A Modern Bayesian Approach to Model Updating of Bridges Considering Measurement Uncertainty

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Contents

Why Finite Element Model Updating?

- Uncertainties
- Model error
- Structural health monitoring (SHM) data

Bayesian Inference

- Prior
- Data
- Likelihood
- Posterior

$$P(\theta \mid data) = \frac{P(data \mid \theta)P(\theta)}{P(data)}$$

$$p(\text{data}) = \int p(\text{data}|\theta) \cdot p(\theta) \; d\theta$$

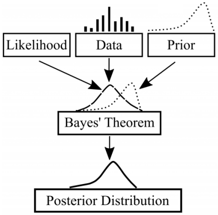

Figure 1: Bayesian inference

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Bayesian is hard because integration is hard!

Closed-form Solution (conjugate priors)

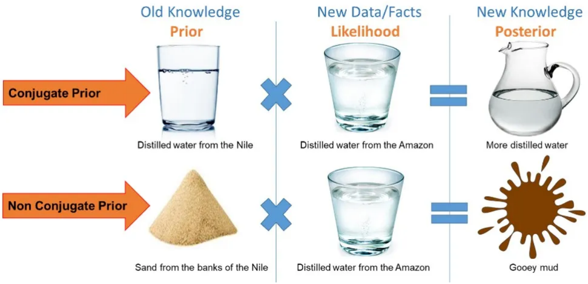

Figure 2: Conjugate and non-conjugate priors (ucanalytics.com)

Sampling Methods

- Metropolis-Hasting Algorithm
  - Moves to a new near state randomly!

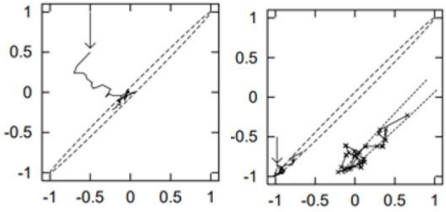

Figure 3: Sampling using MH (MacKay,2003)

Sampling Methods

- Hamiltonian Monte Carlo (HMC)
  - Using the geometry of the log-likelihood model, momentum, and gradient!

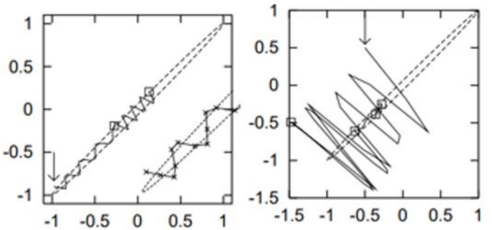

Figure 4: Sampling using HMC (MacKay,2003)

Diagnostic and Convergence

Gelman Rubin statistic

$$\hat{R} = \sqrt{\frac{N-1}{N} + \frac{1}{N}\frac{B}{W}}$$

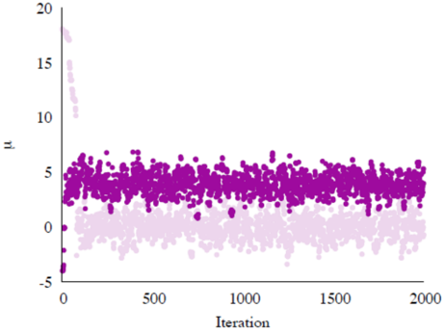

Figure 5: Converging between and within chains (Betancourt, 2014)

Effective Sample Size

MH may get Stuck!

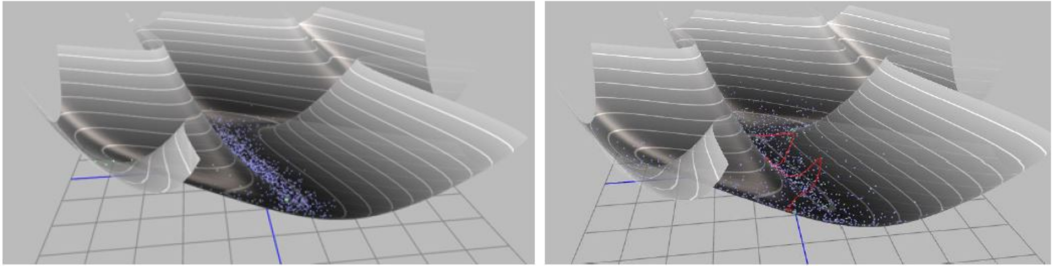

Figure 6: Exploring using MH and HMC (Rogozhnikov, 2016)

Principled Bayesian Workflow Using HMC

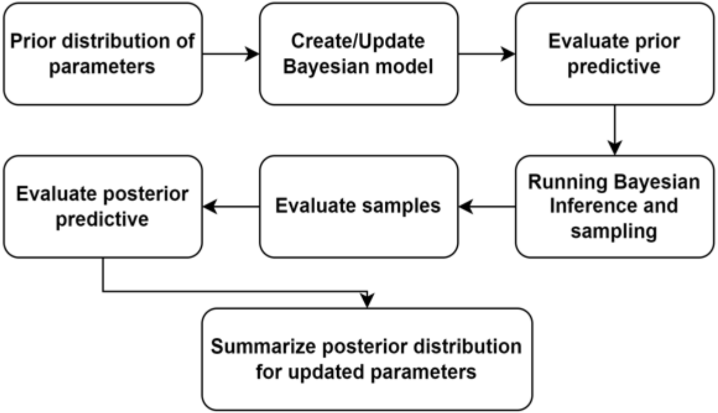

Figure 7: Bayesian workflow

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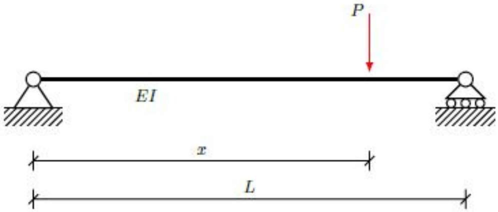
Application

Simple Beam with Moving Point Load



$$\delta = \frac{PL^3}{6EI}\Big[a^3(1-x) - \langle a-x\rangle^3 + a(1-x)^3 - a(1-x)\Big]$$

Function and data results plot for deflection

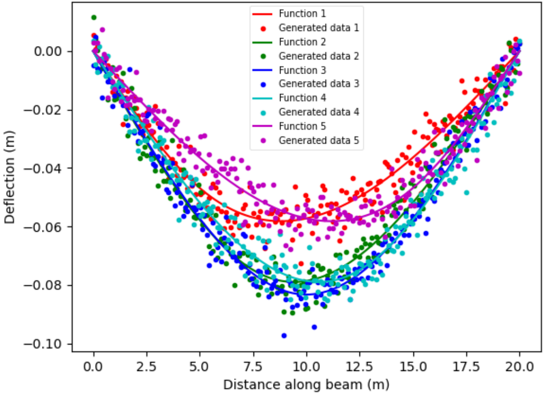


Figure 8: Function and data results plot for deflection in five sensor locations

Prior distributions and probabilistic model structure

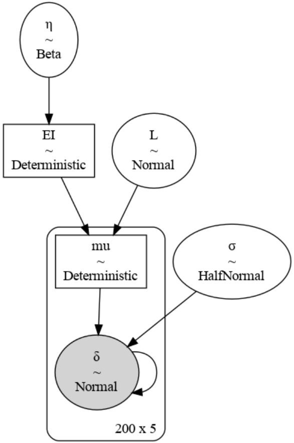


Figure 9: Probabilistic model structure

Parameter	Distribution	Parameters	Mode	Support
EI (MNm^2)	Beta	$\alpha = 2, \beta = 3$	183.3	$(150, 250)$
L (m)	Normal	$\mu = 20, \sigma = 0.01$	20	$(-\infty, +\infty)$

Prior predictive check

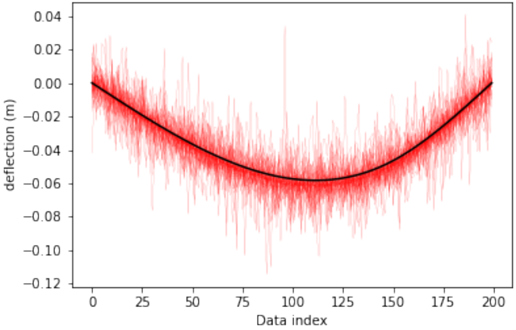


Figure 10: Prior predictive check for the sensor at 0.75L

Posterior density and trace plots of each chain

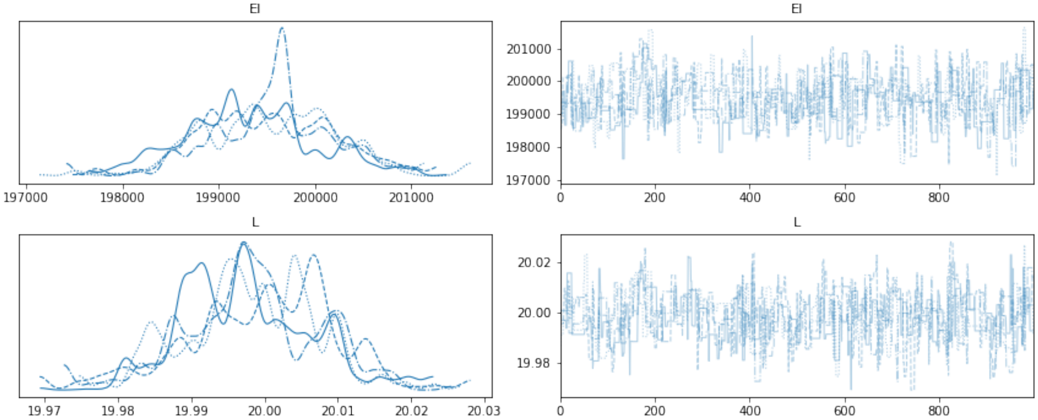


Figure 11: MH

Posterior density and trace plots of each chain

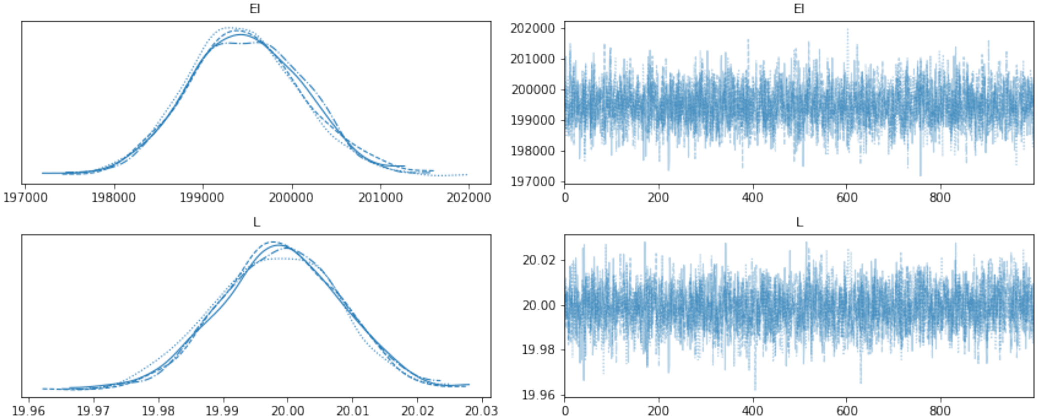


Figure 12: HMC

Comparing Results

Diagnostic and Convergence ($\hat{R}$)

Parameter	HMC	MH
EI	1	1.03
L	1	1.01

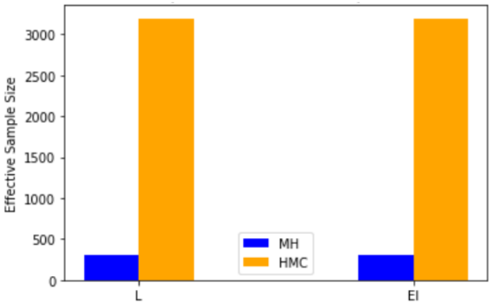


Figure 13: Comparison of effective sample size

Posterior plots (calculate highest density interval (HDI))

The wider HDI from HMC suggests that it might be better at capturing the tails of the posterior distribution, where less probable but still plausible values reside.

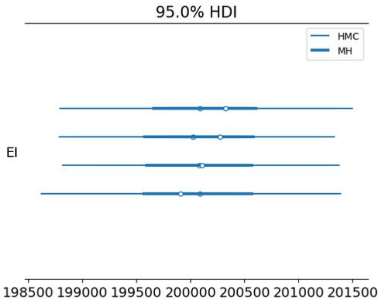


Figure 14: Posterior results comparison

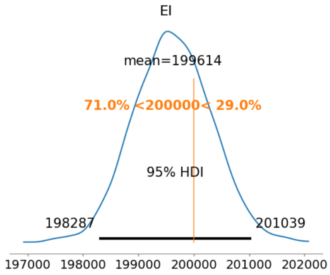


Figure 15: Posterior plot

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Conclusion

- Recent developments in Bayesian methods can help structural engineers improve model updating.
- HMC sampling allows flexibility with non-conjugate distributions and can have important diagnostics.
- Prior and posterior predictive checks validate the model and provide accurate parameter estimates.
- The updating procedure was done without any user input settings hyperparameters for HMC.
- The updated model with posterior parameter distributions can better predict the uncertain real structure behaviour.

Thank You!

- Thank you for your kind attention
- Please email me at Fatemeh.Shaker@monash.edu with any thoughts on this