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plot3logit: Ternary Plots for Interpreting Trinomial Regression Models

Flavio Santi
University of Verona

Maria Michela Dickson
University of Trento

Giuseppe Espa
University of Trento

Diego Giuliani
University of Trento

Abstract

This paper presents the R package **plot3logit** which enables the covariate effects of trinomial regression models to be represented graphically by means of a ternary plot. The aim of the plots is helping the interpretation of regression coefficients in terms of the effects that a change in values of regressors has on the probability distribution of the dependent variable. Such changes may involve either a single regressor, or a group of them (composite changes), and the package permits both cases to be handled in a user-friendly way. Moreover, **plot3logit** can compute and draw confidence regions of the effects of covariate changes and enables multiple changes and profiles to be represented and compared jointly. Upstream and downstream compatibility makes the package able to work with other R packages or applications other than R.

Keywords: plotting software, ternary diagrams, R, **plot3logit**.

1. Introduction

The interpretation of the covariate effect on the probability distribution of the dependent variable of a multinomial regression model is usually neither immediate nor easy. In case of multinomial logit regression, the coefficient of a covariate x referred to the category $\nu^{(m)}$ of the dependent variable determines the average effect of a unitary change in the value of x on the log-odds of category $\nu^{(m)}$ with respect to a reference category $\nu^{(1)}$ of the dependent variable. This modelisation entails that the relation between the covariate coefficients and the probability distribution of the dependent variable is non-linear and depends also on covariate coefficients of other regressors (see Santi, Dickson, and Espa 2019, equations 5 and 6).

The interpretive difficulty of multilogit model coefficients is the reason why the coefficient estimates are usually complemented by some estimates or graphical representations of covariate marginal effects. In both cases, measures are computed and graphs are plotted conditionally to some specific values of the covariates, thus they cannot exhaustively describe the effect of a covariate over the whole space of regressors.

In order to overcome this limitation, Tutz and Schauberger (2013) proposed a reparametrisation of the multinomial logit model (based on the symmetric side constraint) and a graphical representation which allows for a representation of the direction (increase vs decrease) and the relative magnitude of the conditional effect of covariates on the probability distribution of the dependent variable.

In case of multinomial logit models where the dependent variable can take only three values (i.e., the trinomial logit models), Santi *et al.* (2019) show that it is possible to represent the effects of covariates in terms of changes in the probability distribution of the dependent variable by means of a vector field drawn over a ternary plot. The representation of covariate effects can be made both conditionally and unconditionally to the values of the covariates and it can be obtained for changes involving two or more covariates (composite changes).

The graphical representation proposed in Santi *et al.* (2019) is implemented in R (R Core Team 2020) trhough package **plot3logit** (Santi, Dickson, and Espa 2020), available on CRAN since January 2019.

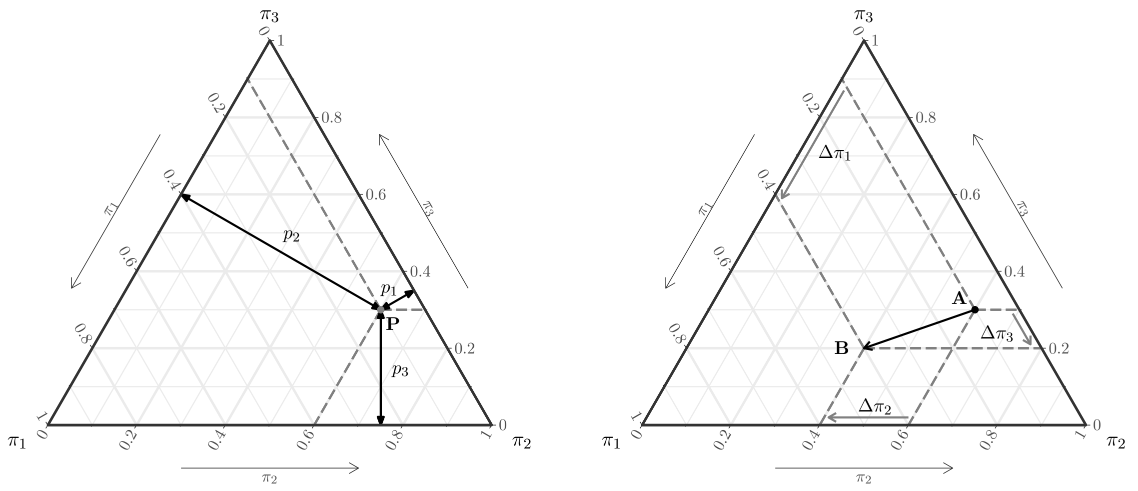
Package **plot3logit** can read the results of both categorical and ordinal trinomial logit regression fitted by various functions (see Section 3) and creates a `field3logit` object which may be represented by means of functions either based on standard R graphics or based on the grammar of graphics (Wilkinson 2005). Composite changes and multiple changes of covariates can be easily represented through a simple and flexible syntax, whereas the analysis proposed in Santi *et al.* (2019) has been extended by including functions for adding confidence regions of the covariate effects to the plots, in order to enrich and improve the interpretation of the results.

The paper is organised as follows. Section 2 briefly shows how to read ternary plots and how the effects of covariate changes on the probability distribution of a trinomial dependent variable can be represented through vector fields and arrows on a ternary plot. Section 3 summarises the features of the package **plot3logit**. Section 4 illustrates how **plot3logit** reads estimates from fitted models, and how the vector fields can be customised, computed and represented graphically. Section 5 illustrates how confidence regions are computed and drawn. Section 6 introduces some wrappers. Finally, Section 7 concludes.

2. Ternary plots and trinomial logit regression

Ternary diagrams were firstly proposed in Bancroft (1897) as a method for representing sets of three numbers from bounded non-negative intervals subject to a constraint on their sum. This is the case of composition data as well as the probabilities of a trinomial random variable. Here we briefly sum up how ternary diagrams work, whereas a more detailed illustration is available in Santi *et al.* (2019). Howarth (1996) offers a valuable and intriguing historical reconstruction of the ternary diagrams.

Consider a random element N which takes values in a set of three labels $\{\nu^{(1)}, \nu^{(2)}, \nu^{(3)}\}$ with probability $\pi_m \equiv \mathbb{P}[N = \nu^{(m)}]$, $m = 1, 2, 3$. The probability distribution of N can be



(a) (b)

Figure 1: Figure (a) shows how the coordinates of a point $P = (p_1, p_2, p_3)$ can be read in a ternary diagram. Figure (b) shows how a change in the probability distribution of a trinomial random variable from $A = (0.1, 0.6, 0.3)$ to $B = (0.4, 0.4, 0.2)$ can be represented, and decomposed in terms of changes of ternary coordinates. Both graphs are taken from Santi *et al.* (2019).

represented through the triplets $(\pi_1, \pi_2, \pi_3) \in [0, 1]^3$, however the parameter space is actually 2-dimensional, as the sum $\pi_1 + \pi_2 + \pi_3$ is constrained to equal one, thus if π_1 and π_2 are given, π_3 automatically equals $1 - \pi_1 - \pi_2$.[1] Mathematically, triplets (π_1, π_2, π_3) which are valid probability distributions define a 2-dimensional simplex in the 3-dimensional space $[0, 1]^3$ which is denoted by S in the rest of the paper. Formally:

$$S = \{(\pi_1, \pi_2, \pi_3) \in [0, 1]^3 \colon \pi_1 + \pi_2 + \pi_3 = 1\}\,. \tag{1}$$

The simplex S is the equilateral triangle which constitutes the ternary diagram (see Figure 1).

Figure 1a shows how the Cartesian coordinates of a point $P = (p_1, p_2, p_3)$ in the 3-dimensional space $[0, 1]^3$ are transposed over the 2-dimensional simplex (the ternary diagram). Note that the value of a coordinate of the point P (say, p_3) is the distance between P and the side opposite the vertex labelled with that component (that is, π_3).

Since all (and only) the admittable probability distributions of a trinomial random variable can be drawn as a point of the simplex of the ternary diagram, a change in any probability distribution can be represented through an arrow starting from a reference starting point A towards a final point B, as shown in Figure 1b.

The ternary diagram in Figure 1b is the basis for representing the effect of a change in one or

[1]Random element N is typically modelled by means of a random vector which is distributed according to a single-trial multinomial law and it is defined through indicator functions. See Santi *et al.* (2019) for this formalisation of the problem, Johnson, Kemp, and Kotz (2005) (pp. 505–524) on the multinomial probability distribution, and Agresti (2013) on the modellisation of categorical responses.

more covariates on the probability distribution of the dependent variable of a trinomial logit regression.

Consider a trinomial logit regression on p covariates $x = (x_1, x_2, \ldots, x_p)$ (including a constant term) and a profile $x_0 \in \mathcal{X} \subseteq \mathbb{R}^p$, so that $(\pi^{(1)}_{(x_0)}, \pi^{(2)}_{(x_0)}, \pi^{(3)}_{(x_0)})$ is the probability distribution associated to $x = x_0$. It can be shown (see Santi *et al.* 2019, equation 6) that the probability distribution of the dependent variable when $x = x_0 + \Delta$ changes as follows:

$$\pi^{(m)}_{(x_0+\Delta)} = \left[1 - \sum_{h=2}^{3} \left(1 - \mathrm{e}^{\Delta^{\mathrm{T}}\beta^{(h)}}\right) \pi^{(h)}_{(x_0)}\right]^{-1} \mathrm{e}^{\Delta^{\mathrm{T}}\beta^{(m)}} \pi^{(m)}_{(x_0)}\,, \tag{2}$$

with $m = 1, 2, 3$, and where $\Delta \in \mathbb{R}^p$ is the change of covariates, $\beta^{(m)} \in \mathbb{R}^p$ are the regression coefficients, and $\beta^{(1)} = 0 \in \mathbb{R}^p$ by construction (see Santi *et al.* 2019).

As (2) shows, the probability distribution after the covariate change Δ only depends on the probability distribution before change $\pi^{(m)}_{(x_0)}$ ($m = 1, 2, 3$) and the coefficients of the trinomial regression, whereas there is not dependence from x_0 other than through $\pi^{(m)}_{(x_0)}$. Relation (2) is thus the theoretical basis which justifies the graphical method proposed in Santi *et al.* (2019), as it allows one to represent and analyse the regression coefficients $\beta^{(2)}, \beta^{(3)}$ over the (2-dimensional) simplex S, instead of the (k-dimensional) space of regressors $\mathcal{X}$. In the following, an example of the method is provided in order to illustrate some of the capabilities of the package **plot3logit**, which are discussed in depth in the next sections of the paper.

A trinomial regression is fitted on self-reported votes for US presidential elections in 2016. Data are provided in Democracy Fund Voter Study Group (2017), where a broad an detailed questionnaire was administred to a sample consisting of 8000 people. In this paper a dataset where only some information collected by Democracy Fund Voter Study Group (2017) is used for illustration purposes. The dataset is made available through the package **plot3logit** under the name `USvote2016`.

In the following we consider a trinomial logit regression which models the self-reported vote (which may take values “Trump”, “Clinton”, and “Other”) over some voters’ characteristics (education level, gender, race, and decade when the voter was born). Here there are the R commands for fitting the model through the package **nnet**:

```
R> library("nnet")
R> data("USvote2016", package = "plot3logit")
R> modVote <- multinom(vote ~ educ + gender + race + birthyr,
+    data = droplevels(USvote2016), trace = FALSE)
```

and Table 1 shows point estimates and standard errors of regression coefficients.

Consider, for example, the coefficients on the regressor `genderFemale`. As the estimates in Table 1 show, both coefficients are negative and statistically different from zero, meaning that, ceteris paribus, female voters had a preference towards Hillary Clinton. Such a preference results in an increase (with respect to male voters with the same characteristics) of the probability to vote for Hillary Clinton to the detriment of Donald Trump and all other candidates. What is hard to assess is the actual effect of gender on the probability distribution of voter’s choice, Figure 2a helps in that by representing the effect of covariate `genderFemale` through a vector field over a ternary diagram.

Table 1: Trinomial logit regression of voter choice (reference level is "Clinton") based on 7590 observations of the Democracy Fund Voter Study Group (2017). All regressors are qualitative: education (ref: "no high school"), gender (ref: "male"), race/ethnicity (ref: "white"), birth year (ref: $[1920, 1940)$). Table shows point estimates followed by significance symbols (* for p-values between 0.05 and 0.1; ** for p-values between 0.01 and 0.05; *** for p-values smaller than 0.01). Standard errors are reported in parenthesis.

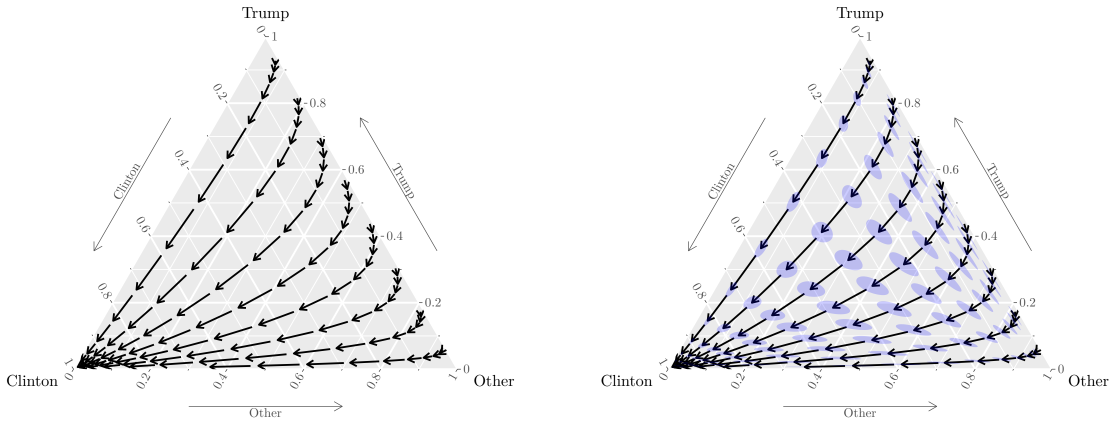
Regressor	*Levels*			
	Trump		Others	
Constant	1.293***	(0.224)	−1.712***	(0.445)
Education				
High school grad.	−0.379*	(0.214)	−0.499	(0.393)
Some college	−0.739***	(0.215)	−0.670*	(0.391)
2-year college	−0.640***	(0.223)	−0.602	(0.405)
4-year college	−0.934***	(0.215)	−0.542	(0.387)
Post-grad	−1.252***	(0.217)	−0.733*	(0.391)
Gender (female)	−0.536***	(0.051)	−0.440***	(0.092)
Race/ethnicity				
Black	−2.560***	(0.150)	−1.087***	(0.191)
Hispanic	−0.479***	(0.114)	−0.004	(0.184)
Asian	−0.912***	(0.246)	−0.296	(0.321)
Mixed	−0.383**	(0.180)	−0.042	(0.281)
Other	0.579***	(0.162)	0.141	(0.303)
Birth year (decade)				
$[1940, 1950)$	−0.058	(0.109)	0.117	(0.291)
$[1950, 1960)$	0.012	(0.104)	0.638**	(0.270)
$[1960, 1970)$	0.081	(0.107)	1.058***	(0.268)
$[1970, 1980)$	−0.302**	(0.123)	1.166***	(0.279)
$[1980, 2000)$	−1.001***	(0.141)	1.365***	(0.280)

The direction of arrows in Figure 2a is consistent with the conclusion outlined before, although the diagram shows also that the direction is not constant over the simplex. On the other hand, arrow lengths enable to assess the magnitude of the effect, which is not constant and cannot be directly appraised from estimates in Table 1.

Figure 2b includes also the 95% confidence regions in order to assess also the degree of uncertainty of the estimates and how uncertainty on regression parameters determines the uncertainty on the effects (note how shapes and sizes of confidence regions changes over the simplex).

Confidence regions are particularly useful when the effect of a covariate change is analysed for some specific profiles (see Figure 3), or when multiple effects are compared with respect to a single (common) profile, as in Figure 4.

Figure 3 shows the effects of gender on five voter profiles distinguished only by the racial/ethnic group they belong to. The graph shows how the magnitude of the gender effect changes



(a) (b)

Figure 2: Vector field on the effect of gender (covariate `genderFemale`) on the probability distribution of voter's choice (Figure 2a). Figure 2b shows the same vector field with 95% confidence regions. Coefficient estimates are reported in Table 1.

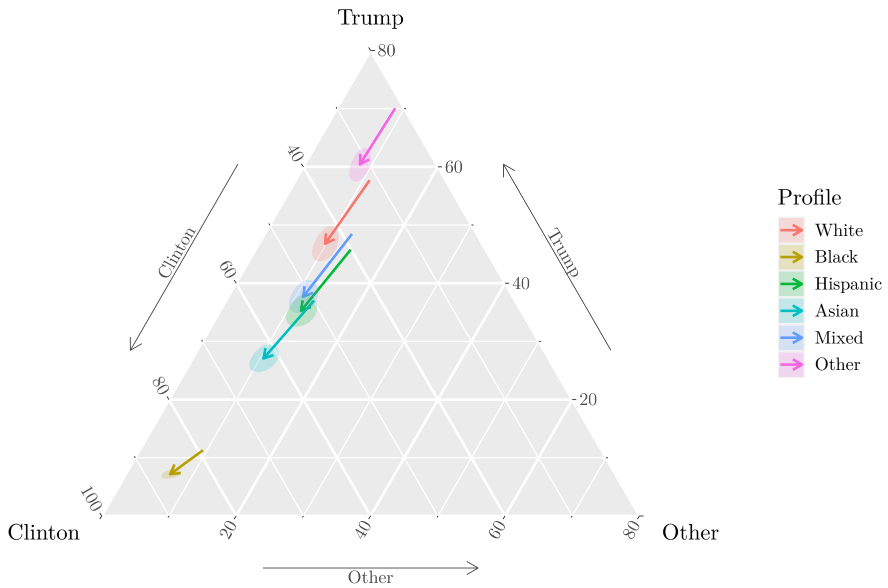


Figure 3: Effect of gender on the probability distribution of voter's choice born in the Seventies and graduated at the high school, distinguished by racial or ethnic group. 95%-confidence regions are drawn. Coefficient estimates are reported in Table 1. Note that only a portion of the simplex is represented in this graph.

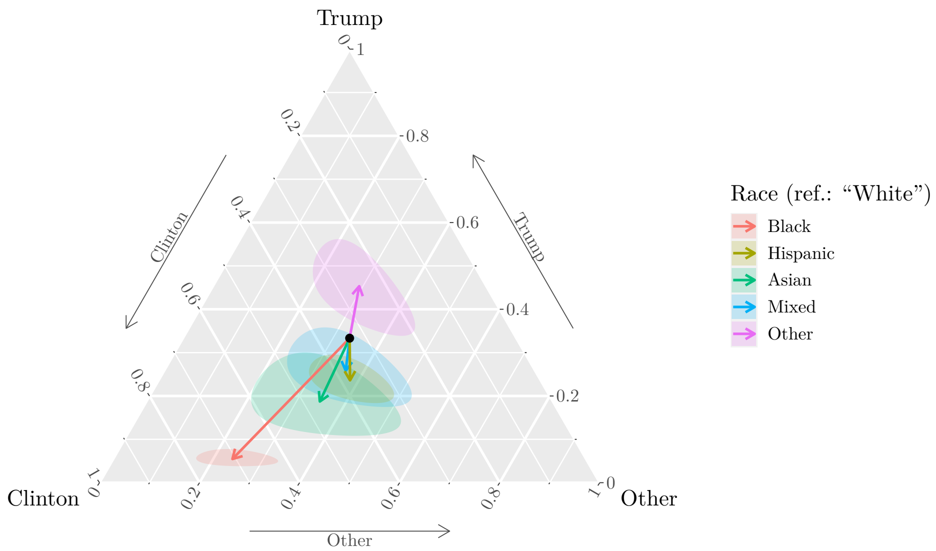


Figure 4: Effect of race on the probability distribution of voter's choice with respect to a white voter having the same probability of choosing Clinton (33.3%), Trump (33.3%) or other candidates (33.3%). 95%-confidence regions are drawn. Coefficient estimates are reported in Table 1.

amongst different groups.

Figure 4 shows the effects of covariates on race with respect to a white voter having the same probability of choosing Clinton (33.3%), Trump (33.3%) or other candidates (33.3%). Ternary diagram enables the reader to assess the direction and the magnitude of differences of voters' preferences by voters' race as well as the degree of uncertainty of the estimates by means of 95% confidence regions.

In the rest of the paper it is illustrated and discussed how diagrams like those in Figure 2, 3, and 4 can be drawn by means of package **plot3logit**.

3. Features

In summary, the package **plot3logit** can:

- read the trinomial logit models fitted by function `multinom` of package **nnet** (Venables and Ripley 2002), function `polr` of package **MASS** (Venables and Ripley 2002), function `mlogit` of package **mlogit** (Croissant 2020), and function `vgam` of package **VGAM** (Yee 2010). Moreover, any other estimates obtained from other packages can be passed explicitly in matrix form and processed by **plot3logit**;
- represent any kind of effect of changes of regressor values involving one or several regressors by means of both vector of changes and mathematical expression involving the names of the covariates;

- work both under standard R graphics paradigm through package **Ternary** (Smith 2017), and under the paradigm of the grammar of graphics (Wilkinson 2005) through packages **ggtern** (Hamilton and Ferry 2018) and **ggplot2** (Wickham 2016). Moreover, methods `as.data.frame`, `as_tibble`, `fortify` and `tidy` enable the graphical data to be easily exported in a standardised format which may be used for drawing ternary fields through any other pacakge or software;
- fully customise any feature of ternary fields, including position, number, and alignment of arrows;
- draw and handle several fields over the same plot, so that the effects of different changes of covariates can be compared;
- compute and draw confidence regions for each effect of covariate change, so that uncertainty about estimates of effects can be shown visually;
- quickly compute and draw ternary fields and confidence regions under standard settings through several wrappers which make the code shorter and easier to write and read.

4. Computation and representation of vector fields

4.1. Computation of vector fields

Function `field3logit` computes the vector field which represents the effects of a change of one or several covariates on the probability distribution of the depedent variable, according to a fitted model. It follows that the two most important arguments of `field3logit` are the parameter estimates of the model (argument `model`) and the change of covariate values (argument `delta`). In this section it is illustrated how they can be set.

Read model estimates

Model estimates are passed to `field3logit` by means of argument `model` and, possibly, arguments `alpha` and `vcov`.

When the trilogit model is fitted through function `multinom` of package **nnet** (Venables and Ripley 2002), function `polr` of package **MASS** (Venables and Ripley 2002), function `mlogit` of package **mlogit** (Croissant 2020), or function `vgam` of package **VGAM** (Yee 2010), `field3logit` automatically extracts all relevant information from the objects returned by those functions, and argument `model` is the only to be specified.

Things are different when estimates are not available as output of the previous functions. In that case, point estimates should be passed to argument `model` and, if an ordinal model is considered, argument `alpha` should be specified too. Finally, argument `vcov` can be optionally passed to `field3logit` if the computation of confidence regions is needed (see Section 5). In the following it is illustrated how these parameters should be set.

Point estimates of coefficients. When the estimates of a trinomial regression cannot be passed through objects returned by the previous functions, argument `model` should consist in a numeric matrix of coefficient point estimates. The number of rows should be equal to the number of covariates and the names of covariates should be added as row names. The intercepts should be included only in case of categorical models, whereas the intercepts of the ordinal models should be passed to argument `alpha`. The columns should be one or two according to whether the model is ordinal or not, respectively. Column names, if provided, are ignored, whereas the labels of the dependent variable should be specified by means of attribute `levels`, as a character vector where the first label is considered as the reference level.

Here it is an example on how the matrix should be defined in case of a categorical trinomial logit regression whith four covariates (a constant term, X_1, X_2 and X_3) and where the dependent variable takes values "Class A" (reference level), "Class B", "Class C":

```
R> library("magrittr")
R> matrix(c(2, 0.3, -0.2, 0.2, 1, 0.1, -0.4, -0.3), 4, 2) %>%
+    set_rownames(c("(Intercept)", "X1", "X2", "X3")) %>%
+    structure(levels = c("Class A", "Class B", "Class C")) -> fittedModel
R> fittedModel
```

```
            [,1] [,2]
(Intercept)  2.0  1.0
X1           0.3  0.1
X2          -0.2 -0.4
X3           0.2 -0.3
attr(,"levels")
[1] "Class A" "Class B" "Class C"
```

Variance-covariance matrix estimates. When point estimates are passed to argument `model`, argument `vcov` can be used for passing the variance-covariance matrix of coefficient estimates to `field3logit`. The variance-covariance matrix should be structured as a numeric matrix (or any coercible object) where the number of rows and columns equals the number of parameters. Rows and columns should be ordered according to the labels of the dependent variable (slower index), and then to the covariates (faster index).

Specification of covariate changes

The change of regressor values may be expressed in two different ways.

Firstly, it may be passed to `field3logit` explicitly as a numeric vector where each component specifies the change of the corresponding regressor. The vector is thus the same denoted by Δ in equation (2).

Consider, for example, the effect of the dummy variable `genderFemale`, which is the seventh covariate (including the constant term) of the model stored in `modVote`. The vector Δ should be defined as follows:

```
R> Delta <- rep(0, 17)
R> Delta[7] <- 1
R> Delta
```

```
 [1] 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
```

then the `field3logit` function enables the vector field in Figure 2a to be computed as follows:

```
R> library("plot3logit")
R> field3logit(model = modVote, delta = Delta)

 Object of class "field3logit"
-------------------------------
Label                      : <empty>
Possible outcomes          : Clinton; Trump; Other
Type of model              : categorical
Effect                     : 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
Model has been read from   : nnet::multinom
Number of stream lines     : 8
Number of arrows           : 106
Covariance matrix          : available
Confidence regions         : not available
```

As an alternative, the change of covariates can be passed to argument `delta` in the form of a character expression in R language. The expression is then evaluated using the covariate names and the implicit vector Δ is computed. For example, the vector field in Figure 2a has been generated through the following command:

```
R> field3logit(model = modVote, delta = "genderFemale")

 Object of class "field3logit"
-------------------------------
Label                      : <empty>
Possible outcomes          : Clinton; Trump; Other
Type of model              : categorical
Effect                     : genderFemale
Explicit effect            : 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
Model has been read from   : nnet::multinom
Number of stream lines     : 8
Number of arrows           : 106
Covariance matrix          : available
Confidence regions         : not available
```

It is worth noting that attribute `Effect` of the `field3logit` object obtained from the former command coincides with attribute `Explicit effect` of the latter `field3logit` object.

The use of R code (passed as a `character`) for expressing changes of covariates makes the `field3logit` function easy to use, especially when changes are fractional or involve several covariates. Consider, for example, the following command where object `fittedModel` has been generated before:

```
R> field3logit(model = fittedModel, delta = "0.5 * X1 + X3 - 2 * X2")
```

```
 Object of class "field3logit"
-------------------------------
Label                        : <empty>
Possible outcomes            : Class A; Class B; Class C
Type of model                : categorical
Effect                       : 0.5 * X1 + X3 - 2 * X2
Explicit effect              : 0 0.5 -2 1
Model has been read from     : matrix
Number of stream lines       : 8
Number of arrows             : 80
Covariance matrix            : not available
Confidence regions           : not available
```

The code is easy-to-read, easy-to-write, and does not depend on the order that covariates have in the formula of the fitted model, unlike what happens when the explicit vector of covariate changes is passed to `field3logit`.

Set up the vector field

In addition to `model` and `delta`, arguments `p0`, `nstreams`, `narrows` and `edge` enable the user to define how many arrows the vector field should consist of, and where they should be placed within the simplex of the ternary plot.

Figure 5 shows four different variations (using package **Ternary** instead of **ggtern** — see Section 4.2) of the field drawn in Figure 2a, and the following is the R code that generated Figure 5:

```
R> ptsAB <- list(A = c(0.3, 0.4, 0.3), B = c(0.5, 0.1, 0.4))
R> par(mfrow = c(2, 2), cex = 0.5, mar = rep(0, 4))
R> # Top-left
R> field3logit(modVote, "genderFemale", edge = 0.1) %>% plot
R> # Top-right
R> field3logit(modVote, "genderFemale", nstreams = 4) %>% plot
R> # Bottom-left
R> field3logit(modVote, "genderFemale", p0 = ptsAB) %>% plot
R> TernaryPoints(ptsAB)
R> TernaryText(ptsAB, labels = names(ptsAB), pos = 1)
R> # Bottom-right
R> field3logit(modVote, "genderFemale", p0 = ptsAB, narrows = 1) %>% plot
R> TernaryPoints(ptsAB)
R> TernaryText(ptsAB, labels = names(ptsAB), pos = 1)
```

The top-left graph in Figure 5 shows the effect of argument `edge`, which sets the minimum distance between the starting and the ending point of each arrow of the field from the sides of the simplex. Vector field represented in Figure 2a has been generated using the default value of `edge` (0.01), whereas the top-left diagram in Figure 5 has been generated with `edge = 0.1`.

As diagram in Figure 2a clearly shows, arrows of ternary fields are arranged along some stream lines. Argument `nstreams` sets the number of stream lines to draw (default value is

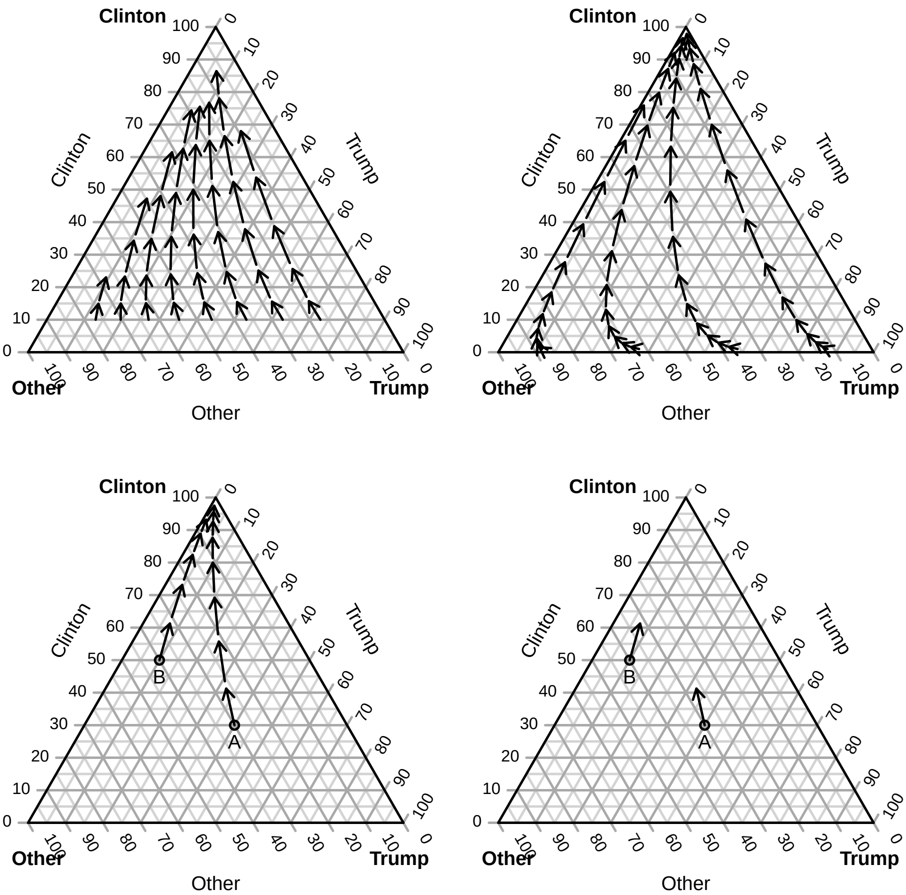


Figure 5: Vector fields on the effect of covariate `genderFemale` (see Figure 2a) generated by `field3logit` with different values of argument `edge` (top-left), `nstreams` (top-right), `p0` (bottom-left), `p0` and `narrows` (bottom-right).

8). `field3logit`, when it generates the field, automatically spreads the stream lines over the simplex in order to produce a field which is graphically optimal. Top-right diagram in Figure 5 shows the vector field on the effect of `genderFemale` (see Figure 2a) where `nstreams = 4`.

Argument `p0` enables one to set the starting points of the stream lines, in order to customise the behaviour of `field3logit`. Argument `p0` should be structured as a `list` whose components are `numeric` vectors of ternary coordinates (see object `ptsAB`, defined before). Bottom-left graph in Figure 5 shows an example where points $A = (0.3, 0.4, 0.3)$ and $B = (0.5, 0.1, 0.4)$ are set as starting points of two stream lines.

Finally, argument `narrows` sets the maximum number of arrows which should be computed for each stream line.[2] Bottom-right graph in Figure 5 shows the same field drawn in the bottom-left graph, where `narrows = 1`. Default value of `narrows` is `Inf`, so that arrows are added to a stream line until the edge set through argument `edge` has been reached.

[2]If the stream line reaches the edge of the simplex, the number of arrows actually drawn may be smaller than `narrows`.

4.2. Representation of vector fields

Vector fields computed by `field3logit` may be represented through functions provided by package **Ternary** (Smith 2017) which is based on standard R graphics, or functions of package **ggtern** (Hamilton and Ferry 2018), which extends package **ggplot2** (Wickham 2016) to ternary diagrams, and it is based on the programming paradigm referred to as "grammar of graphics" (see e.g., Wickham 2016; Wickham and Grolemund 2016) illustrated in Wilkinson (2005).

Plotting by means of package Ternary

Two functions of **plot3logit** enable to draw vector fields of `field3logit` objects through package **Ternary**.

Function `TernaryField` takes a `field3logit` object as first argument and permits the vector field to be added to an existing ternary diagram created by function `TernaryPlot` of package **Ternary**. Both name and argument structure of `TernaryField` are consistent with other functions defined in package **Ternary** (such as `TernaryPoint`, `TernaryPolygon`, . . .).

The S3 method of generic `plot` takes a `field3logit` object as first argument and may either draw the ternary diagram from scratch (if argument `add` is set to `FALSE`), or add the vector field to an existing ternary plot (if `add = TRUE`), and in that case it basically works as a wrapper of `TernaryField`.

Some examples of the graphical rendering of vector fields drawn by means of package **Ternary** are shown in Figure 5.

Clearly, package **plot3logit** does not limit in any way the customisation of the graphs made available by standard R graphics methods and by package **Ternary** (see manuals of **plot3logit** and **Ternary** for details).

Plotting by means of package ggtern

Vector fields of `field3logit` objects can be drawn through package **ggtern** by means of the constructor `gg3logit`, the statistics `stat_field3logit`, `stat_conf3logit`, `stat_3logit`, and the S3 method of generic `autoplot` for class `field3logit`.

As opposed to **ggplot2** (and thus **ggtern**) philosophy, which only accepts `data.frame`s (or any other object of child classes, such as `tibble`) as input for argument `data`, package **plot3logit** handles both `data.frame`s and `field3logit` objects.

This choice has been made in order to make the code simple, as if a `field3logit` object is passed to `gg3logit`, the conversion to a `data.frame` and the initialisation of aesthetic parameters (through the function `aes`) passed to argument `mapping` are carried out automatically.

On the contrary, if a `data.frame` (or any coercible object, including objects of child classes) is passed to argument `data` of `gg3logit`, the following aesthetics must be specified:

- `x`, `y`, `z` are required by:
  - `stat_field3logit` as ternary coordinates of the starting points of the arrows;
  - `stat_conf3logit` as ternary coordinates of the points on the edge of confidence regions (see Section 5);

- `xend`, `yend`, `zend` are required by `stat_field3logit` as ternary coordinates of the ending points of the arrows;
- `group` is always required as it is the identifier of groups of graphical objets (arrows and their confidence regions);
- `type` is always required as it specifies the type of graphical object (arrows or confidence regions);

whereas the following variables of a fortified `field3logit` or a `multifield3logit` object (see next section)[3] may be useful for defining other standard aesthetics (such as `fill`, `colour`, ...):

- `label` identifies a field through a label, thus it is useful for distinguishing the fields in a `multifield3logit` object.
- `idarrow` identifies each group of graphical objets (arrows and their confidence regions) *within* every field. Unlike variable `group`, `idarrow` is not a global identifier of graphical objets.

`multifield3logit` objects and confidence regions are illustrated in depth in the next sections of the paper.

As a first example on function `gg3logit`, it follows the R code for plotting the ternary diagram in Figure 2a:

```
R> fieldFemale <- field3logit(modVote, "genderFemale")
R> gg3logit(fieldFemale) + stat_field3logit()
```

According to the previous code, when a `field3logit` object is passed to `gg3logit`, the syntax is particularly short, as no aestetic has to be set. On the contrary, if a fortified `field3logit` object is passed to `gg3logit`, several aesthetics have to be initialised and the code is longer and less easy to read.

In order to compare the two syntaxes, consider the structure of the fortified `fieldFemale` object:

```
R> field3logit(modVote, "genderFemale") %>% fortify -> fortfieldFemale
R> fortfieldFemale

# A tibble: 106 x 10
   label idarrow group type  Clinton  Trump  Other Clinton_end
   <fct> <fct>   <fct> <fct>   <dbl>  <dbl>  <dbl>       <dbl>
 1 <emp... C1A1    H055  arrow  0.0100 0.928  0.0623      0.0169
 2 <emp... C1A10   H106  arrow  0.736  0.222  0.0419      0.824
```

[3] An object is referred to as *fortified* whenever it is processed by the method `fortify` (see e.g. Wickham 2016) and thus it is structured as a `data.frame` which contains the information available in the original object. By extension, an object may be referred to as *fortified* whenever it is processed through functions such as `as.data.frame`, `as_tibble`, `tidy`.

```
 3 <emp... C1A11   H017  arrow  0.838  0.134  0.0283      0.897
 4 <emp... C1A12   H078  arrow  0.905  0.0765 0.0182      0.941
 5 <emp... C1A13   H005  arrow  0.946  0.0423 0.0113      0.967
 6 <emp... C1A2    H079  arrow  0.0186 0.913  0.0687      0.0312
 7 <emp... C1A3    H036  arrow  0.0343 0.890  0.0752      0.0568
 8 <emp... C1A4    H052  arrow  0.0623 0.857  0.0812      0.101
 9 <emp... C1A5    H051  arrow  0.111  0.804  0.0854      0.174
10 <emp... C1A6    H042  arrow  0.188  0.725  0.0864      0.282
# ... with 96 more rows, and 2 more variables: Trump_end <dbl>,
#   Other_end <dbl>
```

If `fortfieldFemale` is passed to `gg3logit` the code for drawing the diagram in Figure 2a becomes considerably longer:

```
R> gg3logit(fortfieldFemale, aes(
+       x = Clinton, y = Trump, z = Other,
+       xend = Clinton_end, yend = Trump_end, zend = Other_end,
+       group = group, type = type
+  )) +
+    stat_field3logit()
```

The simplicity of the former syntax is apparent, whereas the latter does not provide any practical advantage in terms of greater flexibility, notwithstanding the greater verbosity. This is the reason why the former syntax has been implemented, even though it deviates from orthodox **ggplot2** philosophy that requires that only `data.frame` objects can be passed to argument `data`.

Plotting by means of other packages/software

Besides the integration with packages **Ternary** and **ggtern**, package **plot3logit** guarantees a full downstream compatibility with other R packages or other applications through S3 methods of generics `as.data.frame`, `as_tibble` (package **tibble**, Müller and Wickham 2020), `fortify` (package **ggplot2**, Wickham 2016), and `tidy` (package **broom**, Robinson and Hayes 2020) for classes `field3logit` and `multifield3logit`. All four methods are equivalent, except that `as.data.frame` returns a `data.frame`, whereas others return a `tibble`.

The mentioned methods enable the graphical information (arrows, confidence regions and labels) of a `field3logit` or a `multifield3logit` object to be exported in a standardised table which can be read by any other R package or can be stored on disk through standard R commands (such as `write.csv`, for example) and then read by any other application.

4.3. Handling multiple fields

When the results of a multinomial regression are analysed, the comparison between the effects of various changes in covariate values may be of interest. Figure 4 shows how this kind of comparisons may be carried out by means of ternary plots.

Each Arrow in Figure 4 is associated to a distinct change in the value of one covariate, thus, diagram in Figure 4 may be interpreted as a superimposition of five vector fields consisting of a single arrow each and having the same profile as a reference point. This is actually the way Figure 4 has been generated.

`multifield3logit` is a S3 class which enables `field3logit` objects to be combined, handled, and represented jointly. Besides the standard constructor `multifield3logit`, objects of class `multifield3logit` can be created and combined through the operator `"+"`.[4]

The following code shows how covariate effects of dummies `raceBlack` and `raceHispanic` on a single reference profile where $(\pi_1, \pi_2, \pi_3) = (1/3, 1/3, 1/3)$ are combined in a `multifield3logit` object:

```
R> refprofile <- list(c(1/3, 1/3, 1/3))
R> fieldBlack <- field3logit(
+    model = modVote, delta = "raceBlack", label = "Black",
+    p0 = refprofile, narrows = 1
+  )
R> fieldHispanic <- field3logit(
+    model = modVote, delta = "raceHispanic", label = "Hispanic",
+    p0 = refprofile, narrows = 1
+  )
R> mfieldrace <- fieldBlack + fieldHispanic
R> mfieldrace

 Object of class "multifield3logit"
------------------------------------
Number of fields         : 2
Labels
  1. Black     (dX: raceBlack)
  2. Hispanic  (dX: raceHispanic)
```

The previous example permits also the usage of argument `label` to be clarified, as it is used by graphical functions for distinguishing and labelling the elements of a `multifield3logit` object according to the `field3logit` objects they belong to. This is the reason why, if a single `field3logit` object is defined and used, there is in general no need for initialising the argument `label`, whose default value is simply “`<empty>`”.

The S3 method `"+"` of class `multifield3logit` permits several (two or more) `field3logit` objects to be combined at once, and further `field3logit` objects to be included into an existing `multifield3logit` object:

```
R> fieldAsian <- field3logit(
+    model = modVote, delta = "raceAsian", label = "Asian",
+    p0 = refprofile, narrows = 1
+  )
R> mfieldrace <- mfieldrace + fieldAsian
R> mfieldrace
```

[4]The package makes available also the S3 methods of generics `"["` and `"[<-"` of class `multifield3logit` for extracting and replacing the `field3logit` objects the `multifield3logit` objects consist of. — See the help of **plot3logit** for details and further information.

```
 Object of class "multifield3logit"
------------------------------------
Number of fields         : 3
Labels
  1. Black     (dX: raceBlack)
  2. Hispanic  (dX: raceHispanic)
  3. Asian     (dX: raceAsian)
```

When several vector fields have to be generated and combined in a `multifield3logit` object, the syntax showed above is unnecessary long and in some cases pleonastic. For this reason, it is possible to rely on function `field3logit` by means of the syntax described below.

Assume that we are interested in comparing the effects of all dummies on race in the model on US elections. Let us thus define a list whose elements consist in lists where only varying arguments to be passed to function `field3logit` are specified:

```
R> race_effects <- list(
+    list(delta = "raceBlack", label = "Black"),
+    list(delta = "raceHispanic", label = "Hispanic"),
+    list(delta = "raceAsian", label = "Asian"),
+    list(delta = "raceMixed", label = "Mixed"),
+    list(delta = "raceOther", label = "Other")
+  )
```

If `race_effects` is passed to argument `delta` of `field3logit`, in this way:

```
R> mfieldrace <- field3logit(
+    model = modVote, delta = race_effects,
+    p0 = refprofile, narrows = 1
+  )
R> mfieldrace

 Object of class "multifield3logit"
------------------------------------
Number of fields         : 5
Labels
  1. Black     (dX: raceBlack)
  2. Hispanic  (dX: raceHispanic)
  3. Asian     (dX: raceAsian)
  4. Mixed     (dX: raceMixed)
  5. Other     (dX: raceOther)
```

the function `field3logit` is run once for every element of `race_effects`, and the set of `field3logit` objects are combined into a single object of class `multifield3logit`. When `field3logit` is applied to each element of `race_effects`, the arguments specified in the parent call of `field3logit` are used as default values which are overwritten by those specified in each element of `race_effects`.

The expedient just described enables the `multifield3logit` objects to be generated through a short and efficient syntax even if several `field3logit` objects are involved.

The ways `multifield3logit` objects are graphically represented are similar to those of `field3logit` objects, thus S3 method of generics `plot` draws a `multifield3logit` object through package **Ternary**, whereas functions `autoplot`, `gg3logit`, `stat_field3logit` make the ternary diagrams through the package **ggtern**. The only remarkable difference in case of function `gg3logit` and its statistics is in the variable `label` which enables various aesthetics to be set accordingly to the vector field of the `multifield3logit` object.

For example, the following code generates the diagram of Figure 4 (without confidence regions):

```
R> gg3logit(mfieldrace, aes(colour = label)) +
+    stat_field3logit() +
+    labs(colour = "Race (ref.: White)")
```

5. Confidence regions

Confidence regions for the effects of covariates on the probability distribution of the dependent variable are not considered in Santi *et al.* (2019), however they greatly enrich the information a ternary diagram can provide and help the interpretation of regression results. For these reasons, they have been implemented in package **plot3logit**. Section 5.1 illustrates how they are mathematically derived and how they can be computed through package **plot3logit**, whereas Section 5.2 shows how they can be represented graphically.

5.1. Computation

Let $\eta = [x^T\beta^{(2)}, x^T\beta^{(3)}]^T \in \mathbb{R}^2$ be the vector of linear predictors of the trinomial model, and let $B = [\beta^{(2)}, \beta^{(3)}] \in \mathbb{R}^{k\times 2}$ be the matrix of regression coefficients, whereas the matrix of point estimates is denoted by $\hat{B} \in \mathbb{R}^{k\times 2}$.

The effect of a change $\Delta \in \mathbb{R}^k$ of covariate vector $x \in \mathbb{R}^k$ on $\eta \in \mathbb{R}^2$ can then be expressed through vector $\delta \in \mathbb{R}^2$ as follows:

$$\delta = B^T\Delta = (I_2 \otimes \Delta)^T \, \mathrm{vec}(B)\,,$$

where I_2 is the identity matrix of order 2, $\otimes$ is the Kronecker product, and $\mathrm{vec}(B) \in \mathbb{R}^{2k}$ is the vectorisation of B.

If the point estimate of the variance-covariance matrix of $\mathrm{vec}(B)$ is $\hat{\Xi}$, the variance-covariance matrix of $\hat{\delta} = \hat{B}^T\Delta$ is:

$$(I_2 \otimes \Delta)^T \, \hat{\Xi} \, (I_2 \otimes \Delta)\,,$$

thus the $(1-\alpha)$-confidence region for δ consists of points $\tilde{\delta} \in \mathbb{R}^2$ that satisfy the following condition:

$$(\tilde{\delta} - \hat{\delta})^T [(I_2 \otimes \Delta)^T \, \hat{\Xi} \, (I_2 \otimes \Delta)]^{-1} (\tilde{\delta} - \hat{\delta}) \leq \frac{k}{n-k} \, F_{k,n-k}(1-\alpha)\,, \tag{3}$$

$F_{k,n-k}(1-\alpha)$ being the quantile function of the probability distribution $F_{k,n-k}$.

It follows from this, that the confidence region of the effect of a change $\Delta \in \mathbb{R}^k$ in the values of the covariates for a profile $x_0 \in \mathbb{R}^k$ consists in the following set:

$$\{g^{\leftarrow}(\hat{B}^T x_0 + \tilde{\delta})\colon \tilde{\delta} \text{ satisfies (3)}\}\,, \tag{4}$$

where $g^{\leftarrow}\colon \mathbb{R}^2 \to S$ is the inverse of the link function $g\colon S \to \mathbb{R}^2$ of the tinomial logit model and S is the 2-dimensional simplex (1) of valid ternary probability functions. In particular:

$$g^{\leftarrow}(\eta) = g^{\leftarrow}([\eta_1, \eta_2]^T) = \left[\frac{1}{1+\mathrm{e}^{\eta_1}+\mathrm{e}^{\eta_2}}\,, \quad \frac{\mathrm{e}^{\eta_1}}{1+\mathrm{e}^{\eta_1}+\mathrm{e}^{\eta_2}}\,, \quad \frac{\mathrm{e}^{\eta_2}}{1+\mathrm{e}^{\eta_1}+\mathrm{e}^{\eta_2}}\right]^T\,,$$
$$g(\pi) = g([\pi_1, \pi_2, \pi_3]^T) = \left[\ln\frac{\pi_2}{\pi_1}\,, \quad \ln\frac{\pi_3}{\pi_1}\right]^T\,.$$

Finally, since ternary diagrams conveniently represent the effects of covariates over the space S rather than the space of covariates, the confidence region (4) may be restated with respect to a profile $\pi_0 \in S$, rather than a profile $x_0 \in \mathcal{X}$, thus, the $(1-\alpha)$-confidence region of the effect of change $\Delta \in \mathbb{R}^k$ on profile $\pi_0 \in S$ is the following set of points:

$$\{g^{\leftarrow}(g(\pi_0) + \tilde{\delta})\colon \tilde{\delta} \text{ satisfies (3)}\}\,. \tag{5}$$

Clearly, the edge of the confidence region (5) can be found by considering those points associated to the values $\tilde{\delta}$ which satisfy condition (3) exactly (i.e., with equality instead of inequality).

The package **plot3logit** enables confidence regions to be computed in two ways, by means of function `field3logit` or through function `add_confregions`.

Function `field3logit` computes the confidence regions for all the arrows in the field according to the value passed to argument `conf`. If `conf` is not set or if it is set to `NA` (default value), confidence regions are not computed. Clearly, the computation is possible only if the variance-covariance matrix of the estimates is available. When computed, confidence regions are part of the `field3logit` object returned by `field3logit`.

Function `add_confregions` enables confidence regions to be computed on a `field3logit` or a `multifield3logit` object, if not present. Otherwise it may be used to update confidence regions in `field3logit` or a `multifield3logit` object according to a new confidence level. Since `add_confregions` returns a `field3logit` (or a `multifield3logit`) equipped with confidence regions, it can be run as follows:

```
R> mfieldrace <- add_confregions(mfieldrace)
```

or, alternatively, through package **magrittr**:

```
R> mfieldrace %<>% add_confregions
```

By default, argument `conf` is set to 0.95, thus 95% confidence regions are computed, if not differently specified. As in case of `field3logit`, confidence regions can be computed only if variance-covariance matrix of coefficient estimates is available.

Both function `field3logit` and `add_confregions` have an argument named `npoints` which allows the user to set the number of points used for drawing the edges of confidence regions.

5.2. Representation

Confidence regions can be drawn both through package **Ternary** and **ggtern**.

In the former case, S3 method of generic `plot` works for both `field3logit` and `multifield3logit` objects and creates a new ternary plot if argument `add` is set to `FALSE` (default value), whilst adds a the vector field(s) to an existing ternary plot if `add` is set to `TRUE`. Function `TernaryField` draws also confidence regions of a `field3logit` object (see the help for details).

Confidence regions of `field3logit` and `multifield3logit` objects can be drawn through statistics `stat_conf3logit` which works analogously to `stat_field3logit`. Also in case of confidence regions, several wrappers are available, as illustrated in the next section.

The following code generates the diagram of Figure 4:

```
R> gg3logit(mfieldrace) +
+    stat_field3logit(aes(colour = label)) +
+    stat_conf3logit(aes(fill = label)) +
+    labs(colour = "Race (ref.: White)", fill = "Race (ref.: White)")
```

whereas the following code generates the diagram of Figure 3 from scratch:

```
R> library("tidyverse")
R> tibble(
+    race       = levels(USvote2016$race),
+    educ       = 'High school grad.',
+    gender     = 'Male',
+    birthyr    = '[1970,1980)'
+  ) %>%
+    mutate(delta = 'genderFemale', label = race) %>%
+    group_by(delta, label) %>%
+    nest() %>%
+    mutate(p0 = map(data, ~ list(predict(modVote, .x, type = 'probs')))) %>%
+    select(-data) %>%
+    transpose -> gender_by_race
R> mfieldGbyR <- field3logit(modVote, gender_by_race, narrows = 1, conf = 0.95)
R> gg3logit(mfieldGbyR) +
+    stat_field3logit(aes(colour = label)) +
+    stat_conf3logit(aes(fill = label)) +
+    tern_limits(T = 0.8, R = 0.8) +
+    labs(colour = "Profile", fill = "Profile")
```

6. Wrappers

Package **plot3logit** includes two wrappers which aims at simplifying the syntax when a `field3logit` and `multifield3logit` object is drawn through package **ggtern**.

The first wrapper is `stat_3logit` which is a wrapper for:

```
R> stat_field3logit() + stat_conf3logit()
```

`stat_3logit` has arguments `mapping_field` and `mapping_conf` which enables one to specify the aesthetic mappings for `stat_field3logit` and `stat_conf3logit` respectively, whereas arguments `params_field` and `params_conf` allow one to set the graphical parameters of the two layers.

The second wrapper is `autoplot` which is a wrapper for:

```
R> gg3logit() + stat_3logit()
```

and thus for

```
R> gg3logit() + stat_field3logit() + stat_conf3logit()
```

Just like in case of `stat_3logit`, `autoplot` has arguments `mapping_field`, `mapping_conf`, `params_field`, `params_conf` with the same role described before.

In order to provide an example, the code for drawing graph in Figure 4 is reported both with and without wrappers.

The following command:

```
R> gg3logit(mfieldrace) +
+    stat_field3logit(aes(colour = label)) +
+    stat_conf3logit(aes(fill = label))
```

is then equivalent to the following:

```
R> gg3logit(mfieldrace) +
+    stat_3logit(aes(colour = label), aes(fill = label))
```

which, in turn, is equivalent to this:

```
R> autoplot(mfieldrace, aes(colour = label), aes(fill = label))
```

7. Conclusions

Package **plot3logit** implements the ternary diagrams proposed in Santi *et al.* (2019) for interpreting the coefficient estimates of a trinomial logit regression. The package has been implemented so as to make it easy to use without losing flexibility. Upstream and downstream compatibility of the package enables the user to read model estimates whatever is the package/software that computed them, whereas the implementation of graphical functions based both on standard R graphics and **ggplot2**-based graphics, as well as the export methods (`as.data.frame`, `as_tibble`, `fortify`, `tidy`), provides several graphical tools for drawing the random fields, but does not prevent the user to adopt other graphical packages or applications other than R.

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Affiliation:

Flavio Santi
Department of Economics, University of Verona
Via Cantarane 24
37129 Verona (VR), Italy
E-mail: flavio.santi@univr.it

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