Задача 10.5

10.5. Привести до простішого вигляду рівняння $u_t = a^2\left(u_{xx} + \alpha u_x\right) + cu$.

$a^2 u_{xx} + \alpha a^2 u_x - u_t + cu = 0$

$u(x,t) = u(\xi(x,t), \eta(x,t)) = e^{\lambda\xi + \mu\eta} V(x,t)$

$u_x = (\lambda \xi_x + \mu \eta_x) e^{\lambda\xi + \mu\eta} V_x$

$u_t = (\lambda \xi_t + \mu \eta_t) e^{\lambda\xi + \mu\eta} V_t$

$u_{xx} = (\lambda \xi_{xx} + \mu \eta_{xx} + \lambda^2 \xi_x + \mu^2 \eta_x) e^{\lambda\xi + \mu\eta} V_{xx}$

$a^2(\lambda \xi_{xx} + \mu \eta_{xx} + \lambda^2 \xi_x + \mu^2 \eta_x) e^{\lambda\xi + \mu\eta} V_{xx} + \alpha a^2 (\lambda \xi_x + \mu \eta_x) e^{\lambda\xi + \mu\eta} V_x - (\lambda \xi_t + \mu \eta_t) e^{\lambda\xi + \mu\eta} V_t + c e^{\lambda\xi + \mu\eta} V = 0$

$\xi = x + t, \quad \eta = x - t$

$a^2(\lambda^2 + \mu^2) V_{xx} + \alpha a^2 (\lambda + \mu) V_x - (\lambda - \mu) V_t + cV = 0.$

$\lambda + \mu = 0 \rightarrow \lambda = -\mu.$

$a^2((-\mu)^2 + \mu^2) V_{xx} + \cancel{\alpha a(-\mu + \mu) V_x} - (-\mu - \mu) V_t + cV = 0.$

$2\mu V_t + 2a^2\mu^2 V_{xx} + cV = 0.$

$V_t = -a^2 \mu V_{xx} - \frac{c}{\mu} V.$