Bonus points sheet

Winter term 2020/2021

Computational Economics
(version: 15. Oktober 2020)

- 1. The deadline for this sheets ends on **January 31, 2021** (end of day). Submission is via Ilias.
- 2. Handwritten program code is not allowed! Submission must be a PDF with proper R-Code!
- 3. This sheet contains in total **7 questions**. All questions cover in total **7 pages**.
- 4. You receive **9 points** (i.e., 10%) as bonus points for the exam if **50**% (**or more**) are correctly answered. However, these points are only added if you pass the exam without bonus points. **The bonus points cannot prevent you from failing the exam!**
- 5. Group work is not allowed. Plagiarism can lead to exclusion from the exam.
- 6. Please label the submission document with your name, matriculation number, and program of study.

Good luck!

Ex. 1	Ex. 2	Ex. 3	Ex. 4	Ex. 5	Ex. 6	Ex. 7	Total	Pass
7	8	15	4	8	4	9	55	

Exercise 1: Visualization

- Create a colored plot with contours of the function $f(x,y) = \frac{x^2}{2} + \sin x + \frac{y^2}{2} \cos y$. Both x- and y-dimension should range from -10 to 10.
- 3

b) Prepare a 3D plot of the function $f(x, y) = \sin^2 x - \cos^2 y$.

Exercise 2: Dataframes

- - 8

- a) Load the csv file *persons2.csv* into a dataframe,
 - Insert a column *bmi* which indicates the body mass index (BMI) of the person which is defined as $bmi = \frac{weight \ in \ kg}{(height \ in \ m)^2}$
 - A BMI between 19 and 24 is considered to be normal. Which person does not have a normal BMI?
 - Create a histogram of the BMI with step size one. Choose reasonable scales and labels for x and y-axis.

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Exercise 3: Control Flow



a) Write a for-loop that calculates the following sum

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$$s(n) = \sum_{i=0}^{n} \frac{1}{2^{i}}.$$

Test your loop with n = 10.

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Write a function that tests if a positive integer n can be written as sum of two squared integers, i. e. $n=i^2+j^2$ with $i,j\in\mathbb{N}_{>0}$. Test your function for the integer n=100.

Exercise 4: Linear Algebra



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a) The solution of a linear system

$$Ax = b$$

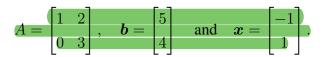
is often rewritten as minimizing a function

$$F(\boldsymbol{x}) = \|A\boldsymbol{x} - \boldsymbol{b}\|^2.$$

We can calculate the gradient to

$$\nabla F(\boldsymbol{x}) = 2A^T (A\boldsymbol{x} - \boldsymbol{b}).$$

Write a matrix expression in R to compute $\nabla F(x)$ for



Exercise 5: Numerical Analysis

- 1
- 7
- a) Convert the number 243043 from base 10 into the hexadecmal. State the corresponding R code.
- Visualize the function $f(x) = e^x + e^{-x}$ and its Taylor approximation for

 $x_0 = 1$

Use an x-axis ranging from -5 to 5 and y-axis from 0 to 20.

Exercise 6: Optimality Conditions



a) Verify that $x = \begin{bmatrix} -\frac{9}{22} \\ -\frac{2}{11} \end{bmatrix}$ is a minimum of the function

$$f(x_1, x_2) = 3 + 2x_1 + 3x_2 + 2x_1^2 + 2x_1x_2 + 6x_2^2.$$

State the corresponding R code.

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Exercise 7: Optimization

a) Implement the one-dimensional Newton's method to find a local extreme point of a function f(x) given a starting point $x^{(0)}$ and a tolerance ε . Stop the algorithm, when $|x^{(i)}-x^{(i-1)}|<\varepsilon$. Complete the following R code.

```
library(Deriv)
f = function(x) x**2 * sin(x-3) * exp(-0.5*x)
f.prime = # TODO
f.double.prime = # TODO

newton = function(f.prime, f.double.prime, x0, tol) {
    # TODO
}
newton(f.prime, f.double.prime, 5, 10**-6)
```

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