

### Note: time unit conversions

- $1\text{ ns} = 1\text{e-}6\text{ ms} = 0.000001\text{ millisecond}$
- $1\text{ millisecond (ms)} = 0.001\text{ seconds}$
- $1\text{ microsecond} = 0.001\text{ milliseconds}$
- alternatively
  - $10\text{e-}9 = \text{nanosecond (ns)}$
  - $10\text{e-}6 = \text{microsecond (mu s)}$
  - $10\text{e-}3 = \text{millisecond (ms)}$

### Note: interpreting timings

- `user` + `sys` tell us how much actual CPU time the process has used. `user` is the time taken on the CPU and `sys` is additional time taken such as allocating memory, which the code cannot do from "user" mode.
- I think wall time is `real`, and could be confounded by other processes

## Computing $\pi$

We now start with the first parallelization tasks. We will

- see how a python application can be parallelized
- discuss **data parallelism** and **task parallelism**

We use the example of computing  $\pi$  because it is a simple example that still takes some time to compute, so perfect for our instructional purposes.

The way this works is with **Monte-Carlo** methods--this means approximating exact results with random numbers and simulation.

## The geometry behind

add figure from online material

- [link to online material](#)
- assume a unit radius
- then the surface of the square is  $4r^2$
- the surface of the circle is  $\pi r^2$
- if we can compute the surfaces, we can call them `M` for the circle and `N` for the square
- then we have  $M / N = \pi r^2 / 4 r^2$
- solving for  $\pi$  we get  $\pi = 4 M / N$

How do we get `M` and `N`?

### Exercise: implement the algorithm (time?)

- using only standard python and the `random.uniform` function

```
In [1]: import random
import math

def calc_pi(N):
    "Compute pi using N random samples"
    M = 0
    for i in range(N):
        # Simulate random coordinates
        x = random.uniform(-1, 1)
        y = random.uniform(-1, 1)
        if x**2 + y**2 < 1: # don't need sqrt b/c 1**2 = 1
            M += 1

    return 4*M/N
```

```
In [2]: time_forloop = %timeit -o calc_pi(10**6)
```

536 ms ± 995 µs per loop (mean ± std. dev. of 7 runs, 1 loop each)

Before starting to parallelize, we should make the inner function (the for loop) as efficient as possible. We discuss two options

1. vectorization with `numpy`
2. native code generation with `numba`

Vectorization works by applying the computation over an array of numbers, instead of each number individually.

What does `numba` do?

- when we execute python code, the line is *interpreted* and directly performed. the speed of this depends on how the programming language is designed (*interpretation*)
- an alternative is *compiled* code, where we write in some language and then translate the code to machine code. Only after this we execute the program. (*ahead-of-time compilation*)
- numba sits in between the two and performs *just-in-time compilation*. It generates optimized machine code---that is faster than python code---, at runtime. This means that our code is compiled to machine code as we define the function.
  - this is faster than python code but has additional overhead from compiling

## Numpy

```
In [3]: # can use this to show how to get to the function below
import numpy as np
np.random.uniform(-1, 1, (1, 1))
points = np.random.uniform(-1, 1, (2, 100))
(points**2).sum(axis=0)
(points**2).sum(axis=0) < 1
np.count_nonzero((points**2).sum(axis=0) < 1)

M = np.count_nonzero((points**2).sum(axis=0) < 1)

4 * M / 100
```

Out[3]: 3.0

```
In [4]: def calc_pi_numpy(N):
        "Compute pi using N samples with numpy"
        points = np.random.uniform(-1, 1, (2, N))
        # M = np.count_nonzero((points**2).sum(axis=0) < 1)
        M = np.count_nonzero((points**2).sum(axis=0) < 1)
        return 4 * M / N
```

```
In [5]: calc_pi_numpy(10**6)
```

Out[5]: 3.143388

```
In [6]: time_numpy = %timeit -o calc_pi_numpy(10**6)
```

15.6 ms ± 19.7 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)

```
In [7]: speedup = time_forloop.average / time_numpy.average
print(f"Using numpy is {speedup:.2f} times faster than using the for loop")
```

Using numpy is 34.25 times faster than using the for loop

This is a *vectorized* version of the algorithm: It demonstrates **data parallelism** where a single operation is applied over a collection of data.

This contrasts to **task parallelism** where different independent procedures are applied in parallel. Example: cutting vegetables while simmering the split peas. In general, everything today will be data parallel -- we will always apply the same operation to a collection of data. For task parallelism, consider for instance dask (tomorrow).

### Discussion: Is this all better?

What are the downside of the vectorized implementation, and of data parallelism in general?

- it uses more memory
- it is perhaps less intuitive
- it is more monolithic and it cannot easily be broken up into parts -- in contrast to the example below.

## Exercise: Daskify

Write `calc_pi_dask` to make the numpy version parallel. Compare speed and memory performance with the numpy version. Remember that `dask.array` mimics the numpy API.

```
In [8]: #can use this to illustrate how get to to the function below
import dask.array as da
points = da.random.uniform(-1, 1, (2, 100))
(points**2).sum(axis=0).compute() < 1
work = da.count_nonzero((points**2).sum(axis=0) < 1)
work.compute()
```

Out[8]: 84

```
In [9]: def calc_pi_dask(N):
```

```

"Compute pi using N samples with dask"
points = da.random.uniform(-1, 1, (2, N))
work = da.count_nonzero((points**2).sum(axis=0) < 1)

# using the physical number of cores, as we found out in previous episode
M = work.compute(num_workers=4)
return 4 * M / N

```

In [10]: `calc_pi_dask(10**6)`

Out[10]: 3.141612

In [11]: `time_dask = %timeit -o calc_pi_dask(10**6)`

22.1 ms ± 156 µs per loop (mean ± std. dev. of 7 runs, 10 loops each)

In [12]: `speedup = time_forloop.average / time_dask.average`  
`print(f"Using dask is {speedup:.2f} times faster than using the for loop")`

Using dask is 24.27 times faster than using the for loop

### Compare memory and performance

In [13]: `from memory_profiler import memory_usage`  
`import matplotlib.pyplot as plt`

In [14]: `# Little wrappers to make life easy`  
`N = 10**8`  
`def pi_with_numpy():`  
 `return calc_pi_numpy(N)`  
  
`def pi_with_dask():`  
 `return calc_pi_dask(N)`

In [15]: `memory_numpy = memory_usage(pi_with_numpy, interval=0.01)`  
`time_numpy = %timeit -o pi_with_numpy()`

2.82 s ± 283 ms per loop (mean ± std. dev. of 7 runs, 1 loop each)

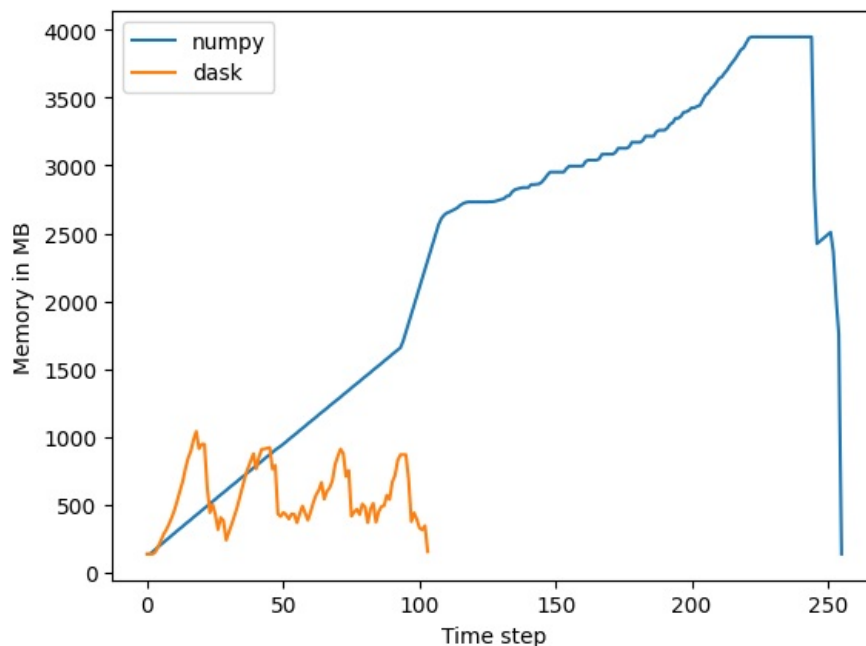
In [16]: `memory_dask = memory_usage(pi_with_dask, interval=0.01)`  
`time_dask = %timeit -o pi_with_dask()`

1.01 s ± 131 ms per loop (mean ± std. dev. of 7 runs, 1 loop each)

In [17]: `speedup = time_numpy.average / time_dask.average`  
`print(f"Using dask is {speedup:.2f} faster than using numpy")`

Using dask is 2.79 faster than using numpy

In [18]: `plt.plot(memory_numpy, label="numpy")`  
`plt.plot(memory_dask, label="dask")`  
`plt.xlabel("Time step")`  
`plt.ylabel("Memory in MB")`  
`plt.legend()`  
`plt.show()`



Using numba to accelerate python code

numba makes it easy for us to build accelerated functions. We can call the decorator `numba.jit`. What does it do? -- It sends the function, to which we apply the decorator, to machine code when we execute the cell

```
In [19]: import numba

@numba.jit
def sum_range_numba(a):
    "Compute the sum of the numbers in the range [0, a)"
    x = 0
    for i in range(a):
        x += 1
    return x
```

Let's time three different versions of this.

```
In [20]: ## Naive python iterators
time_naive = %timeit -o sum(range(10**7))
```

132 ms ± 11.9 ms per loop (mean ± std. dev. of 7 runs, 10 loops each)

```
In [21]: # numpy
time_numpy = %timeit -o np.arange(10**7).sum()
```

17.3 ms ± 1.09 ms per loop (mean ± std. dev. of 7 runs, 10 loops each)

```
In [22]: # numba
time_numba = %timeit -o sum_range_numba(10**7)
# sometimes a warning appears "[could be that ..] intermediate result is being cached".
# re-running usually helps
```

148 ns ± 4.71 ns per loop (mean ± std. dev. of 7 runs, 10,000,000 loops each)

```
In [23]: speedup = time_numpy.average / time_numba.average
print(f"Using numba is {speedup:.2f} times faster than using numpy")
```

Using numba is 116846.68 times faster than using numpy

So, numba is 100k times faster (?)

$(14.75 * 10e-3) / (128 * 10e-9) = (17.5 * 10e-3) / (12.8 * 10e-8) = 10e-5$

numba JIT does not work on every python or numpy feature, but a function is a good candidate if written with a python for-loop over a large range of values, as with `sum_range_numba()`.

#### Note on just-in-time compilation

- there may be little or no speed-up when function is called the first time
- similarly, when using `timeit` you may get the following message
  - The slowest run took 14.83 times longer than the fastest. This could mean that an intermediate result is being cached.
- reason: on the first call, the JIT compiler needs to compile the function. on subsequent runs, the function is reused
- **the same function can only be reused if it is called with the same argument types (int, float, etc)**

```
In [24]: %time sum_range_numba(10**7)
```

CPU times: user 6 µs, sys: 2 µs, total: 8 µs  
Wall time: 10.5 µs

Out[24]: 10000000

```
In [25]: %time sum_range_numba(10.**7)
```

CPU times: user 38 ms, sys: 0 ns, total: 38 ms  
Wall time: 37.6 ms

Out[25]: 10000000

```
In [26]: %time sum_range_numba(10.**7)
```

CPU times: user 7 µs, sys: 0 ns, total: 7 µs  
Wall time: 9.78 µs

Out[26]: 10000000

### Challenge: numbify `calc_pi`

Create a numba version of `calc_pi`. Time it.

```
In [27]: # note we're moving back to a function that looks similar to the original function
@numba.jit
def calc_pi_numba(N):
```

```
"Compute pi using N samples with numba"
M = 0
for i in range(N):
    # Simulate random coordinates
    x = random.uniform(-1, 1)
    y = random.uniform(-1, 1)
    if x**2 + y**2 < 1.0: # don't need sqrt b/c 1**2 = 1
        M += 1

return 4*M/N
```

```
In [28]: calc_pi_numba(10**7)
```

```
Out[28]: 3.1412228
```

```
In [29]: time_numba = %timeit -o calc_pi_numba(10**6)
```

```
7.03 ms ± 368 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
```

```
In [30]: speedup = time_dask.average / time_numba.average
print(f"Using numba is {speedup:.2f} times faster than using dask")
```

```
Using numba is 144.01 times faster than using dask
```

- also: random number generator changes

## Conclusion

- measuring - knowing: always profile your code to see which parallelization method works best
- numba often outperforms other methods, but it is not always possible to rewrite code so that one can use numba with it

Processing math: 100%