

Cheat Sheet – Regression Analysis

What is Regression Analysis?

Fitting a function $f(\cdot)$ to datapoints $y_i=f(x_i)$ under some error function. Based on the estimated function and error, we have the following types of regression

1. Linear Regression:

Fits a **line** minimizing the **sum of mean-squared error** for each datapoint.

$$\min_{\beta} \sum_i \|y_i - f_{\beta}^{linear}(x_i)\|^2$$

$$f_{\beta}^{linear}(x_i) = \beta_0 + \beta_1 x_i$$

2. Polynomial Regression:

Fits a **polynomial** of order k ($k+1$ unknowns) minimizing the **sum of mean-squared error** for each datapoint.

$$\min_{\beta} \sum_{i=0}^m \|y_i - f_{\beta}^{poly}(x_i)\|^2$$

$$f_{\beta}^{poly}(x_i) = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \dots + \beta_k x_i^k$$

3. Bayesian Regression:

For each datapoint, fits a **gaussian distribution** by minimizing the **mean-squared error**. As the number of data points x_i increases, it converges to point estimates i.e. $n \rightarrow \infty, \sigma^2 \rightarrow 0$

$$\min_{\beta} \sum_i \|y_i - \mathcal{N}(f_{\beta}(x_i), \sigma^2)\|^2$$

$$f_{\beta}(x_i) = f_{\beta}^{poly}(x_i) \text{ or } f_{\beta}^{linear}(x_i)$$

$$\mathcal{N}(\mu, \sigma^2) \rightarrow \text{Gaussian with mean } \mu \text{ and variance } \sigma^2$$

4. Ridge Regression:

Can fit either a **line**, or **polynomial** minimizing the **sum of mean-squared error** for each datapoint and the **weighted L2 norm** of the function parameters beta.

$$\min_{\beta} \sum_{i=0}^m \|y_i - f_{\beta}(x_i)\|^2 + \sum_{j=0}^k \beta_j^2$$

$$f_{\beta}(x_i) = f_{\beta}^{poly}(x_i) \text{ or } f_{\beta}^{linear}(x_i)$$

5. LASSO Regression:

Can fit either a **line**, or **polynomial** minimizing the **sum of mean-squared error** for each datapoint and the **weighted L1 norm** of the function parameters beta.

$$\min_{\beta} \sum_{i=0}^m \|y_i - f_{\beta}(x_i)\|^2 + \sum_{j=0}^k |\beta_j|$$

$$f_{\beta}(x_i) = f_{\beta}^{poly}(x_i) \text{ or } f_{\beta}^{linear}(x_i)$$

6. Logistic Regression:

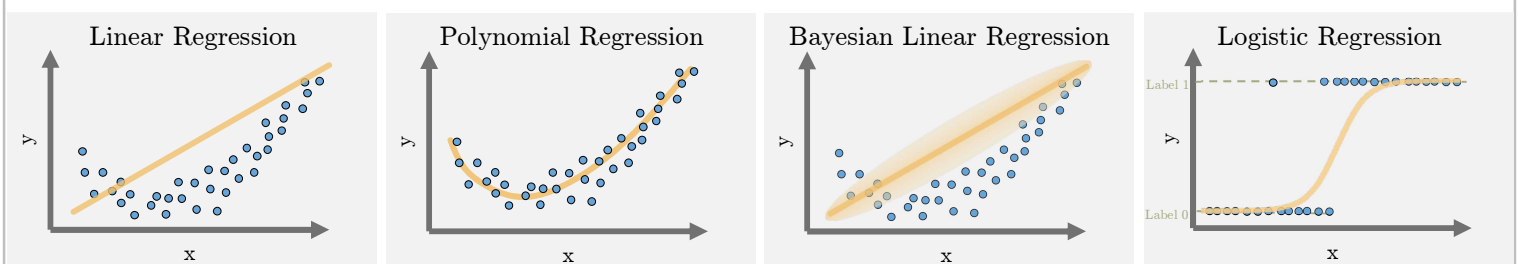
Can fit either a **line**, or **polynomial with sigmoid activation** minimizing the **binary cross-entropy loss** for each datapoint. The labels y are binary class labels.

$$\min_{\beta} \sum_i -y_i \log(\sigma(f_{\beta}(x_i))) - (1 - y_i) \log(1 - \sigma(f_{\beta}(x_i)))$$

$$f_{\beta}(x_i) = f_{\beta}^{poly}(x_i) \text{ or } f_{\beta}^{linear}(x_i)$$

$$\sigma(t) = \frac{1}{1 + e^{-t}}$$

Visual Representation:



Summary:

	What does it fit?	Estimated function	Error Function
Linear	A line in n dimensions	$f_{\beta}^{linear}(x_i) = \beta_0 + \beta_1 x_i$	$\sum_{i=0}^m \ y_i - f_{\beta}(x_i)\ ^2$
Polynomial	A polynomial of order k	$f_{\beta}^{poly}(x_i) = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \dots$	$\sum_{i=0}^m \ y_i - f_{\beta}(x_i)\ ^2$
Bayesian Linear	Gaussian distribution for each point	$\mathcal{N}(f_{\beta}(x_i), \sigma^2)$	$\sum_{i=0}^m \ y_i - \mathcal{N}(f_{\beta}(x_i), \sigma^2)\ ^2$
Ridge	Linear/polynomial	$f_{\beta}^{poly}(x_i) \text{ or } f_{\beta}^{linear}(x_i)$	$\sum_{i=0}^m \ y_i - f_{\beta}(x_i)\ ^2 + \sum_{j=0}^n \beta_j^2$
LASSO	Linear/polynomial	$f_{\beta}^{poly}(x_i) \text{ or } f_{\beta}^{linear}(x_i)$	$\sum_{i=0}^m \ y_i - f_{\beta}(x_i)\ ^2 + \sum_{j=0}^n \beta_j $
Logistic	Linear/polynomial with sigmoid	$\sigma(f_{\beta}(x_i))$	$\min_{\beta} \sum_i -y_i \log(\sigma(f_{\beta}(x_i))) - (1 - y_i) \log(1 - \sigma(f_{\beta}(x_i)))$