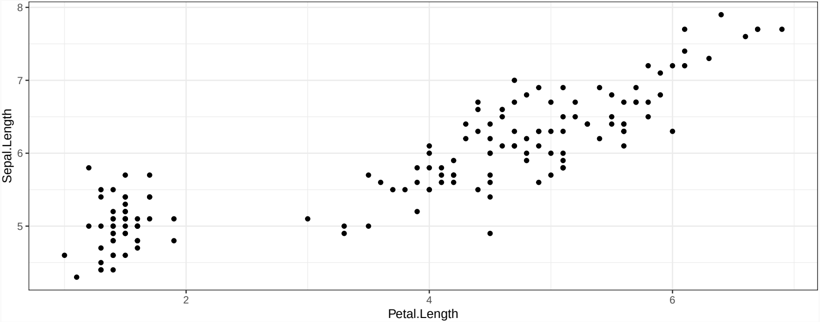
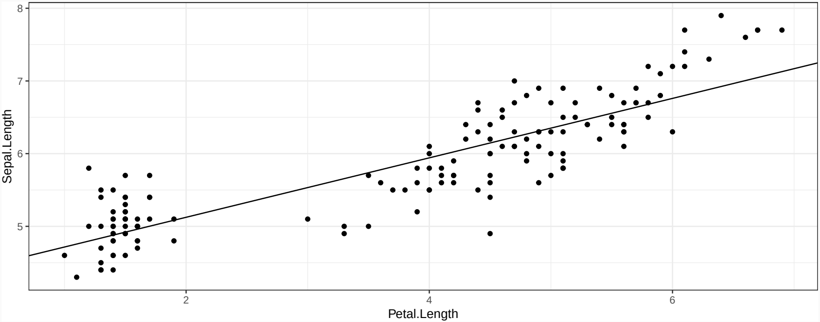
$$E(\text{Sepal.Length}) = \beta_0 + \beta_1 \text{Petal.Length}$$

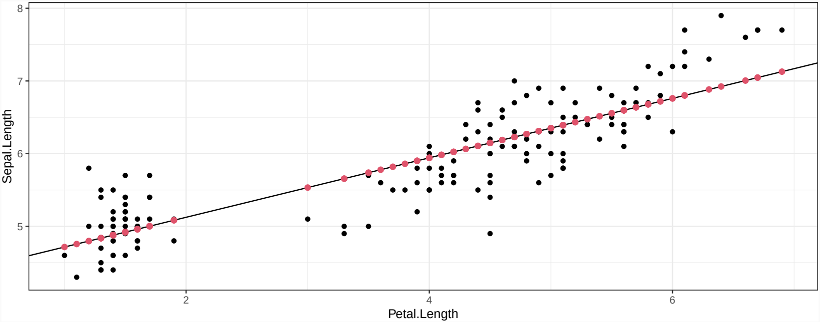
Visualized: observed data



Visualized: regression line



Visualized: expected values (yhat)



Visualized: residuals (epsilon)