

# Zeroth Order Optimization Based Black-box Attacks to Deep Neural Networks

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# Topics

1. Introduction
2. Gradient Smoothing and Zero Order Methods
3. Methods
4. Setup and Evaluation
5. Results
6. Conclusion



# Black-Box vs White-Box

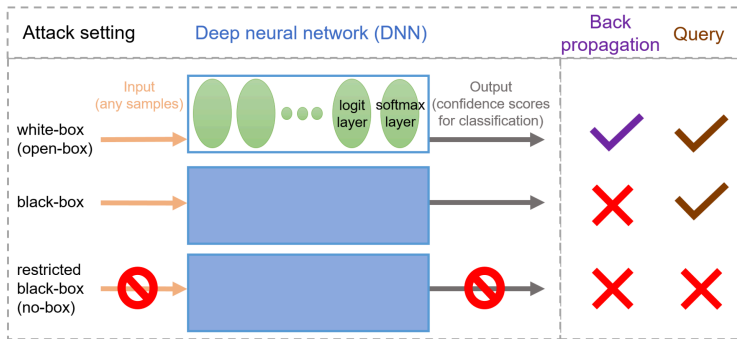


Figure: Types of white-box and black-box attack settings

# Adversarial attack as optimization problem

By defining decision function for model  $\mathcal{M}$  as  $F_{\mathcal{M}}(x) : \chi \rightarrow [0; 1]^m$  and constraint  $C(x) = \arg \min_{i \in \{1, \dots, m\}} F_{\mathcal{M}}(x)$ , we get Carlini and Wagner formulation:

$$\begin{aligned} \min \quad & ||\delta|| \\ \text{sub.to} \quad & C(x + \delta) = t \\ & (x + \delta) \in [0, 1]^n \end{aligned}$$

# Loss functions

New loss function  $f : \mathcal{X} \rightarrow \mathbb{R}$  introduced better suited for optimization algorithms:

- ▶ Targeted attack

$$f_t(x) = \max\{\max_{i \neq t} \log[F_{\mathcal{M}}(x)]_i - \log[F_{\mathcal{M}}(x)]_t, -k\}$$

- ▶ Untargeted attack

$$f(x) = \max\{\log[F_{\mathcal{M}}(x)]_{t_0} - \max_{i \neq t_0} \log[F_{\mathcal{M}}(x)]_i, -k\}$$

# Constraining perturbation set

1. Add a regularization term to the loss function that takes into account the norm of  $\delta$

$$\begin{array}{ll} \min & \|\delta\| + c \cdot f(x + \delta) \\ \text{sub.to} & (x + \delta) \in [0, 1]^n \end{array}$$

2. Modify the feasible region  $\chi = [0; 1]^n$  by doing an intersection with the  $L_p$  Ball of radius set equal to a parameter  $\epsilon$  and centered in the original input  $\mathcal{B}_\epsilon^{(p)}(x)$

$$\begin{array}{ll} \min & f(x + \delta) \\ \text{sub.to} & (x + \delta) \in [0, 1]^n \cap \mathcal{B}_\epsilon^{(p)}(x) \end{array}$$

# General Idea

- ▶ We want to solve the constrained optimization problem overcoming the hard computational requirements it presents.
- ▶ Under regularity hypothesis, the gradient of a function can be approximated using zero-th order information.
- ▶ The loss function may be highly irregular but can be smoothed by considering

$$f_\nu(x) = \mathbb{E}_u[f(x + \nu u)]$$

for some  $\nu \in (0; \infty)$  and  $u \sim \mathcal{N}(0, I_d)$ .



# Gradient Smoothing Approach

We want  $f_\nu(x)$  to be an accurate approximation of  $f(x)$  and to show higher regularity.

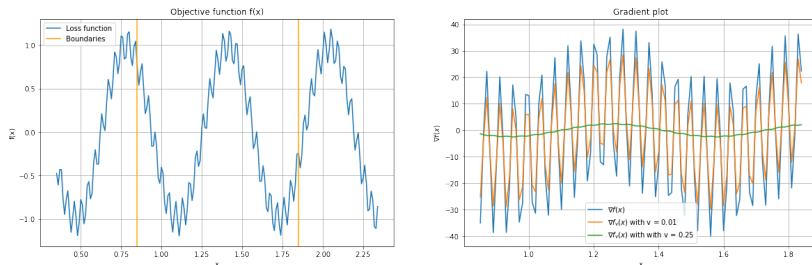
It can be proved that, for any  $\nu > 0$ ,  $f_\nu$  is differentiable and its gradient is Lipschitz-continuous.

In [Nesterov2015] it is also shown that if  $f$  is Lipschitz-continuous with constant  $L$ , then

$$\|f_\nu(x) - f(x)\| \leq \frac{\nu^2}{2}Ln$$
$$\|\nabla f_\nu(x) - \nabla f(x)\| \leq \frac{\nu^2}{2}L(n+3)^{3/2}$$

# The Gaussian smoothing parameter $\nu$ . Pt1

Depending on the choice of  $\nu$ ,  $f_\nu$  can approximate as tightly as wanted the function  $f$  or can show a much more regular behaviour.



**Figure:** Left: the loss function; right: the gradient of  $f(x)$  and the approximated gradient of  $f_\nu(x)$  at different value of  $\nu$

# Zero-th Order Stochastic Oracle - Regularity Assumptions

For any  $x \in \mathbb{R}^d$ , the zero-th order stochastic oracle outputs an estimator  $F(x, \xi)$  of  $f(x)$ . We assume that:

1.  $F(x, \xi)$  is an unbiased estimator of  $f(x)$

$$\mathbb{E}_{\xi}[F(x, \xi)] = f(x)$$

$$\mathbb{E}_{\xi}[\nabla_x F(x, \xi)] = \nabla f(x)$$

$$\mathbb{E}_{\xi}[\|\nabla_x F(x, \xi) - \nabla f(x)\|^2] \leq \sigma^2$$

2. Function  $F$  has LCG with constant  $L$ , almost surely for any  $\xi$ .
3. The feasible set  $\chi$  is bounded such that  $\max_{x, y \in \chi} \|x - y\| \leq D_{\chi}$  for some  $D_{\chi} \geq 0$ .

# Random Gradient-Free Oracles

Given  $u \sim \mathcal{N}(0, I_d)$ , the *stochastic gradient* of  $f$  is:

$$G_\nu(x, \xi, u) = \frac{F(x + \nu u, \xi) - F(x, \xi)}{\nu} u$$

where  $F$  is a zero-th order stochastic oracle of  $f$ . In particular we can use  $F(x, \xi) = f(x)$ .

Thanks to the *Stein Identity*, under Assumption 1, it can be shown that

$$\nabla f_\nu(x) = \mathbb{E}_{u, \xi}[G_\nu(x, \xi, u)].$$

By increasing  $m_k$  the variance of the estimate can be reduced as:

$$\mathbb{E}_{u, \xi}[\|G_\nu^k - \nabla f_\nu(x_{k-1})\|^2] \leq \frac{1}{m_k} \mathbb{E}_\xi[\|G_\nu^{k,j} - \nabla f_\nu(x_{k-1})\|^2].$$

# ZOO

The problem is formulated as:

$$\min_{x \in [0,1]^p} \|x - x_0\|_2^2 + c \cdot f(x, t). \quad (1)$$

An approximated gradient is computed as:

$$\hat{g}_i := \frac{\partial f(x)}{\partial x_i} \approx \frac{f(x + he_i) - f(x - he_i)}{2h}, \quad (2)$$

where  $h$  is a small constant and  $e_i$  is a standard basis vector with only the  $i$ -th component set to 1.

$$\hat{h}_i := \frac{\partial^2 f(x)}{\partial x_{ii}^2} \approx \frac{f(x + he_i) - 2f(x) + f(x - he_i)}{h^2}. \quad (3)$$

# ZOO - Main algorithm (ADAM)

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**Algorithm 1:** ZOO with ADAM

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**Input:**  $\eta, \beta_1, \beta_2, \epsilon$

**Initialize:**  $M = \mathbf{0}, T = \mathbf{0}, v = \mathbf{0}, M, T, v \in \mathbb{R}^p$

**while** *not converged* **do**

    Randomly pick coordinate  $i \in \{1, \dots, p\}$ ;

    Estimate  $\hat{g}_i$ ;

$T_i \leftarrow T_i + 1$ ;

$M_i \leftarrow \beta_1 M_i + (1 - \beta_1) \hat{g}_i$ ;

$v_i \leftarrow \beta_2 v_i + (1 - \beta_2) \hat{g}_i^2$ ;

$\hat{M}_i = \frac{M_i}{(1 - \beta_1^{T_i})}$ ;  $\hat{v}_i = \frac{v_i}{(1 - \beta_2^{T_i})}$ ;

$\delta^* = -\eta \frac{\hat{M}_i}{\sqrt{\hat{v}_i} + \epsilon}$ ;

$x_i \leftarrow x_i + \delta^*$ ;

**end**

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# ZOO - Main algorithm (Newton)

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## Algorithm 2: ZOO with Newton

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**Input:**  $\eta$

**while** *not converged* **do**

    Randomly pick  $B$  coordinates  $i \in \{1, \dots, p\}$ ; Estimate  $\hat{g}_B$   
    and  $\hat{h}_B$ ; **if**  $\hat{h}_B < 0$  **then**

$\delta^* \leftarrow -\eta \hat{g}_B$ ;

**else**

$\delta^* \leftarrow -\eta \frac{\hat{g}_B}{\hat{h}_B}$ ;

**end**

$x_B \leftarrow x_B + \delta^*$ ;

**end**

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# High Dimensional ZOO

For large images we reduce the attack space by introducing a dimension reduction transformation  $D(y)$  where  $y \in \mathbb{R}^m$ ,  $\text{range}(D) \in \mathbb{R}^n$ ,  $m < n$  and using  $D(y)$  to replace  $\delta$ :

$$\min_{x_0 + D(y) \in [0,1]^p} \|D(y)\|_2^2 + c \cdot f(x_0 + D(y), t).$$

To further improve the attack three different techniques are also introduced:

- ▶ Hierarchical Attack;
- ▶ Importance Sampling;
- ▶ ADAM State Reset.



## ZOO - Convergence rate

Considering a non-smooth convex function  $f$  with Lipschitz continuous gradient with constant  $L$ , Nesterov proved:

$$\mathbb{E}_u (||\hat{g}_h(x)||^2) \leq \frac{h^2}{8} L^2 (n+6)^3 + 2(n+4) ||\nabla f(x)||^2$$

Convergence rate for a zeroth-order stochastic gradient method applied to non-smooth

- ▶ convex problems:  $O\left(\frac{n}{\sqrt{k}}\right)$
- ▶ non-convex problems:  $O\left(\sqrt{\frac{n}{k}}\right)$

# Black-Box Frank-Wolfe

- ▶ Variant of Frank-Wolfe optimization algorithm
- ▶ Uses momentum term
- ▶ Projection free algorithm
- ▶ Guaranteed  $O(1/\sqrt{T})$  with  $T$  number of epochs
- ▶ Guaranteed linear query complexity in data dimensions  $d$

# Black-Box Frank-Wolfe - Main algorithm

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**Algorithm 3:** Black-Box Frank-Wolfe algorithm

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**Input:**  $x_0$ ,  $T$  number of iterations,  $\{\gamma_t\}$  list of step sizes,  $b$  number of gradient estimation iterations,  $\delta$  gradient smoothing parameter

**Initialize:**  $m_t = \text{GRAD\_EST}(x_0, b, \delta)$

**for**  $t = 0, \dots, T - 1$  **do**

$q_t = \text{GRAD\_EST}(x_t, b, \delta)$   
     $m_t = \beta \cdot m_{t-1} + (1 - \beta) \cdot q_t$   
     $v_t = \arg \min_{x \in \mathcal{X}} \langle x, m_t \rangle$  // LMO  
     $d_t = v_t - x_t$   
     $x_{t+1} = x_t + \gamma_t d_t$

**end**

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# Black-Box Frank-Wolfe - Gradient estimation

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**Algorithm 4:** Gradient estimation algorithm

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**Input:**  $x$  input variable (e.g. linearized image), number of gradient estimation iterations  $b$ ,  $\delta$  gradient smoothing parameters

**Initialize:**  $q = 0$

**for**  $i = 1, \dots, b$  **do**

**Option 1:** sample  $u_i$  uniformly from Euclidean unit sphere with  $\|u_i\|_2 = 1$

$$q = q + \frac{d}{2\delta b} (f(x + \delta u_i) - f(x - \delta u_i)) u_i$$

**Option 2:** sample  $u_i$  uniformly from standard multivariate Gaussian distribution  $N(0, I)$

$$q = q + \frac{1}{2\delta b} (f(x + \delta u_i) - f(x - \delta u_i)) u_i$$

**end**

**Return:**  $q$

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# Classic ZSCG - Main idea

The vanilla Zero-order Stochastic Conditional Gradient it's a variation of the well known *Frank-Wolfe* method. The difference in here, is that we only have an approximation of the gradient computed with the method described in the previous section.

The main steps are:

- ▶ Compute the approximated gradient  $G_{\nu}^k(x, u)$
- ▶ Solve the linear problem  $z_k = \arg \min_{u \in \chi} \langle G_{\nu}^k, u \rangle$
- ▶ Set  $x_k$  as a convex combination of  $x_{k-1}$  and  $z_k$

# Classic ZSCG - Algorithm

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**Algorithm 5:** Vanilla version of ZSCG

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**Input:**  $x_0 \in \chi$ ,  $\nu > 0$ ,  $\alpha_k$ ,  $m_k$ ,  $N \geq 1$ , probability distribution  $P_R(\cdot)$  over  $\{1, \dots, N\}$ .

**for**  $k = 1, \dots, N$  **do**

1. Generate  $u_k = [u_{k,1}, \dots, u_{k,m_k}]$ , where  $u_k \sim \mathcal{N}(0, I_d)$ ;

$$G_v^k = \frac{1}{m_k} \sum_{j=1}^{m_k} \frac{F(x_{k-1} + \nu u_{k,j}, \xi_{k,j}) - F(x_{k-1}, \xi_{k,j})}{\nu} u_{k,j}$$

2. Solve  $z_k = \arg \min_{u \in \chi} \langle G_v^k, u \rangle$
3. Set  $x_k = (1 - \alpha_k)x_{k-1} + \alpha_k z_k$

**end**

Output: Generate  $R$  according to  $P_R(\cdot)$  and output  $x_R$

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# Classic ZSCG - The Frank-Wolfe Gap

In order to provide convergence analysis, since  $f$  is non-convex we can use the Frank-Wolfe Gap defined as:

$$g_{\chi}^k \equiv g_{\chi}(x_{k-1}) := \langle \nabla f(x_{k-1}), x_{k-1} - \hat{z}_k \rangle$$

where  $\hat{z}_k = \arg \min_{u \in \chi} \langle \nabla f(x_{k-1}), u \rangle$ .

This criteria is useful since  $\langle \nabla f(x_{k-1}), x_{k-1} - u \rangle \leq g_{\chi}(x_{k-1})$ ,  $\forall u \in \chi$  which implies that we can minimize  $g_{\chi}^k$  to obtain a stationary point for our optimization problem.

# Classic ZSCG - Convergence rate 1

Using the *Frank-Wolfe Gap* and by using the following parameters  $\forall k \geq 1$ :

$$\nu = \sqrt{\frac{2B_{L\sigma}}{N(d+3)^3}}, \alpha_k = \frac{1}{\sqrt{N}}, m_k = 2B_{L\sigma}(d+5)N$$

where  $B_{L\sigma} \geq \max \left\{ \frac{\sqrt{B^2 + \sigma^2}}{L}, 1 \right\}$  and  $B \geq \|\nabla f(x)\|^1, \forall x \in \chi$ .  
it can be showed that:

$$\mathbb{E}[g_\chi^R] \leq \frac{f(x_0) - f^* + LD^2 + 2\sqrt{B^2 + \sigma^2}}{\sqrt{N}}$$

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<sup>1</sup>  $\|\nabla f(x)\| - \|\nabla f(x^*)\| \leq \|\nabla f(x) - \nabla f(x^*)\| \leq L\|x - x^*\| \leq DL = B - \|\nabla f(x^*)\|$



## Classic ZSCG - Convergence rate 2

Using the fact that  $\mathbb{E}[g_X^R] \leq \frac{c}{\sqrt{N}} \leq \epsilon$ , which implies  $N \geq \frac{c}{\epsilon^2}$ , and that at each step we call the zeroth-order stochastic oracle  $2B_{L\sigma}(d+5)N$  times, we get that the number of calls to the zeroth-order stochastic oracle and linear subproblems required to be solved to find an  $\epsilon$ -stationary points of our problem are, respectively, bounded by:

$$O\left(\frac{d}{\epsilon^4}\right), O\left(\frac{1}{\epsilon^2}\right)$$

# Classic ZSCG for Adversarial Attacks

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**Algorithm 6:** Classic ZSCG for Adversarial Attacks
 

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**Input:**  $x_0 \in \mathcal{X}$ ,  $\nu > 0$ ,  $\alpha_k$ ,  $m_k$ ,  $N \geq 1$ ,  $S$  be the set of accepted solutions

Set  $q = 0$ ,  $k = 0$

**while**  $q = 0$  and  $k \leq N$  **do**

1. Increment  $k$

2. Generate  $u_k = [u_{k,1}, \dots, u_{k,m_k}]$ , where  $u_k \sim \mathcal{N}(0, I_d)$ ;

$$G_v^k = \frac{1}{m_k} \sum_{j=1}^{m_k} \frac{F(x_{k-1} + \nu u_{k,j}, \xi_{k,j}) - F(x_{k-1}, \xi_{k,j})}{\nu} u_{k,j}$$

3. Solve  $z_k = \arg \min_{u \in \mathcal{X}} \langle G_v^k, u \rangle$

4. Set  $x_k = (1 - \alpha_k)x_{k-1} + \alpha_k z_k$

5. Check stopping criterion:  $q = SC\left(x_k, F_M(\cdot, w), S\right)$

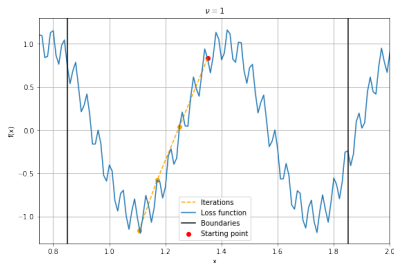
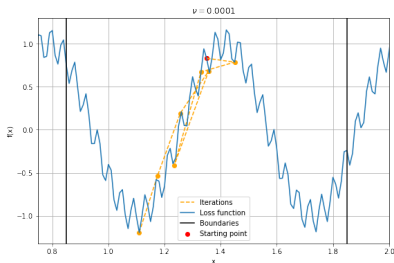
**end**

Return  $x_k$

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## Classic ZSCG - The Gaussian smoothing parameter $\nu$ .

As explained in the Section *Zero-th Order Methods* an important parameter is  $\nu$  which can help to regularize a very unstructured function and increase the convergence rate.



**Figure:** Right: the algorithm has an hard time to converge with a small value of  $\nu$ ; left: A bigger  $\nu$  can be useful to avoid local minima around your  $x$

## Classic ZSCG - The parameter $\alpha$

Another important parameter is  $\alpha$  which regulates the convex combination between the result of the linear programming problem  $z_k$  and the old  $x_{k-1}$ . Small  $\alpha$  can lead to be stuck in the very first local minima, while big  $\alpha$  can mess up with the convergence rate and make you bounce between the boundaries of the feasible set  $\chi$

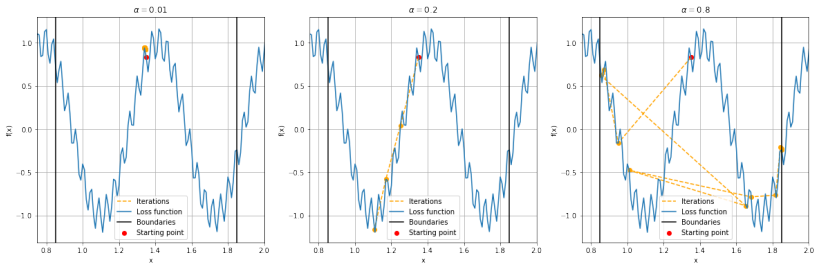
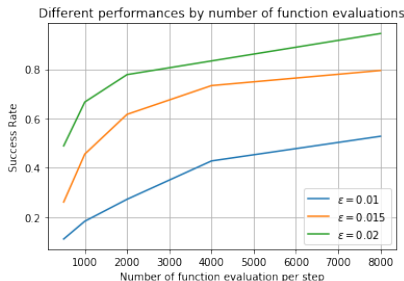


Figure: Different values of  $\alpha$  (0.01, 0.2, 0.8) lead to very different results

## Classic ZSCG - The parameter $m$

The performances can also be very influenced by the number of random Gaussian vector  $u \sim \mathcal{N}(0, I_d)$ ; in fact we can increase the precision of the approximation of the gradient by increasing the number of function evaluations used to estimate it.



**Figure:** We can see that in all the attacks type we can increase the performance by increasing the parameter  $m$

# Inexact ZSCG - Main Idea

The main idea of this ZSCG variation is to update the current estimate  $x_k$  as

$$x_{k+1} = P_{\chi}(x, \xi, \gamma) := \arg \min_{u \in \chi} \left\{ \langle g_k, u \rangle + \frac{\gamma}{2} \|u - x_k\|^2 \right\}$$

where  $g_k$  is the estimated gradient in  $x_k$  and the second term plays the role of an *elastic potential* and forces  $x_{k+1}$  to be close to  $x_k$ .

This step is performed by the *ICG* algorithm.

# Inexact ZSCG - ICG Algorithm

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**Algorithm 7:** Inexact Conditional Gradient method (ICG)

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**Input:**  $x, g, \gamma, \mu$

Set  $\hat{y}_0 = x, t = 1, k = 0$

**while**  $k = 0$  **do**

$$y_t = \arg \min_{u \in \mathcal{X}} \{h_\gamma(u) = \langle g + \gamma(\hat{y}_{t-1} - x), u - \hat{y}_{t-1} \rangle\}$$

**if**  $h_\gamma(y_t) \geq -\mu$  **then**  
|  $k = 1$

**else**

|  $\hat{y}_t = \frac{t-1}{t+1}\hat{y}_{t-1} + \frac{2}{t+1}y_t$  and  $t = t + 1$

**end**

**end**

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# Inexact ZSCG - ICG details

- ▶ Shares the general structure of Conditional Gradient algorithms;
- ▶ Minimizes the convex function:

$$H(u) = \langle g_k, u \rangle + \frac{\gamma}{2} \|u - x_k\|^2;$$

- ▶ Uses *Frank-Wolfe Gap*  $h_\gamma(y_t)$  for the stopping criterion;
- ▶ Hence, the number of linear subproblems solved to find and a  $\mu$ -stationary solution is  $O\left(\frac{1}{\mu}\right)$ .



# Inexact ZSCG - Algorithm

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**Algorithm 8:** Vanilla version of Inexact ZSCG

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**Input:**  $x_0 \in \chi$ ,  $\nu > 0$ ,  $m_k$ ,  $\gamma_k$ ,  $\mu_k$ ,  $N \geq 1$ , probability distribution  $P_R(\cdot)$  over  $\{1, \dots, N\}$ .

**for**  $k = 1, \dots, N$  **do**

1. Generate  $u_k = [u_{k,1}, \dots, u_{k,m_k}]$ , where  $u_k \sim \mathcal{N}(0, I_d)$ ;

$$G_v^k = \frac{1}{m_k} \sum_{j=1}^{m_k} \frac{F(x_{k-1} + \nu u_{k,j}, \xi_{k,j}) - F(x_{k-1}, \xi_{k,j})}{\nu} u_{k,j}$$

2. Set  $x_k = ICG(x_{k-1}, G_v^{*k}, \gamma_k, \mu_k)$

**end**

**Output:** Generate  $R$  according to  $P_R(\cdot)$  and output  $x_R$

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# Inexact ZSCG - Convergence rate

Instead of using the Frank-Wolfe Gap, an alternate convergence criterion is provided by using the *Gradient Mapping Function*

$$GP_{\chi}(x, g, \gamma) = \gamma(x - P_{\chi}(x, g, \gamma)).$$

By properly defining the parameters, and in particular setting  $m_k = 6(d + 5)N$  and  $\mu = \frac{1}{4N}$ , we have a convergence rate of

$$\mathbb{E}[\|GP_{\chi}(x_R, \nabla f(x_R), \gamma_R)\|^2] \leq \frac{8L}{N} (f(x_0) - f^* + L + B^2 + \sigma^2) .$$

The number of calls to the oracle and linear subproblems required to be solved to find an  $\epsilon$ -stationary points of our problem are, respectively, bounded by  $O\left(\frac{d}{\epsilon^2}\right)$ ,  $O\left(\frac{1}{\epsilon^2}\right)$ .

# Inexact ZSCG for Adversarial Attacks

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## Algorithm 9: Inexact ZSCG for Adversarial Attacks

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**Input:** Be  $x_0 \in \chi$ , be  $\nu, m_k, \gamma_k, \mu_k > 0$ , be  $N \geq 1$ , be  $S$  the set of accepted solutions

Set  $q = 0, k = 0$

**while**  $q = 0$  and  $k \leq N$  **do**

1. Increment  $k$

2. Generate  $u_k = [u_{k,1}, \dots, u_{k,m_k}]$ , where  $u_k \sim \mathcal{N}(0, I_d)$ ;  
call zero-th order oracle  $m_k + 1$  times to generate

$$G_v^k \equiv G_v(x_{k-1}, \xi_k, u_k) = \frac{1}{m_k} \sum_{j=1}^{m_k} \frac{F(x_{k-1} + \nu u_{k,j}, \xi_{k,j}) - F(x_{k-1}, \xi_{k,j})}{\nu} u_{k,j}$$

3. Compute new value using ICG:  $x_k = ICG(x_{k-1}, G_v^k, \gamma_k, \mu_k)$

4. Check stopping criterion  $q = SC(x_k, F_M(\cdot, w), S)$

**end**

Return  $x_k$

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# Setup

To execute the code we used Google Colaboratory, running on a Intel Xeon E5 CPU and a NVIDIA Tesla K80 GPU.

The chosen framework is PyTorch, using CUDA to run on the GPU.



**Note:** Results might be slightly inconsistent in terms of computational time due to Colab resources management.

# Datasets

## MNIST:

- ▶ 60.000 images of handwritten digits (0-9)
- ▶ single channel (i.e. black and white)
- ▶  $28 \times 28$  dimension, for a total of 784 pixels
- ▶ interval  $[0,1]$  for each pixel

## Cifar10:

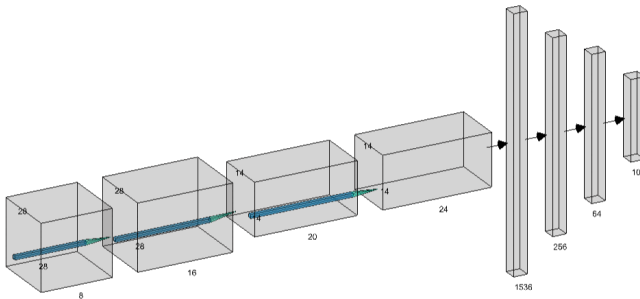
- ▶ 60.000 images divided in 10 classes (0-9)
- ▶ triple channel (i.e. colored)
- ▶  $32 \times 32$  dimension, for a total of 3072 pixels
- ▶ interval  $[0,1]$  for each pixel

# Models

## Ad Hoc

Ad Hoc network trained on MNIST, consisting of:

- ▶ 4 Convolutional layers (kernel =  $2 \times 2$ , ReLU activation)
- ▶ 2 Pooling layers (kernel =  $2 \times 2$ )
- ▶ 3 Feed Forward layers (ReLU activation + Dropouts)



# Models

## VGG16

Used for the harder task of training a network on Cifar10. VGG16 with Batch Normalization is a deep network with the following structure:

- ▶ Feature Extraction Block
- ▶ 3 Feed Forward layers (ReLU activation + Dropouts)

where the Feature Extraction Block is made up of a total of 16 Convolutional layers (ReLU activation + Dropout) and 5 Max Pooling layers.

# Models

## InceptionV3

InceptionV3 is a 48 layers deep network created by Google in 2015. The networks requires an input of  $3 \times 299 \times 299$ , for a total 268203 dimensions.

To use Inception on Cifar10 we had to rescale the images from  $32 \times 32$  to  $299 \times 299$ .



# Code Implementation

Source code:

<https://github.com/04DS2020/final-project-paolofantuz>

## Results - Attacks type

For each attack we evaluate the perturbation set using both  $L_2$  and  $L_\infty$  norms.

- ▶ For the *Ad-Hoc (MNISTNet)* model and *VGG16* we perform both targeted and untargeted attacks.
- ▶ For *InceptionV3*, given the high complexity of the model and of the dimension space, only untargeted attacks have been performed.

# Results - Evaluation metrics

Different evaluation metrics have been used to assess algorithm performance:

**Success Rate** - the percent of successful attacks;

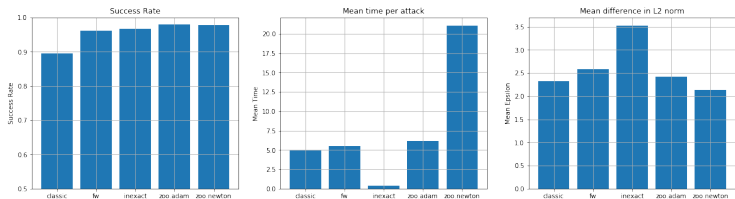
**Average Time** - the average time taken to perform an attack;

**Average Distance** - the average distance between the original image  $x_0$  and the last image found by the optimizer  $\bar{x}$  (only for attacks with  $L_2$ )

$$\langle D \rangle = \frac{1}{N_{\text{examples}}} \sum_{n=1}^{N_{\text{examples}}} \|x_{0,n} - \bar{x}_n\|_2.$$

## Results - MNISTNet

Attacks against *MNIST* with  $L_\infty$  showed that *Inexact ZSCG* and *Frank-Wolfe* achieved the best performances but with the latter being much more slower (from 3 to 20 times slower), while in the case of  $L_2$  the algorithm with the best Success Rate is ZOO with both the solvers, but it is also the slowest.



**Figure:** We can see how the *ZOO* algorithm achieve the best performance in term of Success Rate (right) and  $L_2$  distance but not in terms of time (middle) since *Inexact ZSCG* is around 20 times faster.

# Results - VGG16

Attacks against *VGG16* fine-tuned on *Cifar10* confirmed the robustness of *Frank-Wolfe* which had good performances in all the attacks against this model. It also showed a change of trajectory in the case of Inexact ZSCG, which had worse performances of its classic version (Classic ZSCG).

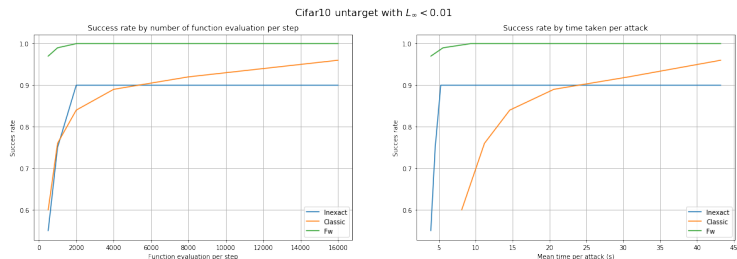


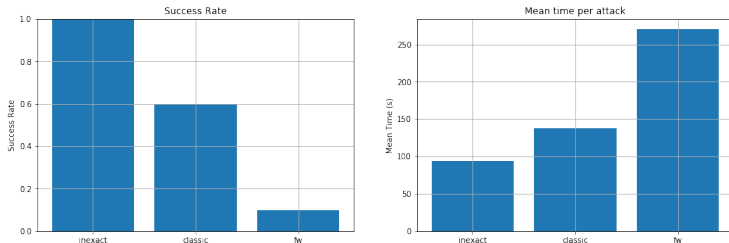
Figure: Untargeted attack against VGG16-Cifar10 with  $L_\infty$

# Results - InceptionV3 - pt.1

In case of attacks against InceptionV3 a maximum time to perform an attack has been given to each algorithm: 300 seconds in case of attacks with  $L_\infty$  and 900 seconds in case of  $L_2$ .

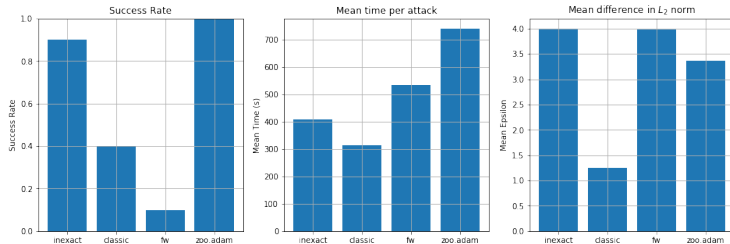
This kind of time limitations didn't influence *Inexact ZSCG* very much, being the fastest algorithm but instead had a major effect on *Frank-Wolfe* which could perform an successful attack only once in ten (both with  $L_\infty$  and  $L_2$ ). On the other hand *ZOO* achieved perfect Success rate but in almost twice the time of *Inexact ZSCG*.

## Results - InceptionV3 - pt.2



**Figure:** Untargeted attack against InceptionV3-Cifar10 with  $L_\infty$ . Right: Success Rate for each algorithm; left: Average Time taken by each algorithms. We can see how *Inexact* ZSCG outperform *Frank-Wolfe* and *Classic* ZSCG both in terms of Success Rate and Average Time.

# Results - InceptionV3 - pt.3



**Figure:** Untargeted attack against InceptionV3-Cifar10 with  $L_2$ . Right: Success Rate for each algorithm; middle: Average Time taken by each algorithms; left: Average  $L_2$  distance. We can see how ZOO and Inexact ZSCG outperform Frank-Wolfe and Classic ZSCG in terms of Success Rate, but with ZOO being the slowest



## Conclusion - Feasibility of the attacks

In the end, simulations, showed that performing an adversarial attack against a black box DNN is feasible but can be challenging, particularly if:

- ▶ the resolution of the image is high (hundreds of thousands to millions of dimensions);
- ▶ the number of (targeted) attacks to perform is of the order of thousand (e.g. an entire dataset);
- ▶ the restrictions on the feasible set  $\chi$  are too strict (e.g.  $\epsilon$  must be very small).

## Conclusion - Methods comparison

Moreover if we analyse the results of the algorithms we can see that a clear winner does not exist, however some final points can be made:






- ▶ if the model to attack is fairly simple or the number of attacks to perform is low, using *ZOO* or *Frank-Wolfe* could be the best choice, since they proved to be the most accurate in terms of Success Rate if enough time is given;
- ▶ if an high number of attacks must be performed or the model attacked is very complex, *ZSCG* algorithms, particularly *Inexact ZSCG*, seems to be the best since the major factor to take in consideration is the time.

# No Free Lunch Theorem?

It seems like we have a sort of No Free Lunch Theorem...



# Bibliography

-  Chen Pin-Yu, Zhang Huan, Sharma Yash. *ZOO: Zeroth Order Optimization based Black-box Attacks to Deep Neural Networks without Training Substitute Models.*
-  Jinghui Chen, Dongruo Zhou, Jinfeng Yi, Quanquan Gu. *A Frank-Wolfe Framework for Efficient and Effective Adversarial Attacks.*
-  Krishnakumar Balasubramanian, Saeed Ghadimi. *Zeroth-order Nonconvex Stochastic Optimization: Handling Constraints, High-Dimensionality and Saddle-Points.*
-  Nicholas Carlini, David Wagner *Towards Evaluating the Robustness of Neural Networks.*
-  Nesterov Yurii, Spokoiny Vladimir. *Random Gradient-Free Minimization of Convex Functions*