The local energy for the helium molecule trial wavefunction. The trial wavefunction has the form

$$\Psi(_{1,2}) = e^{-2r_1} e^{-2r_2} e^{\frac{r_12}{2(1+\alpha r_{12})}}.$$

We abbreviate this as follows

$$\Psi(_{1,2}) = \phi_1 \phi_2 f_{12}$$

where $\phi_1 = e^{-2r_1}$ etcetera.

Now we use the following rules

$$\nabla_1 \phi_1 = \hat{}_1 \phi_1',$$

where $\phi'(1) = \partial \phi_1 / \partial r_1$, and $\hat{}_1$ is a unit vector along $_1$.

Furthermore

$$\nabla^2 \phi_1 = \frac{2}{r_1} \phi_1' + \frac{\partial^2 \phi_1}{\partial r_1^2}.$$

Similarly, we have

$$\nabla_1 f_{12} = \hat{}_{12} f'(r_{12}) and$$

$$\nabla^2 f_{12} = \frac{2}{r_{12}} f_{12}' + \frac{\partial^2 f_{12}}{\partial r_{12}^2}.$$

We now can write immediately

$$\left(\nabla_1^2 + \nabla_2^2\right) \psi(_{1,2}) = \frac{2}{r_1} \phi_1' \phi_2 f_{12} + \frac{2}{r_2} \phi_1 \phi_2' f_{12} + \phi_1'' \phi_2 f_{12} + \phi_1 \phi_2'' f_{12} + 2\left[\left(\hat{}_1 \phi_1' \phi_2 - \hat{}_2 \phi_1 \phi_2'\right) \cdot \hat{}_{12}\right] f_{12}' + \frac{4}{r_{12}} \phi_1 \phi_2 f_{12}' + 2 \phi_1 \phi_2 f_{12}''.$$

Now work out the derivatives:

$$\phi_1' = -2\phi_1; \qquad \phi_1'' = -2,$$

and similarly for ϕ_2. Furthermore

$$
\begin{aligned}
f'_{12} &= \frac{1}{2(1+\alpha r_{12})^2} f_{12} \\
f''_{12} &= \left[\frac{1}{4(1+\alpha r_{12})^4} - \frac{\alpha}{(1+\alpha r_{12})^3}\right] f_{12}.
\end{aligned}
$$

Putting all the pieces together, and putting the factor $-1/2$ in front of the expression, we then obtain for the kinetic energy K:

$$
K = \frac{2}{r_1} + \frac{2}{r_2} - 4 + \frac{(\hat{}_1 - \hat{}_2) \cdot \hat{}_{12}}{(1+\alpha r_{12})^2} - \frac{1}{r_{12}(1+\alpha r_{12})^3} - \frac{1}{4(1+\alpha r_{12})^4}.
$$