DATA / /

1) $a_N = 2a_{N-1} + a_{N-2} - 2a_{N-3}$

$a_0 = a_1 = 1 \qquad a_2 = 3$

$$U_N = \begin{pmatrix} a_{N+1} \\ a_N \\ a_{N-1} \end{pmatrix} = \begin{pmatrix} 2 & 1 & -2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} a_{N-1} \\ a_{N-2} \\ a_{N-3} \end{pmatrix}$$

$$U_1 = \begin{pmatrix} a_{N+1} \\ a_N \\ a_{N-1} \end{pmatrix} = \begin{pmatrix} a_2 \\ a_1 \\ a_0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$$

Autovetores:

$\det(M - \lambda I) = 0$

$$\det \begin{pmatrix} 2-\lambda & 1 & -2 \\ 1/2 & -\lambda & 0 \\ 0 & 1 & -\lambda \end{pmatrix} = (2-\lambda)(-\lambda)(-\lambda) + \lambda =$$

$$-\lambda^{3} + 2\lambda^2 + \lambda - 2$$

Com vetores adivinhados (chute) encontramos $\lambda = 1$

Briot - Ruffini

$$\begin{array}{c|cccc} 1 & -1 & 2 & 1 & -2 \\ \hline & -1 & 1 & 2 & \end{array}$$

DATA / /

$-\lambda^2 + \lambda + 2$

As raízes são $\lambda = 2$ e $\lambda = -1$

Autovetores =

Para $\lambda_1 = -1$

$$\left(\begin{array}{ccc|c} 3 & 1 & -2 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \times \tfrac{1}{3} \to \left(\begin{array}{ccc|c} 1 & 1/3 & -2/3 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \times(-1)$$

$$\left(\begin{array}{ccc|c} 1 & 1/3 & -2/3 & 0 \\ 0 & 2/3 & 2/3 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \times \tfrac{3}{2} \to \left(\begin{array}{ccc|c} 1 & 1/3 & -2/3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \times(-1)$$

$$\left(\begin{array}{ccc|c} 1 & 1/3 & -2/3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \times\left(-\tfrac{1}{3}\right) \quad \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$X_1 = X_3$
$X_2 = -X_3$
X_3 = variável livre

Para X_3 : $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

Para $\lambda_2 = 1$

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right) \xrightarrow{\times(-1)} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right) \times(-1/2)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right) \xrightarrow{\times(-1)} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$X_1 = X_3$

$X_2 = X_3$

$X_3 =$ variável livre

Para $X_3 = 1$ $\quad X = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Para $\lambda_3 = 2$

$$\left(\begin{array}{ccc|c} 0 & 1 & -2 & 0 \\ 1 & -2 & 0 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right) \xrightarrow{\text{mudando a ordem}} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & -2 \\ 0 & 1 & -2 \end{pmatrix} \times(-1)$$

$$\left(\begin{array}{ccc|} 1 & -2 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array}\right) \xrightarrow{\times(2)} \left(\begin{array}{ccc|c} 1 & 0 & -4 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$X_1 = 4x_3$

$X_2 = 2x_3$

$X_3 =$ variável livre

Para $X_3 = 1$

$X = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$

DATA / /

Então:

$$S = \begin{pmatrix} 1 & 1 & 4 \\ -1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

Inversa:

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 4 & 1 & 0 & 0 \\ -1 & 1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \times 1 \rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 4 & 1 & 0 & 0 \\ 0 & 2 & 6 & 1 & 1 & 0 \\ 1 & 1 & 1 & -1 & 0 & 1 \end{array}\right) \times(-1)$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 4 & 1 & 0 & 0 \\ 0 & 2 & 6 & 1 & 1 & 0 \\ 0 & 0 & -3 & -1 & 0 & -1 \end{array}\right) \times\left(\tfrac{1}{2}\right) \quad \left(\begin{array}{ccc|ccc} 1 & 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 3 & \tfrac{1}{2} & \tfrac{1}{2} & 0 \\ 0 & 0 & -3 & -1 & 0 & 1 \end{array}\right) \times(-1)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & \tfrac{1}{2} & -\tfrac{1}{2} & 0 \\ 0 & 1 & 3 & \tfrac{1}{2} & \tfrac{1}{2} & 0 \\ 0 & 0 & -3 & -1 & 0 & 1 \end{array}\right) \times \tfrac{1}{3} \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & \tfrac{1}{2} & -\tfrac{1}{2} & 0 \\ 0 & 1 & 3 & \tfrac{1}{2} & \tfrac{1}{2} & 0 \\ 0 & 0 & 1 & \tfrac{1}{3} & 0 & -\tfrac{1}{3} \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \tfrac{1}{6} & -\tfrac{1}{2} & \tfrac{1}{3} \\ 0 & 1 & 0 & -\tfrac{1}{2} & \tfrac{1}{2} & 1 \\ 0 & 0 & 1 & \tfrac{1}{3} & 0 & -\tfrac{1}{3} \end{array}\right) \rightarrow S^{-1} \begin{pmatrix} \tfrac{1}{6} & -\tfrac{1}{2} & \tfrac{1}{3} \\ -\tfrac{1}{2} & \tfrac{1}{2} & 1 \\ \tfrac{1}{3} & 0 & -\tfrac{1}{3} \end{pmatrix}$$

$$S^{-1} = \frac{1}{6} \begin{pmatrix} +1 & -3 & 2 \\ -3 & 3 & 6 \\ 2 & 0 & -2 \end{pmatrix}$$

$$U_N = \begin{pmatrix} a_N \\ a_{N-1} \\ a_{N-2} \end{pmatrix} = (S\Delta S^{-1})(S\Delta S^{-1})\dots = S\Delta^{N+1}S^{-1}$$

$$U_N = \begin{pmatrix} a_{N+1} \\ a_N \\ a_{N-2} \end{pmatrix} = S\Delta^{N-1}S^{-1}u_1 =$$

$$\begin{pmatrix} 1 & 1 & 4 \\ -1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{6} & -\frac{1}{2} & \frac{1}{3} \\ -\frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{3} & 0 & -\frac{1}{3} \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} (-1)^{N-1} & 1 & 2^{N+1} \\ (-1)^{N} & 1 & 2^{N} \\ (-1)^{N-1} & 1 & 2^{N-1} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{6} & -\frac{1}{2} & \frac{1}{3} \\ -\frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{3} & 0 & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} =$$

$$\begin{pmatrix} \left[\frac{1}{6}(-1)^{N-1} - \frac{1}{2} + \frac{1}{3}(2^{N+1})\right] \cdot \left[-\frac{1}{2}(-1)^{N-1} + \frac{1}{2}\right] \left[\frac{1}{3} \cdot (-1)^{N-1} + 1 - \frac{1}{3}(2^{N+1})\right] \\ \left[\frac{1}{6}(-1)^{N} - \frac{1}{2} + \frac{1}{3}(2^{N})\right] \left[-\frac{1}{2}(-1)^{N} + \frac{1}{2}\right] \left[\frac{1}{3}(-1)^{N} + 1 - \frac{1}{3}(2^{N})\right] \\ \left[\frac{1}{6} \cdot (-1)^{N-1} - \frac{1}{2} + \frac{1}{3}(2^{N-1})\right] \left[-\frac{1}{2} \cdot (-1)^{N-1} + \frac{1}{2}\right] \cdot \left[\frac{1}{3}(-1)^{N-1} + 1 - \frac{1}{3}(2^{N-1})\right] \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \left[\frac{1}{2} \cdot (-1)^{N-1} - \frac{3}{2} + 2^{N+1} - \frac{1}{2} \cdot (-1)^{N-1} + \frac{1}{2} + \frac{1}{3}(-1)^{N-1} + 1 - \frac{1}{3}(2^{N+1})\right] \\ \left[\frac{1}{2}(-1)^{N} - \frac{3}{2} + 2^{N} - \frac{1}{2} \cdot (-1)^{N} + \frac{1}{2} + \frac{1}{3}(-1)^{N} + 1 - \frac{1}{3}(2^{N})\right] \\ \left[\frac{1}{2}(-1)^{N-1} - \frac{3}{2} + 2^{N-1} - \frac{1}{2}(-1)^{N-1} + \frac{1}{2} + \frac{1}{3}(-1)^{N-1} + 1 - \frac{1}{3}(2^{N-1})\right] \end{pmatrix}$$

$$\begin{pmatrix} \frac{2}{3}(2^{N+1}) + \frac{1}{3}(-1)^{N-1} \\ \frac{2}{3}(2^{N}) + \frac{1}{3}(-1)^{N} \\ \frac{2}{3}(2^{N-1}) + \frac{1}{3}(-1)^{N-1} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2^{N+2} + (-1)^{N-1} \\ 2^{N+1} + (-1)^{N} \\ 2^{N} + (-1)^{N-1} \end{pmatrix} = \begin{pmatrix} a_{N+1} \\ a_N \\ a_{N-1} \end{pmatrix}$$

$$a_N = \frac{1}{3}\left[2^{N+1} + (-1)^{N}\right]$$