UCB EE120: Assignment #1

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f1a3h

Problem 1

a) Let $\mu = t - \tau$, then we have

$$\begin{aligned}(x * y)(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_{\infty}^{-\infty} -x(t-\mu)h(\mu)d\mu \\ &= \int_{-\infty}^{\infty} x(t-\mu)h(\mu)d\mu \\ &= (y * x)(t)\end{aligned}$$

b)

$$\begin{aligned}x * (h_1 + h_2)(t) &= \int_{-\infty}^{\infty} x(\tau)(h_1(t-\tau) + h_2(t-\tau))d\tau \\ &= \int_{-\infty}^{\infty} x(\tau)h_1(t-\tau)d\tau + \int_{-\infty}^{\infty} x(\tau)h_2(t-\tau)d\tau \\ &= (x * h_1)(t) + (x * h_2)(t)\end{aligned}$$

c)

$$\begin{aligned}(x * (h_1 * h_2))(t) &= \int_{-\infty}^{\infty} x(\mu)\left(\int_{-\infty}^{\infty} h_1(t-\tau-\mu)h_2(\tau)d\tau\right)d\mu \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\mu)h_1(t-\tau-\mu)h_2(\tau)d\tau d\mu \\ &= \int_{-\infty}^{\infty} h_2(\tau)\left(\int_{-\infty}^{\infty} x(\mu)h_1(t-\tau-\mu)d\mu\right)d\tau \\ &= \int_{-\infty}^{\infty} h_2(\tau)(x * h_1)(t-\tau)d\tau \\ &= h_2 * (x * h_1)(t) \\ &= ((x * h_1) * h_2)(t)\end{aligned}$$

Problem 2

a) stable: $\sum h[n] = \sum_{i=n-9}^{n} u[i] < \infty$
causal: When $n < 0, h[n] = 0$
FIR: $h[n] = \sum_{i=n}^{n+10} \delta[i]$

b) unstable: $\sum h[n] = \sum 2^n u[n] = 2^n - 1$,When $n \to \infty, \sum h[n] \to \infty$
causal: When $n < 0, h[n] = 0$
IIR: A FIR system must be stable.

c) stable: $\sum h[n] = \sum_{i=0}^{\infty} 2^{-i} = 2 < \infty$
non-causal: When $n < 0$, $h[n] = 2^n > 0$
IIR: Since that when $n \leq 0, h[n] > 0$ and that when $n > 0, h[n] = 0$, so $y[0] = \sum_{k=0}^{\infty} x[k]h[-k]$. That is to say, $y[n]$ depends on infinite number of $x[k]$.

d) stable: $\sum h[n] = \sum_{k=1}^{\infty} k(0.8)^k < \sum_{k=1}^{\infty} (1.2)^k (0.8)^k = \sum_{k=1}^{\infty} 0.96^k < 24 < \infty$
causal: When $n < 0, h[n] = 0$
IIR: Since that when $n > 0, h[n] > 0$, that means $y[n]$ depends on infinite number of $x[k]$.

Problem 3

a) Omitted.
b) Omitted.
c) If a_i is in the form of $c_i n$, then $a_i y[n-i]$ should be a time-varying system. Thus any linear combination of $a_i y[n-i]$ cannot be a time-invarying system.
d) Consider the case when $n \geq 0$. Obviously, $y[n] = n + 2$. After $x[n]$ is scaled by a constant α, $y[n]$ becomes $\alpha n + 1 + \alpha$ instead of $\alpha(n+2)$.
e) Yes.

Problem 4

a) $h[n] = 0.5\delta[n] + 0.25\delta[n+1] + 0.25\delta[n-1]$
b) $H(e^{j\omega}) = \sum_{-\infty}^{\infty} h[k]e^{-j\omega k} = \frac{1}{2} + \frac{e^{j\omega}+e^{-j\omega}}{4} = \frac{1+\cos\omega}{2} = \cos^2\frac{\omega}{2}$
b) Low-pass filter.

Problem 5

a)
$$\begin{aligned} RC\frac{dy(t)}{dt} + y(t) &= x(t) \\ RC \cdot H(j\omega)j\omega e^{j\omega t} + H(j\omega) &= e^{j\omega t} \\ RC \cdot H(j\omega)j\omega + H(j\omega) &= 1 \end{aligned}$$
Finally, we get $H(j\omega) = \frac{1}{1+RC(j\omega)}$.
b) $|H(j\omega)| = \frac{1}{|1+RC(j\omega)|} = \frac{1}{\sqrt{1+(RC\omega)^2}}$.
Plotting is omitted.