Analytical probability to have splitting

In[∘]:= $$\text{Anal}[\sigma\_] = \text{Integrate}\left[\text{Cos}\left[\frac{x}{2}\right]^2 \text{PDF}[\text{NormalDistribution}[0,\ \sigma],\ x],\ \{x,\ -\infty,\ \infty\},\ \text{Assumptions} \to \sigma > 0\right] \text{ // FullSimplify}$$

$$\text{Integrate}\left[\text{Sin}\left[\frac{x}{2}\right]^2 \text{PDF}[\text{NormalDistribution}[0,\ \sigma],\ x],\ \{x,\ -\infty,\ \infty\},\ \text{Assumptions} \to \sigma > 0\right] \text{ // FullSimplify}$$

Out[∘]= $$\frac{1}{2}\left(1 + e^{-\frac{\sigma^2}{2}}\right)$$

Out[∘]= $$\frac{1}{2} - \frac{1}{2} e^{-\frac{\sigma^2}{2}}$$

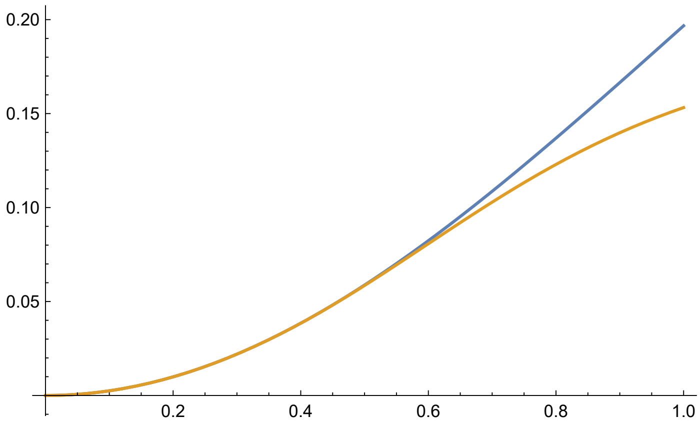
Numerical probability to have splitting

In[∘]:= $$\text{Num}[\sigma\_] = \frac{2}{\text{Erf}\left[\frac{\pi}{\sqrt{2\sigma^2}}\right]\sqrt{2\pi\sigma^2}} \text{Integrate}\left[\text{Cos}\left[\frac{x}{2}\right]^2 \left(\text{Exp}\left[-\frac{x^2}{2\sigma^2}\right] + \text{Exp}\left[-\frac{(x-\pi)^2}{2\sigma^2}\right]\right),\ \left\{x,\ 0,\ \frac{\pi}{2}\right\},\ \text{Assumptions} \to \sigma > 0\right] \text{ // FullSimplify}$$

Out[∘]= $$\frac{1}{4}\left(2 + \frac{e^{-\frac{\sigma^2}{2}}\left(2\,\text{Erf}\left[\frac{\pi - 2\,i\,\sigma^2}{2\sqrt{2}\,\sigma}\right] + \text{Erfc}\left[\frac{\pi - i\,\sigma^2}{\sqrt{2}\,\sigma}\right] + \text{Erfc}\left[\frac{\pi + i\,\sigma^2}{\sqrt{2}\,\sigma}\right] - 2\,\text{Erfc}\left[\frac{\pi + 2\,i\,\sigma^2}{2\sqrt{2}\,\sigma}\right]\right)}{\text{Erf}\left[\frac{\pi}{\sqrt{2}\,\sigma}\right]}\right)$$

In[∘]:= `Plot[{1 - Anal[x], 1 - Num[x]}, {x, 0, 1}] // Quiet`

Out[∘]=



In[∘]:= `Pshift[σ_] := 1 - Anal[σ]`

Parameters

```
shots = 10000 - 2500; (*Number of shots (minus filtered ones)*)
Nch = 100;(*Number of states we want (measure of sparsity)*)
Nq = 40; (*Number of qubits*)
```

In[ ]:=
```
nsp = Ceiling[Log2[Nch]];
eq = Sum[PDF[BinomialDistribution[Nq, Pshift[√Σ]], i], {i, 0, nsp}] // FullSimplify
sol = NSolve[{eq == 1 - 1/shots, Σ > 0}, {Σ}][[1]]
Print["σ = ", √Σ //. sol]
```

Out[ ]=

$$\frac{1}{1\,099\,511\,627\,776} e^{-20\,\Sigma} \left(1 + e^{\Sigma/2}\right)^{33} \left(-15\,380\,937 + e^{2\,\Sigma} \left(-668\,185\,595 + 1\,056\,757\,944\,\mathrm{Cosh}\left[\frac{\Sigma}{2}\right] - 509\,621\,244\,\mathrm{Cosh}[\Sigma] + 136\,429\,960\,\mathrm{Cosh}\left[\frac{3\,\Sigma}{2}\right] - 203\,926\,866\,\mathrm{Sinh}\left[\frac{\Sigma}{2}\right] + 206\,121\,630\,\mathrm{Sinh}[\Sigma] - 89\,945\,882\,\mathrm{Sinh}\left[\frac{3\,\Sigma}{2}\right]\right)\right)$$

Out[ ]= $\{\Sigma \to 0.162257\}$

σ = 0.402811