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"""XC density kernels for response function calculations."""
from abc import ABC, abstractmethod
from dataclasses import dataclass
import numpy as np
from gpaw.response import ResponseGroundStateAdapter, ResponseContext
from gpaw.response.pair functions import SingleQPWDescriptor
from gpaw.response.localft import LocalFTCalculator
from gpaw.response.fxc_kernels import AdiabaticFXCCalculator
@dataclass
class DensityXCKernel(ABC):
    gs: ResponseGroundStateAdapter
    context: ResponseContext
    functional: str
    def __post_init__(self):
        assert self.gs.nspins == 1
    @staticmethod
    def from_functional(gs, context, functional, **kwargs):
        """Factory function creating DensityXCKernels.
        Choose between ALDA, Bootstrap and LRalpha (long-range kernel), where
        alpha is a user specified parameter (for example functional='LR0.25').
        if functional[0] == 'A':
            return AdiabaticDensityKernel(gs, context, functional, kwargs)
        elif functional[:2] == 'LR':
            return LRDensityKernel(gs, context, functional)
        elif functional == 'Bootstrap':
            return BootstrapDensityKernel(gs, context, functional)
        raise ValueError(
             'Invalid functional for the density-density xc kernel:'
            f'{functional}')
         call (self, qpd: SingleQPWDescriptor, chi0 wGG=None):
        self.context.print(f'Calculating {self}')
        return self.calculate(qpd=qpd, chi0_wGG=chi0_wGG)
    @abstractmethod
    def __repr__(self):
    """String representation of the density xc kernel."""
    @abstractmethod
    def calculate(self, *args, **kwargs):
        """Calculate the xc kernel.
        Since the exchange-correlation kernel is going to be rescaled according
        to the bare Coulomb interaction,
        K_xc(q) = v^{-1/2}(q) K_xc(q) v^{-1/2}(q)
        and that the Coulomb interaction in the optical q→0 limit leaves the
        long-range q-dependence out, see gpaw.response.coulomb kernels,
        v(q) = 4\pi/|G+q| \rightarrow 4\pi for G==0 and 4\pi/|G| otherwise if q==0,
        we need to account to account for it here. That is,
        For q\rightarrow 0, the head and wings of the returned K xc(q) needs to be
        rescaled according to K \times C(q) \rightarrow q \times K \times C(q) q
@dataclass
class AdiabaticDensityKernel(DensityXCKernel):
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rshe_kwargs: dict
    def __post_init__(self):
        super().__post_init__()
        localft_calc = LocalFTCalculator.from_rshe_parameters(
            self.gs, self.context, **self.rshe_kwargs)
        self.fxc_calc = AdiabaticFXCCalculator(localft_calc)
         repr (self):
        return f'{self.functional} kernel'
    def calculate(self, qpd: SingleQPWDescriptor, **ignored):
        fxc kernel = self.fxc calc(self.functional, '00', qpd)
        Kxc_GG = fxc_kernel.get_Kxc_GG()
        if qpd.optical_limit:
            Kxc\_GG[0, :] = 0.0
            Kxc GG[:, 0] = 0.0
        return Kxc GG
@dataclass
class LRDensityKernel(DensityXCKernel):
    def post init (self):
        super().__post_init__()
        self.alpha = float(self.functional[2:])
    def __repr__(self):
    return f'LR kernel with alpha = {self.alpha}'
    def calculate(self, gpd: SingleQPWDescriptor, **ignored):
        Kxc_sGG = calculate_lr_kernel(qpd, alpha=self.alpha)
        return Kxc_sGG[0]
@dataclass
class BootstrapDensityKernel(DensityXCKernel):
         _repr__(self):
        return 'Bootstrap kernel'
    def calculate(self, qpd, *, chi0_wGG):
        Kxc sGG = get bootstrap kernel(gpd, chi0 wGG, self.context)
        return Kxc_sGG[0]
def calculate_lr_kernel(qpd, alpha=0.2):
    """Long range kernel: fxc = \alpha / q+G/2"""
    assert qpd.optical limit
    f_G = np.zeros(len(qpd.G2_qG[0]))
    f^{-}G[0] = -alpha
    f_G[1:] = -alpha / qpd.G2_qG[0][1:]
    return np.array([np.diag(f_G)])
def get_bootstrap_kernel(qpd, chi0_wGG, context):
    """ Bootstrap kernel (see below) ""
    if context.comm.rank == 0:
        chi0 GG = chi0 wGG[0]
        if context.comm.size > 1:
            # If size == 1, chi0_GG is not contiguous, and broadcast()
            # will fail in debug mode. So we skip it until someone
            # takes a closer look.
            context.comm.broadcast(chi0_GG, 0)
    else:
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nG = qpd.ngmax
        chi0_GG = np.zeros((nG, nG), complex)
        context.comm.broadcast(chi0_GG, 0)
    return calculate_bootstrap_kernel(qpd, chi0_GG, context)
def calculate_bootstrap_kernel(qpd, chi0_GG, context):
    """Bootstrap kernel PRL 107, 186401"""
    p = context.print
    if qpd.optical_limit:
        v_G = np.zeros(len(qpd.G2_qG[0]))
        v_{G}^{-}[0] = 4 * np.pi
        v_{G[1:]} = 4 * np.pi / qpd.G2_qG[0][1:]
        v_G = 4 * np.pi / qpd.G2_qG[0]
    nG = len(v_G)
    K GG = np.\overline{diag}(v G)
    Kxc_GG = np.zeros((nG, nG), dtype=complex)
    dminv_GG = np.zeros((nG, nG), dtype=complex)
    for iscf in range(120):
        dminvold_GG = dminv_GG.copy()
        Kxc\_GG = K\_GG + Kxc\_GG
        chi_GG = np.dot(np.linalg.inv(np.eye(nG, nG)
                                        - np.dot(chi0_GG, Kxc_GG)), chi0_GG)
        dminv_GG = np.eye(nG, nG) + np.dot(K_GG, chi_GG)
        alpha = dminv_GG[0, 0] / (K_GG[0, 0] * chi0_GG[0, 0])
        Kxc_GG = alpha * K_GG
p(iscf, 'alpha =', alpha, flush=False)
        error = np.abs(dminvold_GG - dminv_GG).sum()
        if np.sum(error) < 0.1:
            p('Self consistent fxc finished in %d iterations !' % iscf)
            break
        if iscf > 100:
            p('Too many fxc scf steps !')
    return np.array([Kxc_GG])
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