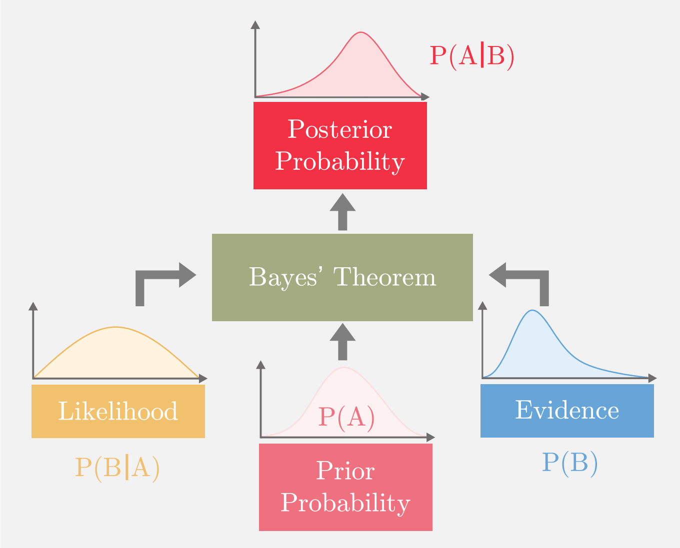
Cheat Sheet – Bayes Theorem and Classifier

What is Bayes' Theorem?

- Describes the probability of an event, based on prior knowledge of conditions that might be related to the event.

$$P(A|B) = \frac{P(B|A)(likelihood) \times P(A)(prior)}{P(B)(evidence)}$$



- How the probability of an event changes when we have knowledge of another event

P(A) ⟶ P(A|B)

Usually, a better estimate than P(A)

Example

- Probability of fire P(F) = 1%
- Probability of smoke P(S) = 10%
- Prob of smoke given there is a fire P(S|F) = 90%
- What is the probability that there is a fire given we see a smoke P(F|S)?

$$P(F|S) = \frac{P(S|F) \times P(F)}{P(S)} = \frac{0.9 \times 0.01}{0.1} = 9\%$$

Maximum Aposteriori Probability (MAP) Estimation

The MAP estimate of the random variable y, given that we have observed iid (x_1, x_2, x_3, …), is given by. We try to accommodate our prior knowledge when estimating.

$$\hat{y}_{MAP} = argmax_y \quad P(y) \prod_i P(x_i|y)$$

y that maximizes the product of prior and likelihood

Maximum Likelihood Estimation (MLE)

The MAP estimate of the random variable y, given that we have observed iid (x_1, x_2, x_3, …), is given by. We assume we don't have any prior knowledge of the quantity being estimated.

$$\hat{y}_{MLE} = argmax_y \prod_i P(x_i|y)$$

y that maximizes only the likelihood

MLE is a special case of MAP where our prior is uniform (all values are equally likely)

Naïve Bayes' Classifier (Instantiation of MAP as classifier)

Suppose we have two classes, y=y_1 and y=y_2. Say we have more than one evidence/features (x_1, x_2, x_3, …), using Bayes' theorem

$$P(y|x_1, x_2, x_3, \ldots) = \frac{P(x_1, x_2, x_3, \ldots |y) \times P(y)}{P(x_1, x_2, x_3, \ldots)}$$

Naïve Bayes' theorem assumes the features (x_1, x_2, …) are i.i.d. i.e $P(x_1, x_2, x_3, \ldots |y) = \prod_i P(x_i|y)$

$$P(y|x_1, x_2, x_3, \ldots) = \prod_i P(x_i|y) \frac{P(y)}{P(x_1, x_2, x_3, \ldots)}$$

$$\hat{y} = y_1 \ if \ \frac{P(y_1|x_1, x_2, x_3, \ldots)}{P(y_2|x_1, x_2, x_3, \ldots)} > 1 \quad else \ \hat{y} = y_2$$

Source: https://www.cheatsheets.aqeel-anwar.com