№1.2 Відповідь $X_n(x) = C\sin\left(\frac{\pi x}{2\ell}(2n+1)\right)$

Перевірка:

$X'' + \lambda X = 0.$

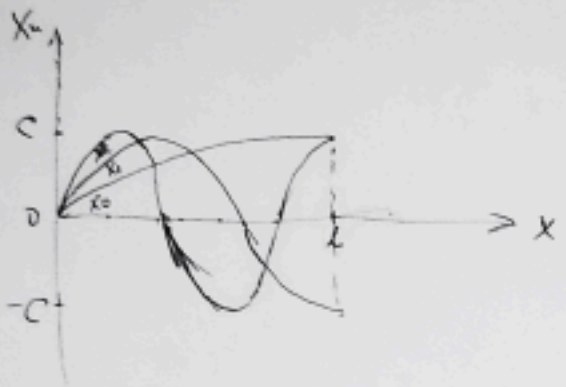
$X_n'' + \left(\frac{\pi}{2\ell}(2n+1)\right)^2 X_n = 0.$

Рахуємо похідну X_n:

$X_n' = \left(C\sqrt{\lambda}\cos(\sqrt{\lambda}x)\right)' = -C\lambda\sin\sqrt{\lambda}x = -\lambda X_n = -\left(\frac{\pi}{2\ell}(2n+1)\right)^2 X_n$

$X_n'' + \left(\frac{\pi x}{2\ell}(2n+1)\right)^2 X_n = 0.$, що збігається з заданим рівнянням

відповідає



$X_0 = C\sin\left(\frac{\pi x}{2\ell}\right)$

$X_1 = C\sin\left(\frac{3\pi x}{2\ell}\right)$

$X_2 = C\sin\left(\frac{5\pi x}{2\ell}\right)$

№1.3

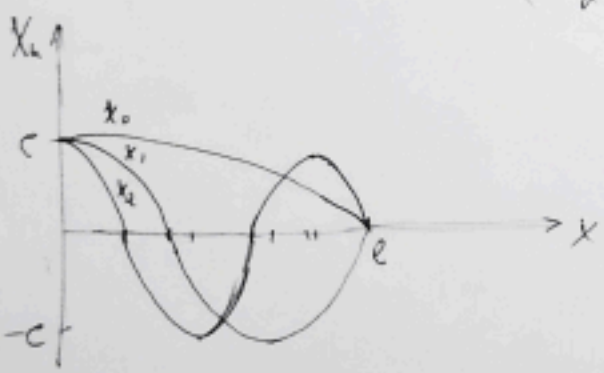
$X_n(x) = C\cos\left(\frac{\pi x}{2\ell}(2n+1)\right)$, $\lambda_n = \left(\frac{\pi(2n+1)}{2\ell}\right)^2$

$X'' + \lambda X = 0.$

$X_n = C\sqrt{\lambda_n}\cos(\sqrt{\lambda_n}x)$; $X_n'' = -C\lambda_n\cos\sqrt{\lambda}$

$X_n' = -C\sqrt{\lambda_n}\sin(\sqrt{\lambda_n}x)$; $X_n'' = -C\lambda_n\cos(\sqrt{\lambda_n}x) = -C\left(\frac{\pi(2n+1)}{2\ell}\right)^2\cos\left(\frac{\pi x(2n+1)}{2\ell}\right)^2$

$X'' + \lambda X = -C\left(\frac{\pi(2n+1)}{2\ell}\right)^2\cos\left(\frac{\pi x(2n+1)}{2\ell}\right) + \left(\frac{\pi(2n+1)}{2\ell}\right)^2 C\cos\left(\frac{\pi x(2n+1)}{2\ell}\right)^2 \equiv 0$



$X_0 = C\cos\left(\frac{\pi x}{2\ell}\right)$

$X_1 = C\cos\left(\frac{3\pi x}{2\ell}\right)$

$X_3 = C\cos\left(\frac{5\pi x}{2\ell}\right)$

№2.1

$X_0 = C$

$X_n = C\cos\left(\frac{\pi n}{\ell}x\right)$, $n > 0$ $\lambda_n = \left(\frac{\pi n}{\ell}\right)^2$

$X_n'' + \lambda X_n = 0$

$X_n' = -\frac{\pi n}{e} \cdot c \sin\left(\frac{\pi n x}{e}\right)$; $X_n'' = -c\left(\frac{\pi n}{e}\right)^2 \cos\left(\frac{\pi n x}{e}\right)$

$X_n'' + \lambda X_n = -c\left(\frac{\pi n}{e}\right)^2 \cos\left(\frac{\pi n x}{e}\right) + \left(\frac{\pi n}{e}\right)^2 \cdot c \cos\left(\frac{\pi n x}{e}\right) \equiv 0.$

$X'' + 0 \cdot X = 0.$

$X'' = 0 \qquad X = c_1 x + c_2, \; X' = c_1, \; X'' = 0.$