

See discussions, stats, and author profiles for this publication at: <http://www.researchgate.net/publication/257218429>

Influence of inter-vehicle communication on peak hour traffic flow

ARTICLE in PHYSICA A: STATISTICAL MECHANICS AND ITS APPLICATIONS · MARCH 2012

Impact Factor: 1.72 · DOI: 10.1016/j.physa.2011.11.027

CITATIONS

9

DOWNLOADS

91

VIEWS

70

2 AUTHORS:



Florian Knorr

University of Duisburg-Essen

11 PUBLICATIONS 38 CITATIONS

SEE PROFILE



Michael Schreckenberg

University of Duisburg-Essen

206 PUBLICATIONS 6,483 CITATIONS

SEE PROFILE

Influence of inter-vehicle communication on peak hour traffic flow

Florian Knorr^{a,*}, Michael Schreckenberg^a

^a*Physik von Transport und Verkehr, Universität Duisburg-Essen, 47048 Duisburg, Germany*

Abstract

Mixed traffic flow consisting of vehicles equipped with wireless inter-vehicle communication devices and non-equipped vehicles is analyzed using bidirectionally coupled network traffic and road traffic simulators in a peak hour scenario. For equipped vehicles a strategy to stabilize traffic flow and to reduce travel time is proposed. The strategy comprises rules to determine both how and when to change driving behavior. Vehicles that detect perturbations downstream try to keep a larger gap to their predecessor by which they aim to compensate traffic inhomogeneities. Improvement of traffic flow was observed even for a ratio of equipped vehicles as low as five percent.

Keywords: IVC, Traffic flow, Traffic efficiency, VANET, Congested flow

1. Introduction

Inter-vehicle communication (IVC) is considered an integral component of future Intelligent Transportation Systems (ITS). Providing a higher level of road safety certainly is the most important aspect as to the application of IVC-based services. Improved traffic information and forecasting systems as well as the optimization of traffic flow are regarded as additional fields of application of IVC in the context of ITS. Dedicated short range communication (DSRC) devices enable cars to form wireless networks to exchange and broadcast messages. This decentralized approach of vehicular ad hoc networks (VANETs) offers various advantages compared to alternative communication channels: low cost as no coordinating infrastructure is necessary, low time delays, extended operating distance (200 m to 500 m [1, 2]) compared to local sensors (collision sensors, driver's line of sight).

For both practical and economic reasons most of the work in this very popular field of research is performed via computer simulations as a large number of equipped vehicles would be necessary in real world experiments. For VANET simulations, both a realistic traffic flow model and a communication model have to be combined and interaction between both models has to be taken into account. In contrast, many current VANET simulations rely on static vehicle traces generated by a traffic simulator which they pass to a network simulator, e.g. ns2 [3], JiST/SWANS [4–6] or Shawn [7], to model communication. The shortcoming of this approach is that vehicles cannot change their driving behavior or route choice in response to the messages received.

Kerner et al. [8] recently presented a testbed for wireless vehicle communication which they used to study the influence of IVC on congested traffic patterns. They showed that if all vehicles were communicating vehicles IVC can considerably increase traffic efficiency. Kesting et al. [9] found even few vehicles equipped with adaptive cruise control (ACC) systems can increase traffic capacity, and expect further improvements by providing additional traffic information via IVC.

The question of which rate of communicating vehicles is necessary to stabilize traffic flow and increase road capacity certainly is interesting from a theoretical as well as from a practical point of view (see, e.g., [10]). In this paper we present a strategy to increase traffic efficiency solely by means of IVC and show this

*Corresponding author. Tel.: +49 203 379-3901, fax : +49 203 379-6564

Email addresses: knorr@ptt.uni-due.de (Florian Knorr), schreckenberg@ptt.uni-due.de (Michael Schreckenberg)

strategy becomes effective even at low rates of equipped vehicles. The results were obtained by a VANET simulator which allows feedback between the network stack and a cellular automaton based mobility model. Vehicles detect the local traffic state by evaluating the messages they received and adapt their driving behavior if necessary. However, the challenge remains how the proposed change in driving behavior can be achieved in real cars with human drivers.

2. Modeling vehicle communication

Our network simulations used the JiST/SWANS platform [4–6, 11] which combines a high-performance discrete event simulation engine with a *scalable wireless ad hoc network simulator* (SWANS). Extensions by Ibrahim and Weigle [12] enable the network simulator to interact with the underlying mobility model which will be presented in the following section.

Vehicle communication is simulated in a wireless ad hoc network using the IEEE 802.11b standard. Radio propagation is modeled by the two ray ground model. All radio equipped vehicles are assumed to have an identical set of physical and MAC layer parameters which are summarized in table 1.

In our simulations vehicles with a radio device periodically broadcast a status message, a so called beacon

PHY	
frequency	2.4 GHz
channel bandwidth	20 MHz
interference model	cumulative noise & capturing effect
signal-to-noise ratio	10
fading	none
pathloss model	two-ray
transmission power	15 dBm
received power threshold	−77 dBm
carrier sense threshold	−71 dBm
thermal noise	−101 dBm
antenna gain	0 dB
max. sensing range	< 300 m
MAC	
slot time	20 μ s
DIFS	50 μ s
transmission rate	2 Mbit s ^{−1}
broadcast frequency	4 Hz
message size	500 B

Table 1: Summarized properties of radio channel

(message). These messages carry a vehicle’s current geographical position, speed, acceleration, heading and a unique vehicle identifier.

From the information encoded in the beacons a receiving vehicle may analyze the local traffic state. We now propose some simple rules for communicating vehicles to decide when to change and how to change driving behavior. Using the position and the speed encoded in a beacon message a receiving vehicle n calculates the average velocity of all downstream vehicles $\bar{v}_n^{\text{ds}}(t)$ during the last second and via the encoded accelerations the average velocity during the preceding second $\bar{v}_n^{\text{ds}}(t-1)$. If both $\bar{v}_n^{\text{ds}}(t)$ and $\bar{v}_n^{\text{ds}}(t-1)$ are below a threshold velocity v_T the vehicle changes its driving behavior (for other possible incentives, see [13]). Moreover, it appends this information about the change in driving behavior to its own status messages. The binary variable jw_n indicates whether the n -th vehicle applies the standard ($\text{jw}_n = 0$) or the changed ($\text{jw}_n = 1$) driving behavior. In the case of changed driving behavior this additional information also comprises a time-stamp jam^{time} and a position jam^{pos} indicating when and where the average velocity dropped below v_T . For simplicity, the position jam^{pos} is assumed to be half the maximum sensing range

ahead:

$$\begin{aligned} & \text{if } (\overline{v_n^{\text{ds}}}(t-1) < v_{\text{T}} \wedge \overline{v_n^{\text{ds}}}(t) < v_{\text{T}}) \text{ then:} \\ & \quad \text{jw}_n \leftarrow 1 \\ & \quad \text{jam}^{\text{pos}} \leftarrow x_n(t) + (\text{sensing range})/2 \\ & \quad \text{jam}^{\text{time}} \leftarrow t \end{aligned}$$

The symbols \wedge and \vee stand for logical conjunction and logical disjunction, respectively. Even if the above condition does not hold true for a vehicle m , it may change its driving behavior provided it has received a beacon from a vehicle n further downstream that has already changed its behavior, i.e. $\text{jam}_n = 1$. In this case, it checks if it is close to the position jam^{pos} and if the message is still valid. A message remains valid as long as the difference between current local time t and jam^{time} is below a temporal threshold lt :

$$\begin{array}{l} \text{jw}_m \leftarrow 0 \\ \text{if } (t - \text{jam}^{\text{time}} < \text{lt} \wedge 0 < \text{jam}^{\text{pos}} - x_m(t) < \text{r}): \\ \quad \text{jw}_m \leftarrow 1 \end{array}$$

If both conditions are fulfilled vehicle m changes its driving behavior and appends this information to its own beacons. The interval and the distance within such an information remains valid are given by l_t and r , respectively.

3. Modeling vehicular traffic

As vehicular network simulations require realistic vehicle traces only microscopic vehicle traffic models should be adopted for this purpose [14]. In this section we present the rules of vehicle motion to mimic the complex dynamics of vehicular traffic. For the highway scenario which will be discussed in the following section we also show how open boundaries and on-ramps are modeled.

3.1. Vehicle motion

Vehicle motion is simulated with an extended version of the NaSch traffic cellular automaton [15]. The extended model [16], called the comfortable driving model (CDM), incorporates anticipatory effects by considering the preceding vehicle's $(n+1)$ velocity v_{n+1} and brake lights when updating vehicle n . Thereby it is able to reproduce the three phases of traffic flow [17] and is in good agreement with empirical data [18].

The CDM is a multi-cell traffic cellular automaton where the number of consecutive cells a vehicle n occupies is given by its length l_n . The space coordinate x_n refers to the vehicle's front end. Brake lights are either on or off ($b_n = 0$ or 1). The effective distance d_n^{eff} is a function of the predecessor's anticipated velocity and the actual distance $d_n = x_{n+1} - l_{n+1} - x_n$ between vehicles n and $n + 1$.

We modified the original rules of motion to allow for communicating vehicles to change their driving behavior. A communicating vehicle that received a message to change its driving behavior ($jw_n = 1$) is assumed to keep a by buf_n increased spatial gap and to exhibit a lower reaction to its predecessor's brake lights if buf_n is sufficiently large. The following parallel update rules describe the transition from the state at time t to $t + 1$ for our extended model. (By skipping step 6, i.e. by setting $\text{buf} \leftarrow 0$ for all vehicles, one obtains the original comfortable driving model.)

1. acceleration:
 $b_n(t+1) \leftarrow 0$
if $(b_n(t) = 0 \wedge b_{n+1}(t) = 0) \vee t_h \geq t_s$ then:
 $v_n(t+1) \leftarrow \min(v_n^{\max}, v_n(t) + 1)$
2. adaptation of buffer:
if $(v_n(t+1) > d_n^{\text{eff}} - \text{buf}_n(t) \wedge \text{buf}_n(t) > 0)$ then:
if $(v_n(t+1) > v_n(t))$ then:
 $v_n(t+1) \leftarrow v_n(t)$
 $\text{buf}_n(t+1) \leftarrow \max(d_n^{\text{eff}} - v_n(t+1), 0)$

3. determination of randomization parameter p :

$$p \leftarrow \begin{cases} p_b, & \text{if } b_{n+1}(t) = 1 \wedge t_h < t_s \wedge (\text{buf}_n(t+1) \leq l_n \vee \text{jw} = 0), \\ p_j, & \text{if } b_{n+1}(t) = 1 \wedge t_h < t_s \wedge \text{buf}_n(t+1) > l_n \wedge \text{jw} = 1, \\ p_0, & \text{if } v_n = 0 \wedge \neg(b_{n+1}(t) = 1 \wedge t_h < t_s), \\ p_d, & \text{otherwise.} \end{cases}$$

4. braking:

$$\begin{aligned} v_n(t+1) &\leftarrow \min(d_n^{\text{eff}} - \text{buf}_n(t+1), v_n(t+1)) \\ b_n(t+1) &\leftarrow 1 - \Theta(v_n(t+1) - v_n(t)) \end{aligned}$$

5. dawdling:

if ($\text{rand}() < p$) then:

$$\begin{aligned} v_n(t+1) &\leftarrow \max(v_n(t+1) - 1, 0) \\ \text{if } p = p_b \text{ then } b_n(t+1) &\leftarrow 1 \end{aligned}$$

6. reaction to warning message:

$$\text{buf}_n(t+1) \leftarrow \begin{cases} \min(2l_n, d_n - v_n^{\max}), & \text{if } \text{jw} = 1 \wedge d_n > v_n^{\max}, \\ 0, & \text{if } \text{jw} = 0 \end{cases}$$

7. car motion:

$$x_n(t+1) \leftarrow x_n(t) + v_n(t+1)$$

The function $\Theta(x)$ is the Heaviside step function which yields 0 if its argument is negative and 1 otherwise. The symbol \neg denotes the logical negation. A random number uniformly distributed in $[0, 1]$ is denoted by $\text{rand}()$.

In step 2 equipped vehicles adapt the additional gap buf_n to avoid braking. Non-equipped vehicles skip this step as for these vehicles the variable buf_n equals 0 at any time. For the same reason only for communicating vehicles can the randomization parameter be set to p_j . The times $t_h = v_n/d_n$ and $t_s = \min(v_n, h)$ serve for comparing the time t_h vehicle n needs to close the gap to its predecessor with a velocity-dependent interaction horizon t_s . (The constant variable h determines the interaction range with brake lights.) The effective distance $d_n^{\text{eff}} = d_n + \max(v_{\text{anti}} - g_{\text{safe}}, 0)$ takes into account the anticipated velocity $v_{\text{anti}} = \min(d_{n+1}, v_{n+1})$ of the leading car. The parameter g_{safe} controls the effectiveness of the anticipation. In step 6 vehicles can increase the additional gap if the distance to the leading vehicle is sufficiently large. The variable's maximum length is limited to twice the vehicle length.

Two distinct types of vehicles were considered which we will refer to as "trucks" and "cars". Trucks have length $l^{\text{truck}} = 10$ cells and maximum velocity $v_{\text{max}}^{\text{truck}} = 15$ cells/time step whereas cars have length $l^{\text{car}} = 5$ cells and maximum velocity $v_{\text{max}}^{\text{car}} = 20$ cells/time step. The remaining variables are set to $p_d = 0.1$, $p_b = 0.94$, $p_0 = 0.5$, $p_j = 0.8p_b$, $h = 6$, $g_{\text{safe}} = 7$, $v_T = 12$ cells/time step, $l_t = 30$ time steps and $r = 2000$ cells.

Each cell corresponds to 1.5 m and each timestep corresponds to 1s. Asymmetric lane changing rules are adopted according to [16].

3.2. Open boundaries and on-ramps

For modeling open boundary conditions we follow a similar approach as Kerner in [19]. However, due to the different vehicle types some modifications are necessary: Let $x_0 = 0$ denote the leftmost cell of the road and x_c the position of the vehicle closest to x_0 ($x_c > x_0$) on the considered lane. With an entrance section of length $l_{\text{entr}} = l^{\text{truck}} + v_{\text{max}}^{\text{truck}}$ a new vehicle starts from $x_i = \min(x_0 + l_{\text{entr}}, x_c - l_c - v_{\text{max}}^{\text{truck}})$, provided that $x_i \geq x_0$. Otherwise, insertion fails and is retried in the subsequent time step. The initial velocity of a newly inserted vehicle is $v_{\text{max}}^{\text{truck}}$. To avoid the formation of plugs at the boundaries trucks must not be inserted in the left lane. Vehicles reaching the downstream boundary are removed.

The on-ramp starts at position x_s and ends at position x_e ($x_s < x_e$). A simple strategy serves for inserting vehicles entering the highway from an on-ramp. A merging vehicle is to be inserted in the largest gap in

the right lane of the on-ramp section. The positions of the vehicles delimiting this gap are given by x_n and x_{n+1} ($x_n < x_{n+1}$). The resulting gap size is $g_n = \min(x_{n+1}, x_e + l_{n+1}) - l_{n+1} - \max(x_n, x_s)$ and the merging vehicle is inserted with equal distance to its predecessor and successor at $x_m = x_n + l_m + \lfloor (g_n - l_m)/2 \rfloor$ ($\lfloor x \rfloor$ denotes the integer part of x). Obviously, insertion has to be aborted if $g_n < l_m$. In addition, we require an additional gap by $g_n - l_m > v_n$. The merging vehicle adopts the velocity of the one following behind (n) and turns its brake lights off if insertion is successful. Otherwise, insertion is retried in the next update step. The adopted on-ramp model yields the same qualitative features around the bottleneck as more sophisticated models (for a comparison see, e.g., [20]).

4. Results

For the simulations a two-lane highway segment of length 18 km (12 000 cells) with a 225 m long on-ramp located 1.5 km from the downstream boundary ($x_s = 11\,000$, $x_e = 11\,150$) is used. The lane width is 4 m. The on-ramp inflow is kept constant at 450 vehicles h^{-1} during the entire simulation time of 6.5 h. To simulate peak hour traffic the demand of the main road is varied as follows: The inflow of the initially empty main road is 1000 vehicles $\text{h}^{-1} \text{ lane}^{-1}$ for the first 30 min. Next, inflow is linearly increased to 1400 vehicles $\text{h}^{-1} \text{ lane}^{-1}$ during a 2 h interval. For the following 3 h inflow rate is linearly decreased to 1000 vehicles $\text{h}^{-1} \text{ lane}^{-1}$ where it is kept constant until the simulation ends. Thus, a total of 15 000 vehicles (2925 vehicles) enters the main road from the open boundary (on-ramp). The maximum value of 1400 vehicles $\text{h}^{-1} \text{ lane}^{-1}$ is reached in all simulations presented below. If vehicles cannot be inserted at the desired rate due to a spontaneous breakdown – which occurs occasionally with no or few communicating vehicles – the simulation run is discarded.

To generate a realistic load on the radio channel we add another two-lane highway section with a constant flow of 1250 vehicles $\text{h}^{-1} \text{ lane}^{-1}$ for the opposite direction during the entire simulation (16 250 vehicles). Note that messages from vehicles traveling in the opposite direction do not influence driving behavior. For both the on-ramp and the open boundaries 10 percent of the inserted vehicles are trucks. The influence of the proposed method on traffic flow is studied for different percentages of communicating vehicles. For each rate the results are averaged over 20 independent runs.

Fig. 1 shows the spatiotemporal dynamics of vehicle velocity for all vehicles moving on the road with the on-ramp. Dark regions mark vehicles moving with a low velocity. One can see the onset of traffic breakdown after approximately 2.5 h in the on-ramp area after main road inflow reaches its maximum. As a result of the decreasing flow after 2.5 h, free flow is restored after maximally 6 h in all cases. Without vehicle-to-vehicle communication (cf. fig. 1(a)) the impact of the on-ramp induced traffic breakdown can be observed even 10 km upstream the on-ramp. With an increasing percentage of equipped vehicles (cf. fig. 1(b), 1(c), 1(d)) both spatial and temporal extent of regions with reduced velocity decrease significantly. That means both the number of vehicles affected by the on-ramp induced breakdown in traffic flow and the degree of affection decrease as well. With 30 percent of all vehicles being equipped the peak hour induced perturbations nearly vanish and become tightly localized around the bottleneck. Due to the merging process and the high vehicle flow we do not expect perturbations to disappear completely.

To quantify the positive impact of IVC on traffic flow we examine travel time. Travel time is a very intuitive measure of a transportation system's performance used by traffic engineers and analysts. For commuters travel time is even the most important determinant of the highway quality of service [21] as they consider travel time as "lost". (For an analytic treatment of travel time cost and peak hour commuting behavior see [22].) The microscopic simulation allows for each vehicle to record the time it took the vehicle to pass the entire stretch of highway considered. Fig. 2 shows the temporal evolution of travel time for different penetration rates.

The averaged single vehicle travel times (cf. fig. 2(a)) demonstrate the impact IVC has on traffic flow: Without IVC travel time more than doubles for some vehicles from approximately 10 min to more than 22 min. With equipment rates of 15 percent and above no vehicle needs longer than 16 min to pass the same distance.

Fig. 2(b) shows the cumulated travel times [23] which serve as performance measure from a less user-centric point of view. Again, the highest values are obtained without communicating vehicles. Already with

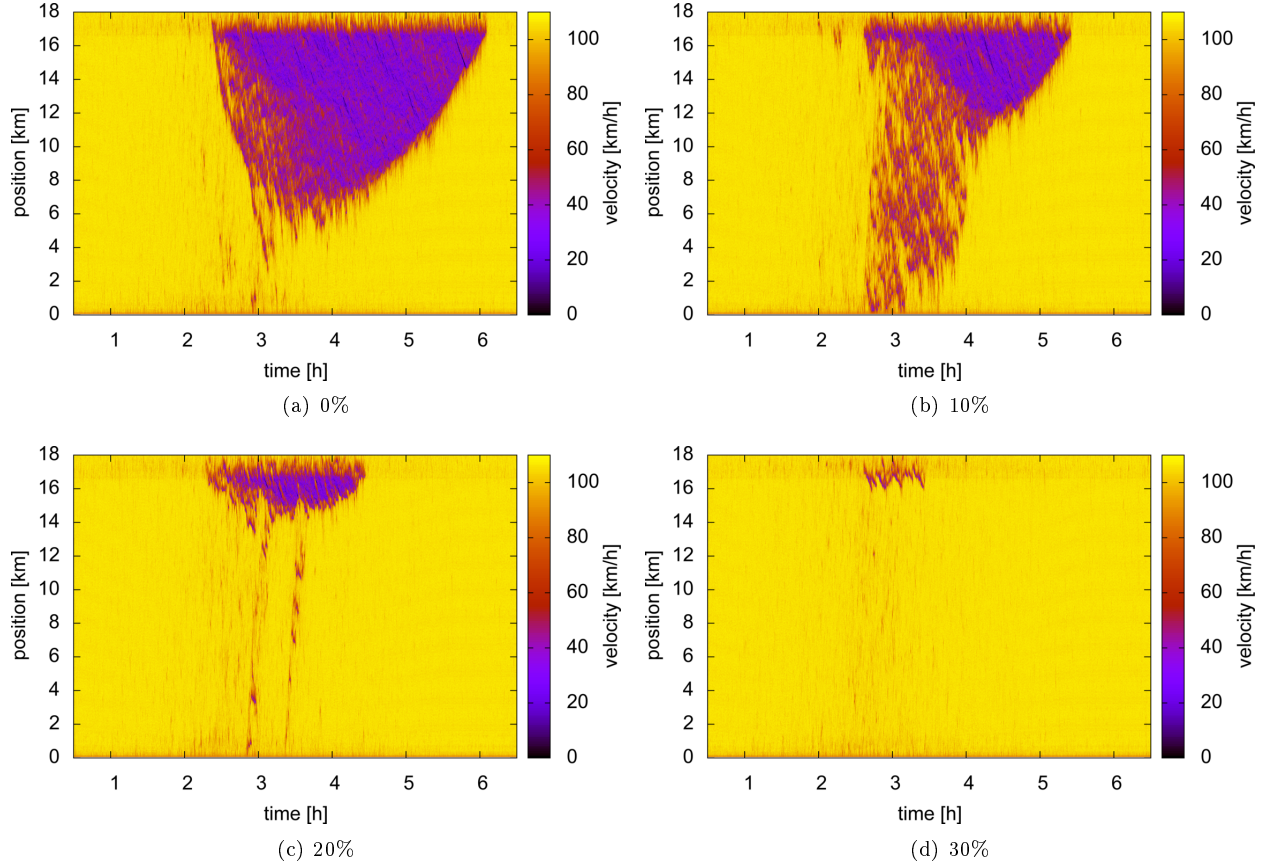


Figure 1: Examples of the spatiotemporal vehicle dynamics for different rates of IVC vehicles with the on-ramp at $x = 16$ km.

only 5 percent communicating vehicles cumulated travel time goes down for 4.5 percent. The difference between the curves for 25 percent and 30 percent is negligible which suggests even higher penetration rates will not further improve the obtained results considerably.

Fig. 3 shows the relation between rate of equipped vehicles and average delay in travel time and maximum congestion length, respectively. As maximum congestion length we define the maximum length of a sequence of vehicles whose velocity does not exceed 10 cells per time step (54 km h^{-1}). The delay in travel time is given by the difference between actual travel time and an ideal travel time of 620 s for the stretch of road considered. Both curves have similar characteristics: With 15 percent communicating vehicles both maximum congestion length and travel time delay reduce to one half of the values obtained without equipped vehicles. With a 30 percent penetration rate the average delay drops to less than 30 s and maximum congestion length is below 1 km.

5. Conclusion

The present paper studied the impact of inter-vehicle communication on traffic flow. Vehicles equipped with a communication device periodically broadcast messages containing their current position and velocity. Other equipped vehicles receive and evaluate these messages. To react on the information received both rules for when to change and how to change driving behavior were presented. Special attention was paid to the relation between equipment rates and impact on traffic flow.

Simulations used a realistic peak hour scenario with open boundaries, variable inflow and heterogeneous traffic. Results show a positive effect on traffic efficiency and stability by the means of inter-vehicle com-

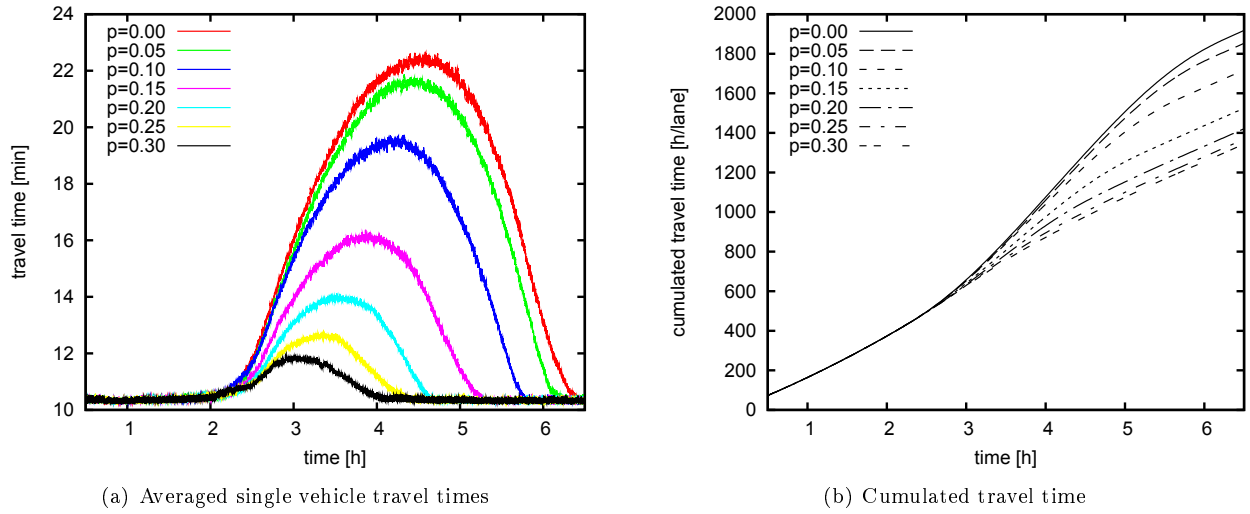


Figure 2: Cumulated and averaged travel times depending on different percentages p of communicating vehicles. Single vehicle travel times were averaged over a moving 20 second interval.

munication even for 5 percent communicating vehicles. A further increase in the percentage of equipped vehicles significantly reduced travel time and congestion for all vehicles on the road. Equipment rates above 30 percent restore free flow except for a narrow region around the on-ramp. The proposed strategy to keep larger gaps might also contribute to an increased traffic safety especially in dense traffic.

The success of this strategy, however, does not depend on cooperation between vehicles, but only on the willingness of vehicles to change their driving behavior under certain circumstances. As this change is beneficial for the vehicle, i.e. driver, we are confident the majority of drivers will follow the recommendation.

Acknowledgments

This work was supported by the state of North Rhine-Westphalia and the European Union within the NRW-EU Ziel 2 program.

References

- [1] J. P. Singh, N. Bambos, B. Srinivasan, D. Clawin, Wireless LAN performance under varied stress conditions in vehicular traffic scenarios, in: Vehicular Technology Conference, Vol. 2, 2002, pp. 743–747.
- [2] J. Ott, D. Kutscher, Drive-thru internet: IEEE 802.11b for "automobile" users, in: INFOCOM, Vol. 1, 2004, pp. 362–373.
- [3] K. Fall, K. Varadhan, The ns Manual, The VINT Project (2005).
- [4] R. Barr, Z. J. Haas, R. van Renesse, Softw. Pract. Exper. 35 (6) (2005) 539–576.
- [5] R. Barr, Z. J. Haas, R. van Renesse, Scalable wireless ad hoc network simulation, in: J. Wu (Ed.), Handb. on theor. and algorithm. aspects of sensor, ad hoc wirel., and peer-to-peer netw., CRC Press, 2005, Ch. 19, pp. 297–311.
- [6] R. Barr, An efficient, unifying approach to simulation using virtual machines, Ph.D. thesis (2004).
- [7] S. Fekete, A. Kroller, S. Fischer, D. Pfisterer, Shawn: The fast, highly customizable sensor network simulator, in: Proc. 4th Int. Conf. on netw. sens. syst., 2007, p. 299.
- [8] B. Kerner, S. Klenov, A. Brakemeier, Testbed for wireless vehicle communication: a simulation approach based on three-phase traffic theory, in: Intell. Veh. Symp., IEEE, 2008, pp. 180–185.
- [9] A. Kesting, M. Treiber, M. Schönhof, F. Kranke, D. Helbing, Jam-avoiding adaptive cruise control (ACC) and its impact on traffic dynamics, in: A. Schadschneider, T. Pöschel, R. Kühne, M. Schreckenberg, D. E. Wolf (Eds.), Traffic and Granular Flow'05, Springer, 2007, pp. 633–643.
- [10] K. Matheus, R. Morich, I. Paulus, C. Menig, A. Lübke, B. Rech, W. Specks, Car-to-Car Communication - Market Introduction and Success Factors, in: 5th Eur. Congr. and Exhib. on Intell. Transp. Syst. and Serv., 2005.
- [11] G. Klot, Technion extensions of the JiST/SWANS simulator, <http://www.cs.technion.ac.il/~gabik/Jist-Swans/> (Sep 2010).

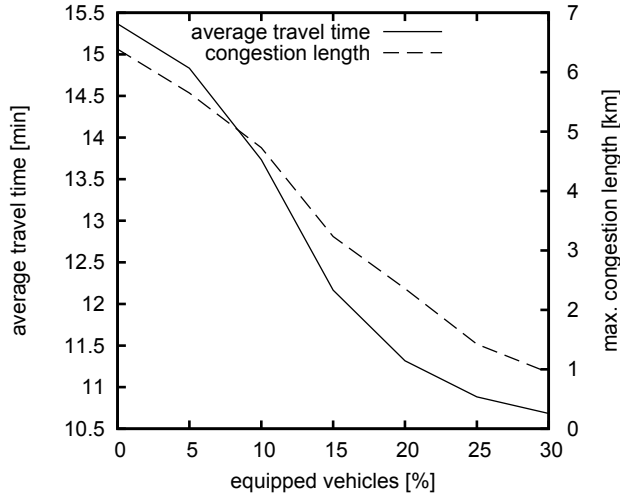


Figure 3: Congestion lengths and travel times decrease with an increasing number of communicating vehicles.

- [12] K. Ibrahim, M. C. Weigle, ASH: Application-aware SWANS with highway mobility (poster), in: Proc. IEEE INFOCOM Workshop on mob. netw. for veh. environ., 2008.
- [13] W.-X. Wang, B.-H. Wang, W.-C. Zheng, C.-Y. Yin, T. Zhou, Phys. Rev. E 72 (6 Pt 2) (2005) 066702.
- [14] M. Fiore, Vehicular mobility models, in: S. Olariu, M. Weigle (Eds.), Vehicular Networks from Theory to Practice, Chapman & Hall/CRC, 2009, Ch. 12.
- [15] K. Nagel, M. Schreckenberg, A cellular automaton model for freeway traffic, J. Phys. I 2 (12) (1992) 2221–2229.
- [16] W. Knospe, L. Santen, A. Schadschneider, M. Schreckenberg, J. Phys. A 35 (15) (2002) 3369–3388.
- [17] B. Kerner, Phys. World 12 (8) (1999) 25–30.
- [18] W. Knospe, L. Santen, A. Schadschneider, M. Schreckenberg, Phys. Rev. E 70 (1) (2004) 016115.
- [19] B. Kerner, The physics of traffic, Springer, 2004.
- [20] X.-G. Li, Z.-Y. Gao, B. Jia, Properties of cellular automaton model for on-ramp system, in: Proc. 9th Int. Conf. on Cellular automata for res. and ind., Springer, 2010, pp. 613–618.
- [21] F. Hall, S. Wakefield, A. Al-Kaisy, Transp. Res. Rec. 1776 (2001) 17–23.
- [22] Q. Tian, H. Huang, H. Yang, An analysis on the morning commuting equilibrium in a corridor network: Based on the fundamental diagram of traffic flow, in: Proc. 12th Int. Conf. on Travel Behav. Res., 2003.
- [23] A. Kesting, Microscopic modeling of human and automated driving: Towards traffic-adaptive cruise control, Ph.D. thesis (2008).