Flow Control Protocols for Integrated Networks with Partially Observed Voice Traffic

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Abstract—**The design of flow control protocols for integrated networks with partially observed voice traffic on the data link level is investigated. A closed Markovian queueing network with two classes of users and a preemptive resume queueing discipline for modeling the integrated link is introduced. The class of admissible flow control policies analyzed maximizes the average data link throughput subject to an average system time delay constraint on a finite horizon. A separation principle between the flow control policy and an estimate of the state of the voice traffic is derived. In particular, it is shown that the optimum control law is bang-bang and the conditional mean estimate of the state of the voice traffic is a sufficient statistic for the optimal control strategy. Since the complexity of the analytical results prohibits finding the explicit flow control policy, a suboptimum and easily implementable *adaptive* window flow control mechanism is proposed. The window size changes dynamically according to the estimated state of the voice traffic at the destination node. The robustness of the estimator, the dynamics of the window size, and the effectiveness of the suboptimum scheme are verified by means of simulations.**

I. Introduction

The growing popular demand for diverse types of communications (e.g., voice, video, and data) is the keystone of the current universal trend towards integrated networks (IN's). The architecture, design, control, and performance aspects of IN's have been under an intense worldwide investigation [6] (see also the references therein). The objective of this paper is to develop the optimum data link flow control protocol for IN's with partially observed voice traffic.

The IN model employed for optimal flow control is based on the proposed implementation of a packet switched overlay on the existing telephone network [9] (see Fig. 1). In this model the data packets are transmitted through the existing trunks of the telephone network in which new switching nodes are integrated. Since there is a real-time constraint in transmitting the voice messages, a preemptive priority discipline is followed for voice calls over the data packets at the switching nodes. The capacity assignment at each of the nodes of this network is assumed to be provided by a synchronous TDM frame with movable boundary. Circuit switching is preferred over packet switching for the voice traffic network-wide. An overlay of packet traffic is superimposed on a given voice traffic demand specified by the blocking probability of setting up a call. It can be shown that the data packets are transmitted through a store-and-forward packet switched network with random link capacities which are correlated with each other [9], [11].

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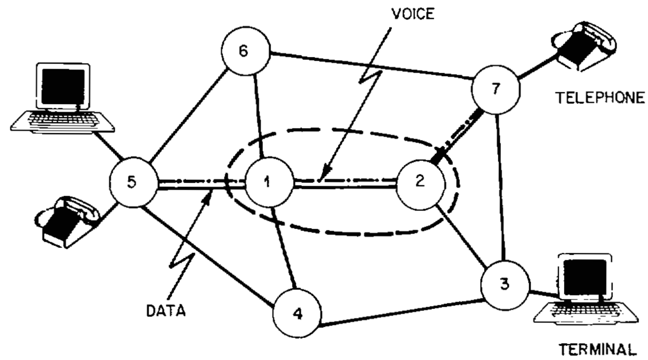

Fig. 1. A model for an integrated network.

Since the available link capacities depend on the voice traffic, the voice traffic statistics play a key role in the design of any optimum data flow control protocol (e.g., data link or end-to-end). To obtain the necessary statistics for flow control purposes, either of the following approaches might be adopted.

1)Any required information about the state of the voice traffic is made available to the controller upon request (this is the complete observation case). The necessary information about the voice traffic is transmitted by an additional data traffic in the network. For end-to-end flow control protocols, it might be required to provide the controller with the state of the voice traffic at all the hops of the corresponding virtual circuit. This approach is, therefore, more suitable for centralized networks. The process of acquisition of appropriate information and the subsequent optimum control actions require a complex software implementation.

2) The controller estimates the state of the voice traffic based on its observation of the state of the data traffic in the system (this is said to be the partial observations case). This approach results in a *distributed* flow control algorithm and is better suited for decentralized networks; it requires each user to transmit a limited number of data packets (real or dummy) through the network. By monitoring the corresponding transmissions and acknowledgments, decentralized controllers (i.e., the end users) could estimate the load of the network and subsequently adopt a "suitable" adaptive control strategy. This concept might be further explored in order to provide truly *distributed* end-to-end flow control protocols for either segregated packet switched networks or IN's.

As a first step towards the design of distributed flow control algorithms for IN's, in this paper we search for the optimum data link flow control protocol of IN's with partial observations. To develop the data link flow control protocol, a pair of adjacent source–destination nodes in isolation is considered (see Fig. 1). The flow control protocol considered regulates the data flow from the source node to the destination node under an appropriate optimality criterion. To establish the optimal control problem, this source–destination pair is modeled as a closed queueing network consisting of two queues [see Fig. 2(b)]. The second queue and its single server model the outgoing data flow (and voice traffic) at

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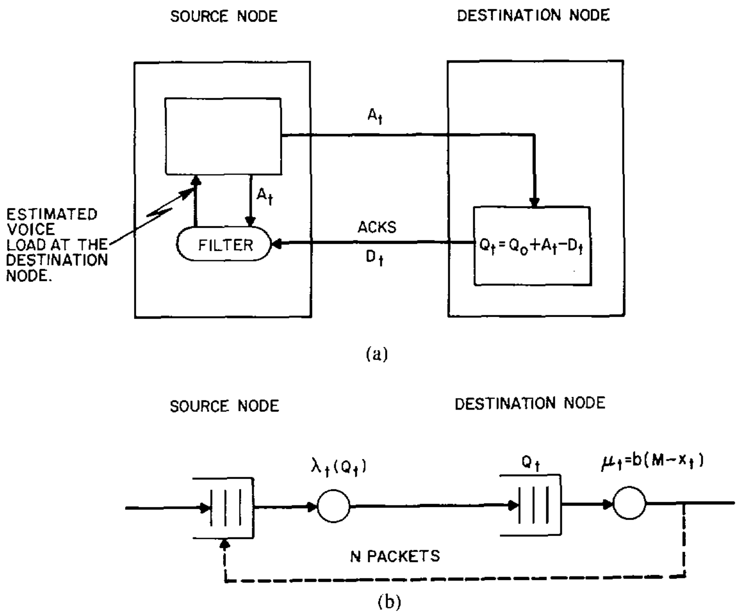

Fig. 2. The block diagram representation and queueing model for the data link flow control with partial observations.

the destination node. The service rate of this queue is random and governed by the statistics of the destination voice traffic. The first queue of Fig. 2(b) represents the source node which performs the control task. This queueing system is represented by a two-dimensional process (X_t, Q_t). The components X_t and Q_t are the state of the voice and data traffic at the destination node, respectively. The unknown service rate of the first queue represents the controlled transmission rate of the data packets being sent from the source node to the destination node.

In our analysis, the service rate of the first queue (i.e., transmission rate from source to destination) is selected as the control parameter. The simulation results (see Section II) indicate that during the voice "overload" periods (i.e., heavy voice traffic) the data queue builds up, while at light voice loads (i.e., voice "underload" periods) the queue length is small. Hereafter, we refer to this phenomenon as the transient behavior of the data traffic. The transient behavior of data traffic is due to the randomness of the allocated capacity to the data traffic at each node. Since the data flow exhibits transient behavior, the stochastic control problem is considered dynamically, and over a finite time interval $(0, S]$. The finite horizon formulation of the problem leads to optimal control actions that conform to the *real-time* (rather than the equilibrium) statistics of the system over each (i.e., "overload" or "underload") period. The control parameter is chosen such that the second queue achieves its maximum average throughput under an average system time delay constraint on a finite horizon [5], [8], [9].

The source node (controller) only keeps a record of the state of the data buffer at the destination node Q_t. This can be obtained by continuously monitoring the associated departure (acknowledgment stream) and arrival (transmission stream) processes. Note that this requires the source node's knowledge of Q_0 (in practice, whenever the system is reinitialized Q_0 is set to zero, i.e., all the data packets are dropped), and the individual acknowledgment of data packets by the destination node. Since no direct information about the destination voice traffic is assumed to be available, it is "natural" to implement a filter at the source node (controller) that estimates the state of the voice traffic at the destination node. The input process to this optimum minimum mean square error (MMSE) filter is the state of the destination data queue, Q_t (i.e., the arrival process A_t and the departure process D_t, associated with the destination node). To develop an appropriate filter structure based on this queueing model, the joint state of the destination voice and data traffic is modeled as a two-dimensional Markov chain (X_t, Q_t). Furthermore, in this model the unobserved component X_t itself constitutes a birth–death process. The output of the filter represents the posterior distribution of the destination voice traffic load [see Fig. 2(a)]. It is shown that this probability distribution is a sufficient statistic for the optimum control law. To be consistent with the literature [7], [12], we refer to this property as the *separation principle*. Moreover, the optimum data link protocol is a *bang-bang* control.

This paper is organized as follows. In Section II the media access protocol at each of the nodes of a generic IN is described in detail. A simple analysis of a typical network node by means of simulation indicates the transient behavior of the data flow and the necessity for dynamic flow control policies. The queueing model for dynamic data link flow control of IN's with partial observations (partially observed voice traffic) is analyzed in detail in Section III. The mathematical statement of the stochastic control problem with partial observations is also presented here. The objective function is given by the average throughput of the data link. The constraints are dictated by the average system time delay and the maximum capacity of the data link at the source node. In Section IV the optimization problem is rephrased in terms of the conditional mean estimate of the voice traffic load. The structure of the optimal filter is explicitly derived. In Section V a separation principle between flow control and estimation is proven and the optimality of a bang-bang flow control policy is also demonstrated. Since the optimum control policy cannot be explicitly specified from the theoretical results, a suboptimum and easily implementable dynamic window flow control scheme is proposed. Finally, in Section VI, the performance of the filter, the robustness of the proposed model for the number of active voice slots, and the effectiveness of a dynamic suboptimal window flow control protocol are extensively investigated by means of simulations.

II. The Media Access Protocol at an Integrated Node

As previously mentioned, we consider an IN consisting of a set of interconnected hybrid circuit-packet switching nodes. At the switching nodes of the network two classes of traffic, voice and data, dynamically share the master frame of a time division multiplexing facility. The voice traffic is circuit switched and the data traffic is packet switched. At each node of this network M

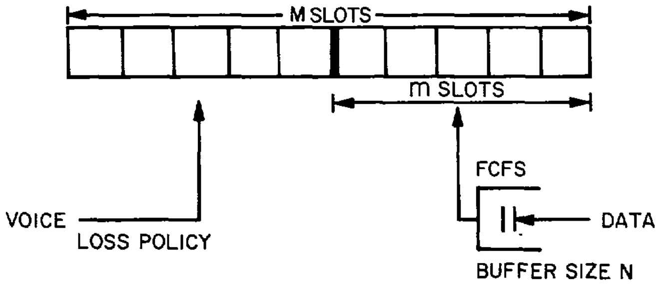

Fig. 3. A TDM frame with moveable boundary.

channels are time multiplexed together. Up to $(M - m)$ channels are allocated to the voice traffic. If all the $(M - m)$ channels are reserved by $(M - m)$ off-hook calls, then new voice calls will be blocked. The remaining m channels are always allocated to the data traffic. Moreover, to increase the efficiency of the resource allocation, the data traffic is also allowed to utilize the idle (unreserved) voice channels and the dedicated voice channels during the silent periods of the corresponding off-hook calls. To guarantee the real-time delivery of the voice messages, a preemptive priority discipline is provided for the voice calls over the data packets at the switching nodes.

The frame structure at a typical node of this network is shown in Fig. 3. The duration of the frame is fixed and consists of a total of M time *slots*. The allocation of a time slot to either type of traffic is assumed to be equivalent to the assignment of b bits/s of the link capacity to that type of traffic. Up to $(M - m)$ slots per frame are assigned to the voice traffic. If $(M - m)$ off-hook voice calls (circuit-switched calls) are present in the system, the additional voice calls are lost. The data (packet-switched) traffic is stored in a queue of buffer size N, and served according to a first-come-first-served (FCFS) priority discipline. In addition to the minimum of m slots in a frame, the data traffic is also allowed to utilize the empty voice slots and the slots of those off-hook voice calls that are in the silent period. For the subsequent theoretical analysis, the maximum buffer size N is assumed to be arbitrarily large.

The maximum allowable number of off-hook voice calls in the system is chosen such that the probability of blocking for new off-hook circuit-switched arrivals (new voice calls) does not exceed a preassigned upper bound β. Let us assume that the off-hook calls are originated according to a Poisson distribution with mean rate δ and the holding time of an off-hook call is exponentially distributed with mean ζ^{-1}. The blocking probability for a new off-hook arrival is given by the Erlang B formula:

$$B\left(\frac{\delta}{\zeta}, M-m\right)=\left\{\frac{\left(\frac{\delta}{\zeta}\right)^{M-m}}{(M-m)!}\right\}\left\{\sum_{j=0}^{M-m}\frac{\left(\frac{\delta}{\zeta}\right)^{j}}{j!}\right\}^{-1} \tag{1}$$

where δ/ζ is the offered circuit switched load in erlangs. The number of allocated voice slots to the voice traffic is chosen such that the constraint

$$B\left(\frac{\delta}{\zeta}, M-m\right)\leq\beta \tag{2}$$

is satisfied.

To model the data flow at the node, we define an active voice slot to be any off-hook voice slot of the TDM frame that is in talkspurt. Let X_t and Q_t be the number of active voice slots and data packets at the node at time t, respectively. The allocation of a time slot to either type of traffic is equivalent to the assignment of b bits/s of the link capacity to that type of traffic. Hence, the total capacity of the digital pipe which is allocated to the data (packet-switched) traffic is given by

$$\mu_t = b(M - X_t). \tag{3}$$

Therefore, the link capacity allocated to the data (packet-switched) traffic on each link of the IN is random. Throughout this paper, the data packet length is assumed to be exponentially distributed. Hence, the data flow at each node can be modeled as a queue with a single random exponential server μ_t. In this model the packet at the head of the queue is allowed to utilize as many data slots as required. If the packet cannot be completely transmitted over the current frame, its residual length will be transmitted over the following frame(s). Since the data transmission rate at each node is random, the data flow in the network is likely to exhibit transient behavior [11]. Consequently, to tailor the optimal control actions to the real-time statistics of the traffic in the network, the control aspects of this network are investigated over a finite horizon.

To reveal the transient behavior of the network, its potential performance deterioration, and the necessity of dynamic data link flow control schemes, a typical node of the IN has been studied by means of simulation. The simulation model consists of four time-multiplexed channels, three for voice and one for data. The capacity of each channel b has been chosen to be 64 kbits/s. The simulator, subsequently referred to as SIMIN (simulator of integrated node), consists of a set of Fortran and C programs.

Throughout this work, the length of the simulation runs is 1023 (about 17 min). The arrival of the off-hook calls to the system is modeled as a Poisson process with rate $\delta = 2$ calls/min. The average holding time of a typical call is drawn from an exponential distribution with mean $\zeta^{-1} = 3$ min. Hence, the voice offered load to the system is 6 erlangs.

Each channel has its own speech activity generator which is activated upon the reservation of the channel by the off-hook voice calls. The duration of successive talkspurt and silent periods of an off-hook voice call are assumed to be independent and exponentially distributed with means θ^{-1} and σ^{-1}, respectively. The average length of the talkspurt and silent periods are, $\theta^{-1} = 1.23$ and $\sigma^{-1} = 1.79$, respectively [1]. The expected number of off-hook voice calls present in the system amounts to

$$6(1-B(6, 3))=2.46. \tag{4}$$

The proportion of time a voice call is in talkspurt is given by

$$\frac{\theta^{-1}}{\theta^{-1}+\sigma^{-1}}=\frac{\sigma}{\sigma+\theta}.$$

Thus, the average number of active voice slots is

$$\frac{6(1-B(6, 3))\sigma}{\sigma+\theta}=1.02. \tag{5}$$

Data packets enter the system according to a Poisson distribution with a rate equal to the maximum data offered load c. Since the packet length is assumed to be exponentially distributed with mean 1024 bits, the average data service rate at the node amounts to $b(M - 1.02)/1024$ packets/s. This yields an average data traffic intensity that amounts to $c/187.38$. The maximum buffer size is $N = 7000$.

The simulation results for the system with maximum offered loads of 160, 180, and 200 packets/s are discussed below. The realization of the simulation results for the system with maximum data offered loads to 160 packets/s is displayed in Fig. 4. The top graph represents the number of off-hook voice calls. The second graph represents the realization of the active off-hook voice calls in the system. The contents of the data buffer are shown in the third graph. The buildup of the data packet queue in the system leads to an intolerable increase of the average time delay and the degradation of the system performance. This is clearly indicated

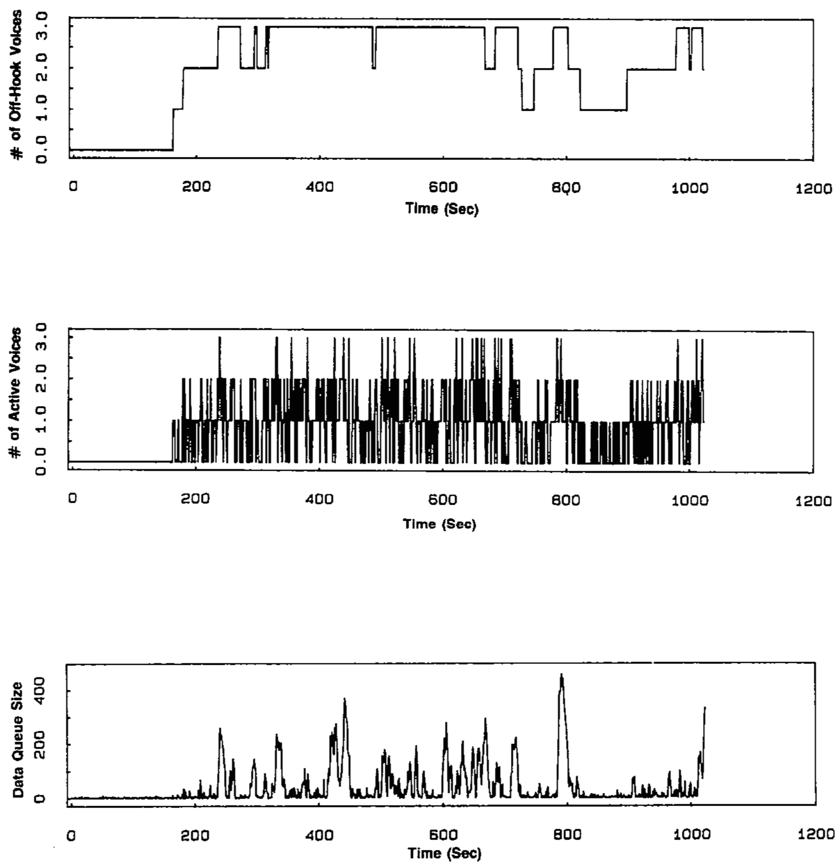

Fig. 4. The performance of the uncontrolled node (c = 160).

in the last graph. Moreover, even though in the case where the data traffic intensity equals 0.85, the last graph of Fig. 4 indicates that the data flow in the system exhibits transient behavior. With 160 packets/s as maximum offered load, the throughput and the time delay of the system are equal to 159.26 packets/s and 259.80 ms, respectively. For c = 180 packets/s, i.e., data traffic intensity 0.95, the throughput and time delay amount to 178.92 packets/s and 1.15, respectively. Upon the increase of the maximum offered load c to 200 packets/s, i.e., data traffic intensity 1.07, the node is saturated. The time delay and throughput of the system are equal to 17.91 s and 191.28 packets/s, respectively. By varying the maximum offered load c, the throughput-offered load and the time delay-offered load performance of the node are obtained and summarized in Table I.

To avoid excessive time delays for data packets, the packet flow between adjacent source–destination nodes has to be controlled according to a suitable optimality criterion (data link flow control). The transient behavior of the data queue, as observed in Fig.4, requires a dynamic optimal data link flow control, i.e., the control actions have to be taken according to the real-time statistical evolution of the system.

III. The Queueing Model for Data Link Flow Control

Let us consider an adjacent source–destination pair of the IN in isolation (see Fig. 1). To avoid excessive time delays at the destination node or subsequent packet loss, the data flow from the source node (node 1) to the destination node (node 2) has to be controlled according to a suitable optimality criterion (data link flow control). To develop a queueing model for the data link flow control protocol of IN's with partial observations, recall that the nodal model (proposed in Section II) consists of a single server queue with an exponential server whose service rate depends on the voice traffic at the destination node. Therefore, the considered source–destination pair can be modeled as a closed queueing system consisting of two queues [see Fig. 2(b)]. X_t and Q_t denote the number of active voice slots and the data queue size at the destination node at time t, respectively.

TABLE I
SUMMARY OF THE SIMULATION RESULTS

C	THROUGHPUT	TIME DELAY (msec)
50	49.74	9.06
120	119.31	45.87
160	159.26	259.80
180	178.92	1150.00
200	191.28	17910.00

In the queueing system of Fig. 2(b), the second queue and its random exponential server $\mu_t = b(M - X_t)$ model the destination node data queue (node 2) and the capacity of its outbound link (or links), respectively. Note that in this model X_t represents the aggregated voice load of all outgoing links at the destination node. One should bear in mind that at the data link level, the source node has no information about the routing strategy of the destination node. In other words, the source node observes the corresponding "gross *effective* service rate" of the destination node. The first queue and its unknown exponential rate $\lambda_t(Q_t)$ represent the data queue at the source node and the controlled transmission rate of the data packets from the source node to the destination node, respectively.

In this model the source node is designated as the controller and its transmission rate $\lambda_t(Q_t)$ represents the control. The control $\lambda_t(Q_t)$ cannot exceed the data capacity of the outbound source link (i.e., link 1). Since the buffer size is assumed to be N, there will be at most N packets in this queueing system. Note that in the closed queueing model set forth, we implicitly assume that the source node has an infinite number of packets (to close the network).

This assumption has no adverse effect on the results. If the traffic load is light and the source has no packets to transmit, the (realized) transmission rate of the source will be less than the optimum rate and the average delay at the destination will be less than the tolerable upper bound.

The interpretation of this model is simple. Whenever a successful transmission is acknowledged by the destination node, the source node (controller) transmits an additional data packet. Therefore, the service rate of the first queue determines the controlled packet arrival rate to the destination node (see also the remark to follow). Due to the lack of on-line information about the state of the voice traffic X_t, it is natural to assume that the controlled arrival rate $\lambda_t(Q_t)$ is a function of the observed component Q_t only. Note that, through the state of the data buffer Q_t, the control $\lambda_t(Q_t)$ implicitly depends on the state of the voice traffic X_t at the destination node. As previously mentioned, the service rate of the first queue, $\lambda_t(Q_t)$, is bounded by the capacity of the outbound source link (link 1) for the data traffic, i.e., the maximum offered load c at the destination node equals the data capacity of link 1. As already mentioned in Section II, this capacity is also random and is correlated with the total data capacity of the outgoing links at the destination node (i.e., with μ_t). Without any loss of generality, the maximum offered load c at the destination node is assumed to be constant (see [9], [10] for the general statement). Moreover, the states of the voice traffic at the source and destination nodes are assumed to be independent.

The transient behavior of the data traffic as observed in Fig. 4 implies the need for controlling the second queue by the first queue over a finite horizon $(0, S]$ under a suitable optimality criterion. The criterion employed maximizes the throughput subject to a maximum allowable time delay T [5]. In practice, the maximum tolerable time delay T represents the time-out period; if a packet is not acknowledged T units of time after its transmission, the source will retransmit a copy of the same packet again. Since for data link flow control protocols the time-out period starts upon the departure of the packet from the source, T represents the upper bound for the average delay of packets at the destination node.

Remark: The incoming data packets at the destination node can be partitioned into two distinct streams. One stream represents packets transmitted from the considered source node, while the other consists of data packets which are received from all other network nodes except the considered source node. We refer to the latter as the "interfering data traffic" at the destination node. Since the source node only monitors its own transmissions and receives acknowledgments for its own packets only, it essentially observes the "effective" queue length of its own packets at the destination node. Therefore, the model implicitly incorporates the interfering data traffic at the destination node. The states of the voice traffic at the source and destination nodes are assumed to be independent.

To establish the general optimization problem for the data link flow control protocol, let $A = (A_t, F_t)$ and $D = (D_t, F_t)$, $0 \leq t \leq S$, defined on the complete probability space (Ω, F, P), be the arrival and departure processes associated with the second queue of Fig. 2(b). The σ-algebra $F_t \subset F$ generated by the process $\{X_u, Q_u, u \leq t\}$ represents the information pattern at the destination node. Then, we have

$$Q_t = Q_0 + A_t - D_t \tag{6}$$

where Q_0 is the initial number of packets in the second queue. Since

$$\left(D_t - \int_0^t \mu_u 1(Q_u > 0)\, du,\ F_t \right)$$

is a Martingale, for all t, $0 \leq t \leq S$ [2], the average number of departures in the interval $(0, S]$ is given by

$$ED_S = E \int_0^S \mu_u 1(Q_u > 0)\, du. \tag{7}$$

Therefore, the average throughput, i.e., the average number of packets leaving the second queue in the time interval $(0, S]$ per unit time, is given by

$$E_{\gamma S} = \frac{1}{S} ED_S$$

and hence

$$E_{\gamma S} = \frac{1}{S} E \int_0^S \mu_u 1(Q_u > 0)\, du. \tag{8}$$

For flow control purposes the analysis also requires an estimate of the time delay. In what follows, the notion of departure system time delay, τ_S, will be introduced. The departure system time delay reflects the amount of time a packet spends in the system. The definition given below is motivated by the definition of the average time delay of a queueing system in equilibrium [5].

The total time delay of the packets in the second queue during the time interval $(0, S]$ is given by [3], [8]

$$\int_0^S Q_u\, du.$$

Since the number of packets departed in $(0, S]$ is D_S, we introduce the following.

Definition 1: The random variable τ_S, given by

$$\tau_S = \frac{\int_0^S Q_u\, du}{ED_S},$$

is said to be the departure system time delay.

Let F_t^Q be the σ-algebra generated by the observed process $\{Q_u, u \leq t\}$.

Definition 2: $\lambda = (\lambda_t(Q_t), F_t^Q)$, $0 \leq t \leq S$, will hereafter denote the control.

The maximum packet switched offered load to the destination node, c, defines the class of admissible controls as in the following.

Definition 3: The class of controls λ such that $0 \leq \lambda_t(Q_t) \leq c$, $0 \leq t \leq S$, where c, $c \in R_+$, is a constant, is called admissible.

The optimality criterion employed for controlling of the second queue maximizes the average throughput such that the average departure system time delay dos not exceed an upper bound T.

Definition 4: The control $\lambda = (\lambda_t(Q_t), F_t^Q)$, $0 \leq t \leq S$, is said to be optimum over the class of admissible controls for a given T, $T \in R_+$, if the maximum of

$$\max_{E\tau_S \leq T} E\gamma_S$$

is achieved.

Thus, we can summarize the general stochastic control problem as follows.

Lemma 1: $\lambda = (\lambda_t(Q_t), F_t^Q)$, $0 \leq t \leq S$, is an optimal control iff it achieves the maximum of

$$E \int_0^S \mu_u 1(Q_u > 0)\, du$$

subject to the set of admissible controls

$$0 \leq \lambda_t(Q_t) \leq c$$

and the system time delay constraint

$$E \int_0^S Q_u\, du - TE \int_0^S \mu_u 1(Q_u > 0)\, du \leq 0.$$

We now focus on the modeling of the voice and data traffic at the destination node. The modeling is carried out in two steps. First, the number of active voice slots X_t is modeled as a birth-death process. Second, a two-dimensional Markov chain model for the joint state of the voice and data traffic at the destination is presented.

Assuming that each off-hook voice call starts with a talkspurt and ends with a silent period, the model for the voice traffic at the destination node is a two-dimensional Markov chain with the state transition diagram shown in Fig. 5(a) [4]. The components l and k of the joint state (l, k) represent the number of *off-hook* and the number of *active* voice slots at the destination node, respectively. This model results, however, in a needless analytical complexity. To simplify it, a simple birth–death model with a state transition diagram shown in Fig. 5(b) is developed. The birth rate (η_j), $0 \leq j < M - m$, and the death rate (ν_j), $0 < j \leq M - m$, of this model are given by

$$\eta_j = \sigma \sum_{l=j+1}^{M-m} (l-j)\pi_l$$

$$\nu_j = j\theta \sum_{l=j}^{M-m} \pi_l$$

for all j, $0 \leq j \leq M - m$, *where* π_l is the probability that there exist l off-hook voice calls at the destination node and is given by

$$\pi_l = \frac{\left(\frac{\delta}{\zeta}\right)^l}{l!} \left\{ \sum_{j=0}^{M-m} \frac{\left(\frac{\delta}{\zeta}\right)^j}{j!} \right\}^{-1}.$$

The $(M - m + 1) \times (M - m + 1)$ matrix $A = [q_{ij}]$ with elements

$$q_{i,i+1} = \eta_i \quad q_{ii} = -(\nu_i + \eta_i) = -q_i$$
$$q_{i,i-1} = \nu_1 \quad q_{ij} = 0, \qquad |i-j| \geq 2 \tag{10}$$

for all i, $0 \leq i \leq M - m + 1$, will hereafter represent the infinitesimal generator of this Markov process. In Section VII the robustness of this model will be verified by means of simulations. The simulation results indicate that the optimum filter developed based on this model provides a good estimate of the voice traffic load at the destination node.

We assume that the joint state of the destination voice and data traffic (X_t, Q_t) constitutes a two-dimensional Markov process. The closed queueing model of the data link flow control [Fig. 2(b)] and the preceding modeling of the voice traffic X_t yield the state transition diagram of Fig. 5(b) for this Markov process with $\zeta_j = b(M - j)$, $0 \leq j \leq M - m$. The rates ν_j and η_j are given in (9).

In summary, for the partial observation case, the two-dimensional Markov process $(X, Q) = \{X_t, Q_t; F_t\}$ with state space $E_1 \times E_2 = \{0, 1, \cdots, M - m\} \times \{0, 1, \cdots, N\}$ and the state transition diagram of Fig. 6 defined on the complete probability space (Ω, F, P), represents the number of active voice slots and the data buffer size at the destination node. As mentioned before, only the component Q_t of this two-dimensional Markov chain is observed. The unobserved component X_t constitutes a birth–death process with the state space $E_1 = \{0, 1, \cdots, M - m\}$ and the state transition diagram of Fig. 5(b).

IV. The Stochastic Control Problem

The general optimization problem for the data link flow control of IN's was presented in Lemma 1. To rephrase the stochastic control problem in terms of the estimated voice traffic load, let $z_t(j)$ denote that there are exactly j active voice calls, i.e.,

$$z_t(j) = 1(X_t = j)$$

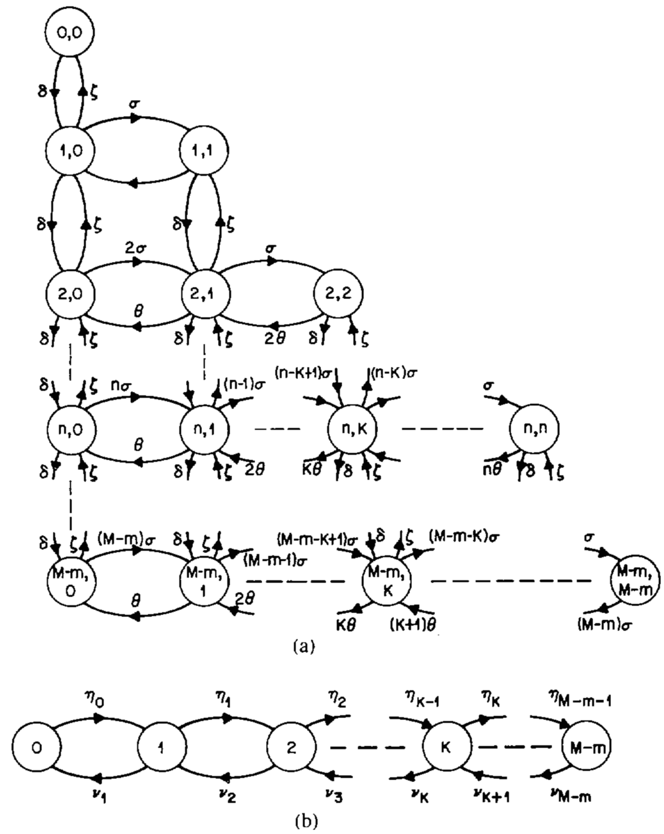

Fig. 5. Modeling of the voice traffic at the destination node.

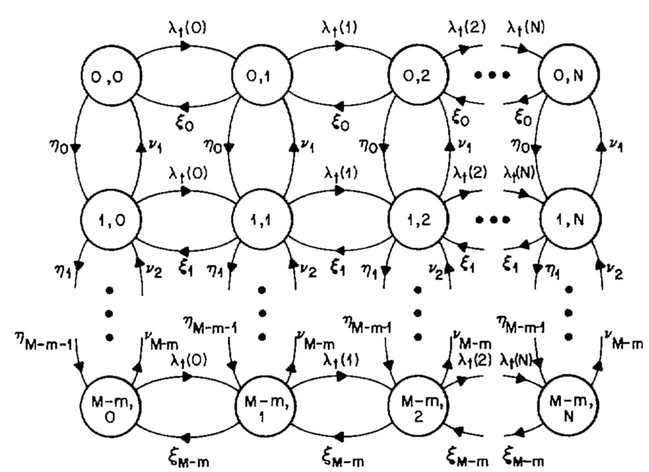

Fig. 6. The two-dimensional Markov model for the state of voice and data traffic at the destination node.

for all $0 \leq j \leq M - m$. Therefore, the state of the voice at the destination node can be uniquely characterized by the $(M - m + 1) \times 1$ vector

$$Z_t = [z_t(0),\ z_t(1),\ \cdots,\ z_t(M-m)]'.$$

Recall that F_t^Q is the σ-algebra generated by the observed process $\{Q_u, u \leq t\}$. Then, the optimization problem can be reformulated as follows.

Theorem 1: $\lambda = (\lambda_t(Q_t), F_t^Q)$, $0 \leq t \leq S$, is an optimal control iff it achieves the maximum of

$$J(\lambda) = E\left\{ \int_0^S \mu \hat{Z}_u 1(Q_u > 0)\ du \right\}$$

subject to

i) the set of state equations

$$\hat{Z}_t=\hat{Z}_0+\int_0^t A'\hat{Z}_u\,du+\int_0^t\left[\frac{H\hat{Z}_{u-}}{\mu\hat{Z}_{u-}}-\hat{Z}_{u-}\right][dD_u-\mu\hat{Z}_u 1(Q_u>0)\,du],$$

ii) the system time delay constraint

$$\int_0^S E\{Q_u-T\mu\hat{Z}_u 1(Q_u>0)\}\,du\le 0,$$

iii) the set of admissible controls

$$0\le\lambda_t(Q_t)\le c,$$

where $\hat{Z}_t = E\{Z_t|F_t^Q\}$, A is the infinitesimal generator of the Markov chain X_t, H is a $(M-m+1)\times(M-m+1)$ diagonal matrix with elements $h_{kk} = b(M-k)$ and $\mu = [\mu_k]$ is a $1\times(M-m+1)$ row matrix with elements $\mu_k = b(M-k)$ for all k, $0\le k\le M-m$, and all t, $0\le t\le S$.

Proof: By applying Fubini's theorem [2], we have

$$J(\lambda)=E\left\{\int_0^S \mu Z_u 1(Q_u>0)\,du\right\}=E\left\{\int_0^S \mu\hat{Z}_u 1(Q_u>0)\,du\right\}$$

and similarly

$$\int_0^S E\{Q_u-T\mu Z_u 1(Q_u>0)\}\,du=\int_0^S E\{Q_u-T\mu\hat{Z}_u 1(Q_u>0)\}\,du\le 0.$$

The posterior distribution $\hat{Z}_t$ verifies the set of equations i). To see this, note that in the preceding section, the state of the voice and data traffic was modeled as a two-dimensional Markov process $\{X_t, Q_t; F_t\}$ with the state transition diagram as shown in Fig. 6. In the partial observation case, only the component Q_t with the state space $E_2 = \{0, 1, \cdots, N\}$ is observed. The unobserved component of the chain X_t is a Markov chain with state space $E_1 = \{0, 1, \cdots, M-m\}$ and infinitesimal generator A. By abuse of language, the MMSE estimate of the state of the voice $\hat{Z}_t$ is given by the conditional expectation [2]

$$\hat{Z}_t=E\{Z_t|F_t^Q\}.$$

Notice that

$$\hat{z}_t(j)=E\{1(X_t=j)|F_t^Q\}=P\{X_t=j|F_t^Q\}.$$

The structure of the optimum filter is based on the following result.

Lemma 2: The optimal filter $\hat{Z}_t$ is given by

$$\hat{Z}_t=\hat{Z}_0+\int_0^t A'\hat{Z}_u\,du+\int_0^t\left[\frac{H\hat{Z}_{u-}}{\mu\hat{Z}_{u-}}-\hat{Z}_{u-}\right][dD_u-\mu\hat{Z}_u 1(Q_u>0)\,du]$$

where A is the infinitesimal generator of the Markov chain X_t and $\hat{Z}_t = [\hat{z}_t(k)]$ is an $(M-m+1)\times 1$ vector with elements $\hat{z}_t(k) = P\{X_t = k|F_t^Q\}$ for all t, $0\le t\le S$. The initial condition $\hat{Z} = [\hat{z}_0(k)]$ is an $(M-m+1)\times 1$ vector with elements $\hat{z}_0(k) = P\{X_0 = k\}$. Finally, H is an $(M-m+1)\times(M-m+1)$ diagonal matrix with elements $h_{kk} = b(M-k)$ and $\mu = [\mu_k]$ is a $1\times(M-m+1)$ row matrix with elements $\mu_k = b(M-k)$ for all $0\le k\le M-m$.

The proof of this lemma is given in Appendix A.

V. The Separation Principle and the Optimal Control

A recursive filter which estimates the state of the voice traffic in the system was obtained in the previous section. This estimate $\hat{Z}_t$ completely characterizes the posterior distribution of X_t. The explicit statement of the stochastic control problem was presented in Theorem 1. It is natural to expect that the estimate $\hat{Z}_t$ of X_t be a sufficient statistic for the optimal control. We refer to this property as the *separation principle.* The optimal control strategy in the partial observations case is a bang-bang control policy. The separation principle and the optimality of the bang-bang control policy for the optimization problem of Theorem 1 are presented in the following.

Theorem 2: There exists a differentiable function $V(\hat{Z}_t, Q_t, t)$ and a control $\lambda = (\lambda_t(Q_t), F_t^Q)$, for all $Q_t = i$, $0\le i\le N$, and for all t, $0\le t\le S$, such that the partial differential equation

$$\begin{aligned}\mu\hat{Z}_t 1(Q_t=i>0)&+\frac{\partial V(\hat{Z}_t,i,t)}{\partial t}+\frac{\partial V(\hat{Z}_t,i,t)}{\partial\hat{Z}_t}\frac{d\hat{Z}_t^c}{dt}\\&+c1([V(\hat{Z}_t,i+1,t)-V(\hat{Z}_t,i,t)]>0)\\&\cdot[V(\hat{Z}_t,i+1,t)-V(\hat{Z}_t,i,t)]-\mu\hat{Z}_t 1(Q_t=i>0)\\&\cdot\left[V\left(\frac{H\hat{Z}_t}{\mu\hat{Z}_t},i-1,t\right)-V(\hat{Z}_t,i,t)\right]=0\end{aligned}$$

with boundary condition

$$V(\hat{Z}_S, i, S)=0$$

is satisfied, where $\hat{Z}_t^c$ is the continuous part of $\hat{Z}_t$. The optimal control policy $\lambda_t(Q_t = i)$, $0\le i\le N$, and $0\le t\le S$, is of bang-bang type and is given by

$$\lambda_t(Q_t=i)=\begin{cases}c & \text{if } V(\hat{Z}_t,i+1,t)-V(\hat{Z}_t,i,t)>0\\ \text{undetermined} & \text{if } V(\hat{Z}_t,i+1,t)-V(\hat{Z}_t,i,t)=0\\ 0 & \text{if } V(\hat{Z}_t,i+1,t)-V(\hat{Z}_t,i,t)<0.\end{cases}$$

Moreover, the maximum throughput given $\hat{Z}_0$ and Q_0 is then

$$\sup_\lambda J(\lambda)=V(\hat{Z}_0, Q_0, 0).$$

Proof: The proof of this theorem is presented in Appendix B.

Remark: The function $V(\hat{Z}_t, Q_t, t)$ represents the *optimal cost to go* and is defined by (see [2])

$$V(\hat{Z}_t, Q_t, t)=E\left\{\int_t^S \mu\hat{Z}_u 1(Q_u>0)\,du\,|\,\hat{Z}_t, Q_t\right\}.$$

This theorem highlights the bang-bang structure of the optimal control policy. The optimum control $\lambda_t(Q_t)$ specifies the instantaneous source transmission rate and *dynamically* adapts to the real-time evolution of the traffic in the network. Since the computation of the function $V(\cdot,\cdot,\cdot)$ appears to be prohibitive, the explicit structure of the optimum control actions cannot be determined from the above theorem.

Theorem 2 indicates that a bang-bang control of the source transmission rate is optimum, i.e., the source node either transmits with the maximum rate or does not transmit at all. It is also known that in the window flow control schemes, the window size represents the number of unacknowledged packets in the system. Unless the number of unacknowledged packets exceeds the window size, the source transmits packets to the destination at the maximum rate. Whenever the number of unacknowledged packets equals the window size, the source does not transmit until the number of unacknowledged packets is reduced below the window size. Hence, any window flow control mechanism is a bang-bang control of the source transmission rate. We have also shown that the optimal data link flow control of IN's with complete observations is also a bang-bang control mechanism [11]. In the complete observation case, an adaptive flow control mechanism has been proposed whose window size is a function of the destination voice load X_t, the state of the destination data queue Q_t, the maximum tolerable time delay T, the length of the control interval S, and the offered load c [11]. To emphasize the dependence of the window size on X_t, we denote the window size for the complete observations case by $L(X_t)$. Since i) the optimal data link flow control of IN's with partial observation is a bang-bang policy, and ii) the estimate $\hat{Z}_t$ is a sufficient statistic for the optimal control, it is natural to consider the class of (suboptimal) bang-bang policies with a window size L_t given by

$$L_t = \sum_{j=0}^{M-m} L(X_t = j)\hat{z}_t(j). \tag{11}$$

The implementation of this window flow control protocol can be done by using an $(N+1) \times (M-m+1)$ table of window sizes for different values of the Q_t's and X_t's. It also requires some computing resource for the filtering process at the source node.

VI. Performance of the Suboptimum Protocol

In this section, we are primarily concerned with the performance of the suboptimum data link flow control protocol proposed in the previous section. Our objective is to address four important issues by means of simulation. They are

i) accuracy of the filter;

ii) robustness of the Markov model of Fig. 5(b) for the number of active voice slots;

iii) the effectiveness of the suboptimum protocol;

iv) the comparison between the performance of the suboptimum protocols with complete and with partial observations.

To accomplish this goal, a simulation study has been carried out in two steps.

Step 1: We focus on the accuracy of the recursive filter. The simulation model employed here differs from the simulator described in Section II (SIMIN) in only one aspect. The difference is that in this model we generate the aggregated destination voice activity according to a Markov chain with the state transition diagram of Fig. 5(b), rather than implementing an independent speaker activity generator on each channel. The optimum filter (Lemma 2) and the suboptimum window flow control mechanism [see (11)], with a maximum tolerable time delay of $T = 16$ ms, have subsequently been implemented in this model. The simulation results for the maximum offered loads of 160, 180, and 200 packets/s are obtained. A set of realizations for maximum offered load of 160 packets/s is depicted in Fig. 7. In this figure, (a) shows the number of active voice slots X_t. The estimated number of active voice slots $\hat{X}_t$, i.e., $\hat{X}_t = \sum_{j=0}^{M-m} j\hat{z}_t(j)$, is represented in (b). The corresponding "window process" and the data queue size are displayed in (c) and (d), respectively.

Despite the rapid variation of the signal X_t, it is clear from Fig. 7(a) and (b) that the filter provides a good estimate $\hat{X}_t$ of the signal X_t. This judgement is subjective, however. To quantify the accuracy of the filter, let us consider its signal-to-noise ratio. If IS denotes the length of each simulation run, the signal-to-noise ratio (SNR) is defined as

$$\mathrm{SNR} = 10 \cdot \log \left[\frac{\int_0^{IS} X_u^2 \, du}{\int_0^{IS} (X_u - \hat{X}_u)^2 \, du} \right].$$

The SNR has been obtained in each of the above trials. The results are summarized in Table II. So far in Step 1, the aggregated voice activity is generated according to the Markov model of Fig. 5b. In addition, the structure of the m.m.s.e filter is also developed based upon the same model. Hence, Table II provides us with a measure of accuracy for the optimum filter. In Step 2, we will take advantage of this measure to experimentally test the robustness of the Markov model of Fig. 5(b) for the number of active voice slots at the destination node.

Step 2: To address ii), iii), and iv), the optimum filter (Lemma 2) and the suboptimum protocol [see (11)] with maximum tolerable system time delay of $T = 16$ ms have been implemented on the original simulator (SIMIN). As in Step 1, the simulation results for the maximum offered loads of 160, 180, and 200 packets/s are obtained and the realization of the results for maximum offered load of 160 packets/s is displayed in Fig. 8. In this figure, the number of off-hook voice calls in the system is shown in (a). The number of active voice slots X_t and its estimate $\hat{X}_t$ are depicted in (b) and (c), respectively. Finally, (d) and (e) display the corresponding realizations of the window process and the data queue size.

A subjective look at the signal X_t and its estimate $\hat{X}_t$ in Fig. 8 indicates that in spite of the rapid fluctuations in the signal, the filter obtains a good estimate of its values. This qualitative argument is quantified by calculating the SNR for all three trials. These are summarized in Table III.

The comparison of the corresponding entries in Tables II and III shows the degree of robustness of the proposed model [see Fig. 5(b)] for the number of active voice slots at the destination node.

The realizations of the "window process" L_t indicate the dynamics of the window control mechanism. In these trials (both steps) the optimum window size has been computed according to (11). As Fig. 8(d) indicates, at any point of time L_t has taken integer values in the interval (0,20]. Particularly, throughout this trial the optimum window size has taken all of the integers in the interval [3, 20]. With a maximum offered load of 160 packets/s, the average throughput and time delay of the dynamically controlled system are equal to 141.81 packets/s and 15.04 ms, respectively. The implementation of the suboptimum control strategy results in 11 percent reduction in the throughput of the system from 159.26 packets/s for the uncontrolled system to 141.81 packets/s for the controlled system. The reduction of the time delay from 259.80 ms before the implementation of the suboptimal flow control policy to 15.04 ms after the adoption of this control strategy, i.e., 18 times reduction of time delay, demonstrates a significant improvement in the performance of the system. For the maximum offered load of 180 packets/s, the average throughput and the average system time delay of the dynamically controlled system dropped to 153.81 packets/s and 16.81 ms from 178.92 packets/s and 1.15 s in the uncontrolled case. This shows that while the average throughput of the dynamically controlled system decreases by about 14 percent, the average system time delay decreases 68 times when compared to the same parameters in the controlled case. The improvement in the system performance is more dramatic for the case of 200 packet/s maximum offered load. In this case the time delay decreases from 17.91 s to 16.73 ms, while the throughput drops from 191.28 to 161.78 packets/s. This is only a 15 percent drop in the throughput of the system. These results are summarized in Table IV.

By varying the maximum tolerable time delay T, the throughput–time delay performance of the system with partial observa-

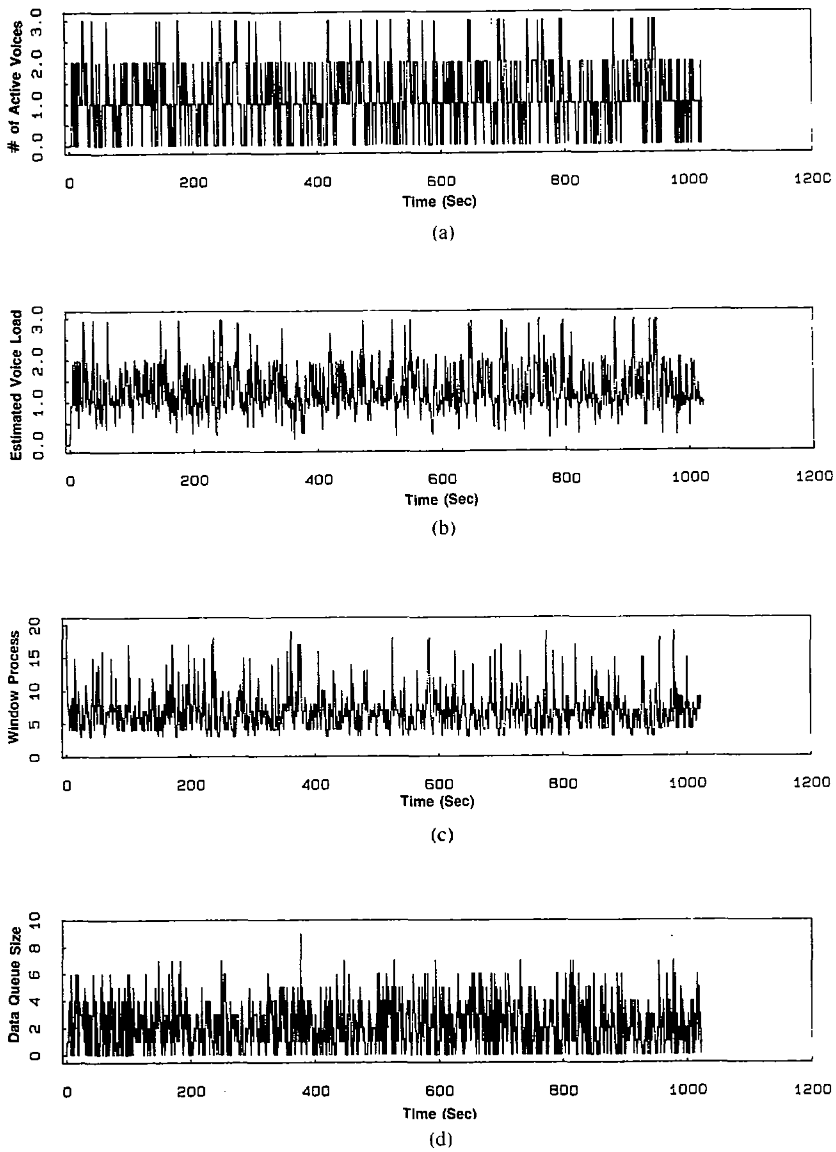

Fig. 7. The performance of the suboptimum protocol with partial observations and $c = 160$ (aggregated voice activity).

TABLE II
THE SIGNAL-TO-NOISE RATIO (AGGREVATED VOICE ACTIVITY)

c	SNR (db)
160	8.65
180	8.89
200	9.00

tions is obtained. Fig. 9 displays this curve alongside its complete observations counterpart (taken from [11]). Due to the estimation error, it is expected that the performance of the optimum control protocol with partial observations be somewhat inferior to the performance of the optimum control protocol in the complete observation case. The simulation results support this argument.

VII. Conclusions

In this paper, a new class of problems arising in the emerging field of integrated networks has been considered. A network protocol already studied by the authors in [9] was used as the basis for a queueing network model with two classes of users. Although only voice and data have been named in our study, the investigations can be easily extended to any information flow that requires real and nonreal time performance characteristics.

An optimization criterion that reflects the expected performance requirements for integrated networks has been developed for data link flow control. The class of control algorithms considered maximizes the link utilization subject to a system time delay constraint on a finite horizon. It has been shown that the optimal flow control protocol is a bang-bang policy and the conditional mean estimate of the voice load at the destination node is a sufficient statistic for optimal control. In practice, in order to obtain an estimate of the destination voice traffic load, an optimum MMSE filter has to be implemented at the source node. The recursive nature of the filter lends itself to a simple real-time

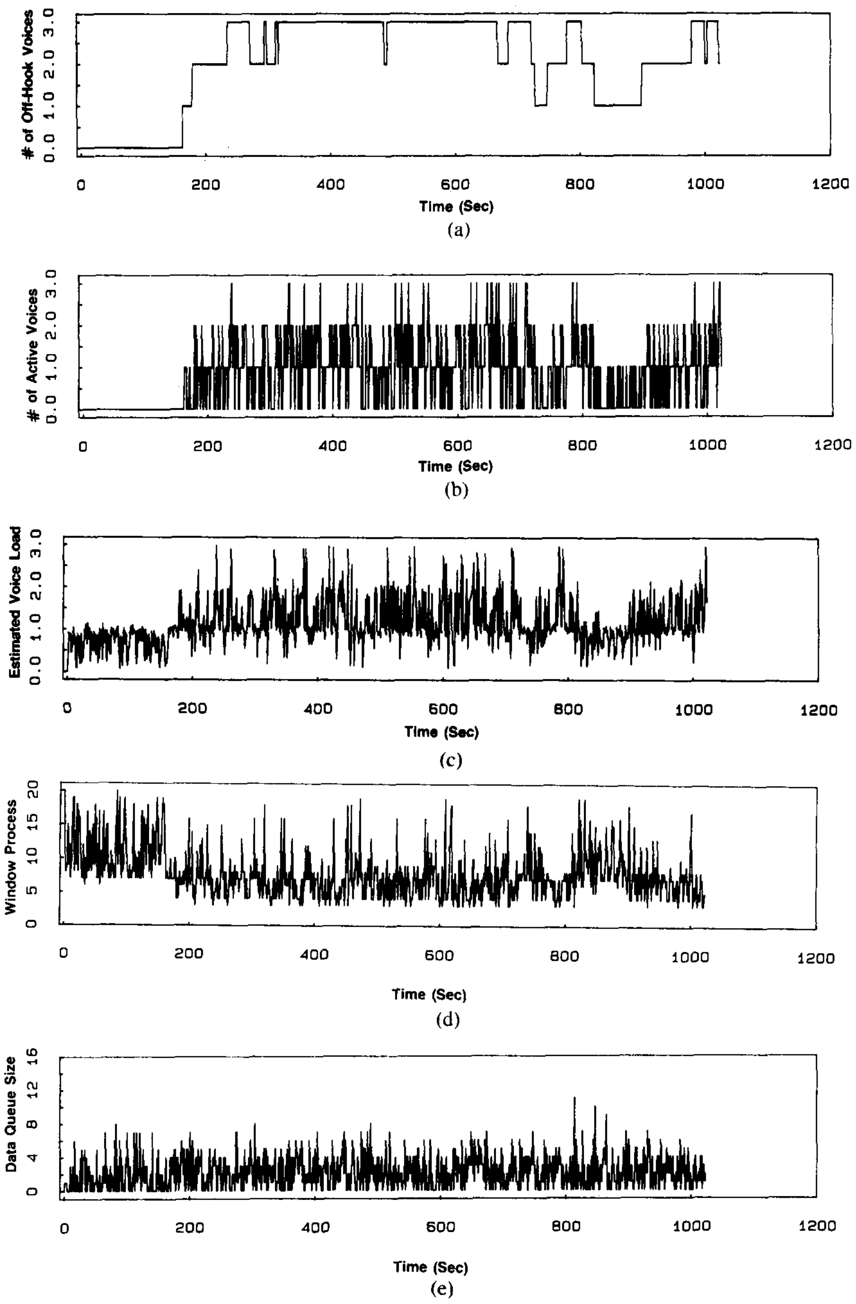

Fig. 8. The performance of the suboptimum protocol with partial observations and $c = 160$ (independent speakers).

TABLE III
THE SIGNAL-TO-NOISE RATIO (INDEPENDENT SPEAKERS)

c	SNR (db)
160	5.81
180	6.73
200	7.20

TABLE IV
THE PERFORMANCE OF THE PROTOCOL

c	THROUGHPUT	TIME DELAY (msec)
50	49.74	9.06
120	114.16	13.39
160	141.81	15.04
180	153.81	16.81
200	161.71	16.73

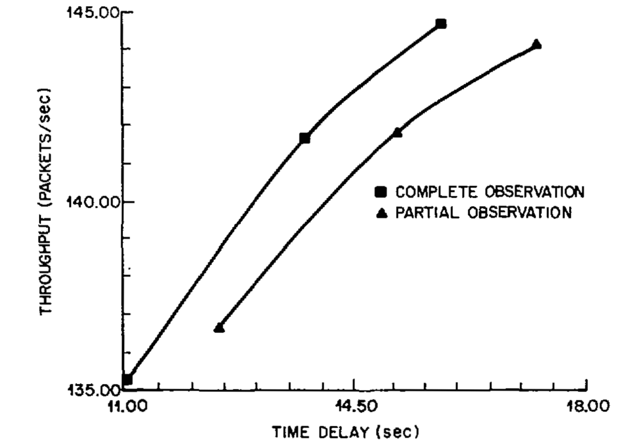

Fig. 9. Throughput-time delay performance of the system with the suboptimum protocol, $c = 160$, complete and partial observations.

implementation. Finally, a practical suboptimal solution (i.e., an adaptive window flow control scheme) has been proposed and studied by means of simulations. The suboptimum window size dynamically adapts to the estimated state of the destination voice traffic. The simulation results indicate the accuracy and robustness of the filter, the time-dependent variation of the window size, and the effectiveness of the proposed window flow control scheme.

Appendix A

Proof of Lemma 2: The proof follows the Martingale approach treated in [2]. First, note that the following semi-Martingale representation can be obtained for $z_t(l)$:

$$z_t(l) = z_0(l) + \int_0^t \left(\sum_{i \in E_1 - \{l\}} z_u(i) q_{il} - q_l z_u(l) \right) du + n_t(l) \quad \text{(A.1)}$$

where $\{n_t(l), F_t\}$ is a Martingale on (Ω, F, P), for all l, $0 \le l \le M - m$.

The arrival process A_t and the departure process D_t [see (6)] admit the intensities $(\lambda_t(Q_t), F_t)$ and $(\mu_t 1(Q_t > 0), F_t)$, respectively, with $\mu_t = \boldsymbol{\mu} \cdot Z_t$. Therefore, from the projection theorem [2] we have

$$\hat{z}_t(l) = \hat{z}_0(l) + \int_0^t \left(\sum_{i \in E_1 - \{l\}} \hat{z}_u(i) q_{il} - q_l \hat{z}_u(l) \right) du$$

$$+ \int_0^t K_u^{(1)}(l)(dA_u - \hat{\lambda}_u \, du)$$

$$+ \int_0^t K_u^{(2)}(l)(dD_u - \hat{\mu}_u 1(Q_u > 0) \, du) \quad \text{(A.2)}$$

where the F_t^Q-predictable processes $K_t^{(i)}(l)$, $i = 1, 2$, are the innovation gains and their calculation completes the proof. The innovation gains can be decomposed in three components as follows [2]:

$$K_t^{(i)}(l) = \Psi_{1,t}^{(i)}(l) - \Psi_{2,t}^{(i)}(l) + \Psi_{3,t}^{(i)}(l)$$

where the ψ's are F_t^Q-predictable processes. Since the processes Z_t, A_t, and D_t have no common jumps, $\Psi^{(i)}(l) = 0$, for $i = 1, 2$. Also, $\Psi_{2,t}^{(i)}(l) = \hat{z}_{t-}(l)$ for $i = 1, 2$ [2]. The process $\Psi_{1,t}^{(1)}(l)$ has the property that the equality

$$E\left\{ \int_0^t C_u z_u(l) \lambda_u \, du \right\} = E\left\{ \int_0^t C_u \Psi_{1,u}^{(1)}(l) \hat{\lambda}_u \, du \right\} \quad \text{(A.3)}$$

is satisfied for all F_t^Q-predictable nonnegative bounded process C_t and for all t, $0 \le t \le S$. Note that since $\lambda_t = \lambda_t(Q_t)$, and $\mu_t = \boldsymbol{\mu} \cdot Z_t = \Sigma_{j \in E_1} b(M - j) z_t(j)$, their respective conditional means are $\hat{\lambda}_t = \lambda_t = \lambda_t(Q_t)$ and $\hat{\mu}_t = \Sigma_{j \in E_1} b(M - j) \hat{z}_t(j)$. Since $\lambda_u = \hat{\lambda}_u = \lambda_u(Q_u)$, (A.3) implies that

$$\Psi_{1,t}^{(1)}(l) = \hat{z}_{t-}(l) \quad \text{(A.4)}$$

for all l, $0 \le l \le M - m$. Similarly, $\Psi_{1,u}^{(2)}(l)$ satisfies

$$E\left\{ \int_0^t C_u z_u(l) \mu_u 1(Q_u > 0) \, du \right\}$$

$$= E\left\{ \int_0^t C_u \Psi_{1,u}^{(2)}(l) \hat{\mu}_u 1(Q_u > 0) \, du \right\} \quad \text{(A.5)}$$

for all F_t^Q-predictable nonnegative bounded processes C_t and for all $0 \le t \le S$, and $0 \le l \le M - m$. We have

$$\mu_u 1(Q_u > 0) z_u(l) = \boldsymbol{\mu} \cdot Z_u z_u(l) 1(Q_u > 0) = b(M - 1) z_u(l) 1(Q_u > 0) \quad \text{(A.6)}$$

and

$$\Psi_{1,t}^{(2)}(l) = \frac{b(M - l)\hat{z}_{t-}(l)}{\sum_{j \in E_1} b(M - j)\hat{z}_u(j) 1(Q_u > 0).} \quad \text{(A.7)}$$

From (A.5)–(A.7) we conclude that

$$\Psi_{1,t}^{(2)}(l) = \frac{b(M - l)\hat{z}_{t-}(l)}{\sum_{j \in E_1} b(M - j)\hat{z}_{t-}(j)} \quad \text{(A.8)}$$

for all l, $0 \le l \le M - m$. Therefore,

$$K_t^{(1)}(l) = 0$$

and

$$K_t^{(2)}(l) = \frac{b(M - l)\hat{z}_{t-}(l)}{\sum_{j \in E_1} b(M - j)\hat{z}_{t-}(j)} - \hat{z}_{t-}(l) \quad \text{(A.9)}$$

for all l, $0 \le l \le M - m$. Note that, since the arrival process A_t does not carry any information about X_t, it is natural to have $K_t^{(1)}(l) = 0$ for all l, $0 \le l \le M - m$. Substituting (A.9) into (A.2), the recursive filter is given by

$$\hat{z}_t(l) = \hat{z}_0(l) + \int_0^t \left(\sum_{i \in E_1 - \{l\}} \hat{z}_u(i) q_{il} - q_l \hat{z}_u(l) \right) du$$

$$+ \int_0^t \left\{ \frac{b(M - l)\hat{z}_{u-}(l)}{\sum_{j \in E_1} b(M - j)\hat{z}_{u-}(j)} - \hat{z}_{u-}(l) \right\}$$

$$\cdot \left\{ dD_u - \left[\sum_{j \in E_1} b(M - j)\hat{z}_u(j) 1(Q_u > 0) \right] du \right\}, \quad \text{(A.10)}$$

for all l, $0 \le l \le M - m$. Rewriting (A.10) in matrix form completes the proof.

Appendix B

The proof of the *separation principle* (i.e., Theorem 2) is based on the following lemma stating that the joint process $(\hat{Z}_t$,

Q_t; F_t^Q) is Markov. Using this result, we conclude the Appendix with the proof of Theorem 2.

Lemma B1: The joint process ($\hat{Z}_t$, Q_t; F_t^Q) defined on (Ω, F, P) with values in $E_3 = [0, 1]^{M-m+1} \times \{0, 1, \cdots, N\}$ is Markov.

Proof: Since the arrival process A_t and the departure process D_t [see (6)] admit the intensities ($\lambda_t(Q_t)$; F_t^Q) and $\mu \hat{Z}_t 1(Q_t > 0)$; F_t^Q), respectively, we can write

$$Q_t = Q_s + \int_s^t [\lambda_u(Q_u)\ du - \mu \hat{Z}_u 1(Q_u > 0)]\ du + \xi_t \quad \text{(B.1)}$$

and

$$\hat{Z}_t = \hat{Z}_s + \int_s^t A' \hat{Z}_u\ du + \int_s^t \left[\frac{H\hat{Z}_{u-}}{\mu \hat{Z}_{u-}} - \hat{Z}_u \right] [dD_u - \mu \hat{Z}_u 1(Q_u > 0)\ du] \quad \text{(B.2)}$$

where (ξ_t; F_t^Q) is a Martingale on (Ω, F, P). Equations (B.1) and (B.2) imply that

$$P\{\hat{Z}_t, Q_t | \hat{Z}_u, Q_u, u \le s\} = P\{\hat{Z}_t, Q_t | \hat{Z}_s, Q_s\},$$

i.e., the process ($\hat{Z}_t$, Q_t; F_t^Q) is Markov.

Proof of Theorem 2: The proof gives the solution of the optimal control problem of Theorem 1. To simplify the optimization problem, let us introduce the auxiliary state variable $\hat{z}_t(M - m + 1)$ by

$$\hat{z}_t(M-m+1) = -\int_t^S E\{Q_u - T\mu \hat{Z}_u 1(Q_u > 0)\}\ du \quad \text{(B.3)}$$

with the initial conditions

$$\hat{z}_0(M-m+1) = 0. \quad \text{(B.4)}$$

Therefore, the auxiliary state $\hat{z}_t(M - m + 1)$ satisfies the differential equation

$$d\hat{z}_t(M-m+1) = E\{Q_t - T\mu \hat{Z}_u 1(Q_u > 0)\}\ du, \qquad \hat{z}_0(M-m+1) = 0. \quad \text{(B.5)}$$

To augment the auxiliary state variable in the optimization problem, we redefine the vectors μ and $\hat{Z}_t$ and the diagonal matrix H as follows. The $(M - m + 2) \times 1$ vector $\hat{Z}_t$ is given by

$$\hat{Z}_t = [\hat{z}_t(0), \hat{z}_t(1), \cdots, \hat{z}_t(M-m), \hat{z}_t(M-m+1)],$$

the $1 \times (M - m + 2)$ row matrix μ is redefined as

$$\mu = [bM, b(M-1), \cdots, b(M-m), 0],$$

and the $(M - m + 2) \times (M - m + 2)$ diagonal matrix H has the elements

$$h_{kk} = \begin{cases} b(M-k) & \text{if } 0 \le k \le M-m \\ 0 & \text{if } k = M-m+1. \end{cases}$$

Hence, in the partial observations case, we arrive at the following optimization problem.

Problem: Find the control $\lambda = (\lambda_t(Q_t), F_t^Q)$, $0 \le t \le S$, that achieves the maximum of

$$J(\lambda) = E\left\{ \int_0^S \mu \hat{Z}_u 1(Q_u > 0)\ du \right\}$$

subject to

i) the set of state equations

$$\hat{z}_t(l) = \hat{z}_0(l) + \int_0^t [\eta_{l-1}\hat{z}_u(l-1) + \nu_{l+1}\hat{z}_u(l+1) - (\eta_l + \nu_l)\hat{z}_u(l)]\ du + \int_0^t \left\{ \frac{b(M-l)\hat{z}_{u-}(l)}{\sum_{j \in E_1} b(M-j)\hat{z}_{u-}(j)} - \hat{z}_{u-}(l) \right\} \cdot \left\{ dD_u - \left[\sum_{j \in E_1} b(M-j)\hat{z}_u(j) 1(Q_u > 0) \right] du \right\}$$

for all $0 \le l \le M - m$, and

$$d\hat{z}_t(M-m+1) = E\{Q_t - T\mu \hat{Z}_t 1(Q_t > 0)\} dt, \qquad \hat{z}_0(M-m+1) = 0$$

ii) the set of admissible controls

$$0 \le \lambda_t(Q_t) \le c.$$

To solve this problem, let us recall that

$$dV(\hat{Z}_t, Q_t, t) = \frac{\partial V(\hat{Z}_t, Q_t, t)}{\partial \hat{Z}_t} d\hat{Z}_t^c + \frac{\partial V(\hat{Z}_t, Q_t, t)}{\partial t} dt + [V(\hat{Z}_t, Q_t, t) - V(\hat{Z}_{t-}, Q_{t-}, t)] dA_t - [V(\hat{Z}_t, Q_t, t) - V(\hat{Z}_{t-}, Q_{t-}, t)] dD_t. \quad \text{(B.6)}$$

Since at an arrival $\hat{Z}_t = \hat{Z}_{t-}$ and $Q_t = Q_{t-} + 1$ and at a departure $\hat{Z}_t = H\hat{Z}_{t-}/\mu \hat{Z}_{t-}$ and $Q_t = Q_{t-} - 1$ (given $Q_{t-} > 0$), (B.6) results in

$$V(\hat{Z}_t, Q_t, t) = V(\hat{Z}_0, Q_0, 0) + \int_0^t \frac{\partial V(\hat{Z}_u, Q_u, u)}{\partial \hat{Z}_u} \frac{d\hat{Z}_u^c}{du} + \frac{\partial V(\hat{Z}_u, Q_u, u)}{\partial u} du + \int_0^t [V(\hat{Z}_{u-}, Q_{u-} + 1, u) - V(\hat{Z}_{u-}, Q_{u-}, u)]\ dA_u - \int_0^t \left[V\left(\frac{H\hat{Z}_{u-}}{\mu \hat{Z}_{u-}}, Q_{u-} - 1, u \right) - V(\hat{Z}_{u-}, Q_{u-}, u) \right] dD_u.$$

Therefore, we have

$$V(\hat{Z}_t, Q_t, t) = V(\hat{Z}_0, Q_0, 0) + \int_0^t \left\{ \frac{\partial V(\hat{Z}_u, Q_u, u)}{\partial \hat{Z}_u} \frac{d\hat{Z}_u^c}{du} + \frac{\partial V(\hat{Z}_u, Q_u, u)}{\partial u} + [V(\hat{Z}_{u-}, Q_{u-} + 1, u) - V(\hat{Z}_{u-}, Q_{u-}, u)] \lambda_u(Q_u) - \left[V\left(\frac{H\hat{Z}_{u-}}{\mu \hat{Z}_{u-}}, Q_{u-} - 1, u \right) - V(\hat{Z}_{u-}, Q_{u-}, u) \right] \mu \hat{Z}_u 1(Q_u > 0) \right\} du + \int_0^t [V(\hat{Z}_{u-}, Q_{u-} + 1, u) - V(\hat{Z}_{u-}, Q_{u-}, u)] \cdot$$

$$\cdot [dA_u - \lambda_u(Q_u)\, du]$$
$$- \int_0^t \left[V\left(\frac{H\hat{Z}_{u-}}{\mu \hat{Z}_{u-}},\ Q_{u-}-1,\ u \right) - V(\hat{Z}_{u-},\ Q_{u-},\ u) \right] [dD_u - \mu \hat{Z}_u 1(Q_u>0)\, du].$$

Since by hypothesis $V(\hat{Z}_S, Q_S, S) = 0$, then

$$\int_0^S \mu \hat{Z}_u 1(Q_u>0)\, du + V(\hat{Z}_S,\ Q_S,\ S)$$
$$= V(\hat{Z}_0,\ Q_0,\ 0) + \int_0^S \left\{ \mu \hat{Z}_u 1(Q_u>0) + \frac{\partial V(\hat{Z}_u,\ Q_u,\ u)}{\partial \hat{Z}_u} \frac{d\hat{Z}_u^c}{du} + \frac{\partial V(\hat{Z}_u,\ Q_u,\ u)}{\partial u} \right.$$
$$+ [V(\hat{Z}_{u-},\ Q_{u-}+1,\ u) - V(\hat{Z}_{u-},\ Q_{u-},\ u)]\lambda_u(Q_u)$$
$$\left. - \left[V\left(\frac{H\hat{Z}_{u-}}{\mu \hat{Z}_{u-}},\ Q_{u-}-1,\ u \right) - V(\hat{Z}_{u-},\ Q_{u-},\ u) \right] \cdot \mu \hat{Z}_u 1(Q_u>0) \right\} du + \int_0^S [V(\hat{Z}_{u-},\ Q_{u-}+1,\ u)$$
$$- V(\hat{Z}_{u-},\ Q_{u-},\ u)][dA_u - \lambda_u(Q_u)\, du]$$
$$- \int_0^S \left[V\left(\frac{H\hat{Z}_{u-}}{\mu \hat{Z}_{u-}},\ Q_{u-}-1,\ u \right) - V(\hat{Z}_{u-},\ Q_{u-},\ u) \right]$$
$$\cdot [dD_u - \mu \hat{Z}_u 1(Q_u>0)\, du].$$

Finally, noting that $[V(\hat{Z}_{u-}, Q_{u-} + 1, u) - V(\hat{Z}_{u-}, Q_{u-}, u)]$ and $[V(H\hat{Z}_{u-}/\mu\hat{Z}_{u-}, Q_{u-} - 1, u) - V(\hat{Z}_{u-}, Q_{u-}, u)]$ are bounded F_t^Q-predictable processes,

$$E\left\{ \int_0^S [V(\hat{Z}_{u-},\ Q_{u-}+1,\ u) - V(\hat{Z}_{u-},\ Q_{u-},\ u)] [dA_u - \lambda_u(Q_u)\, du] \right\} = 0$$

and

$$E\left\{ \int_0^S \left[V\left(\frac{H\hat{Z}_{u-}}{\mu \hat{Z}_{u-}},\ Q_{u-}-1,\ u \right) - V(\hat{Z}_{u-},\ Q_{u-},\ u) \right] \cdot [dD_u - \mu \hat{Z}_u 1(Q_u>0)\, du] \right\} = 0.$$

Therefore, (B.7) reduces to

$$J(\lambda) = V(\hat{Z}_0,\ Q_0,\ 0) + E\left[\int_0^S \left\{ \mu \hat{Z}_u 1(Q_u>0) + \frac{\partial V(\hat{Z}_u,\ Q_u,\ u)}{\partial \hat{Z}_u} \frac{d\hat{Z}_u^c}{du} + \frac{\partial V(\hat{Z}_u,\ Q_u,\ u)}{\partial u} \right. \right.$$
$$+ [V(\hat{Z}_{u-},\ Q_{u-}+1,\ u) - V(\hat{Z}_{u-},\ Q_{u-},\ u)]\lambda_u(Q_u)$$
$$\left. \left. - \left[V\left(\frac{H\hat{Z}}{\mu \hat{Z}_{u-}},\ Q_{u-}-1,\ u \right) - V(\hat{Z}_{u-},\ Q_{u-},\ u) \right] \cdot \mu \hat{Z}_u 1(Q_u>0) \right\} du \right].$$

By hypothesis

$$J(\lambda) \le V(\hat{Z}_0,\ Q_0,\ 0)$$

and $\lambda_u^*(Q_u)$ defined in the theorem is such that

$$J(\lambda^*) = V(\hat{Z}_0,\ Q_0,\ 0).$$

Hence

$$J(\lambda^*) = \sup_{\lambda} J(\lambda) = V(\hat{Z}_0,\ Q_0,\ 0)$$

and the proof is complete.

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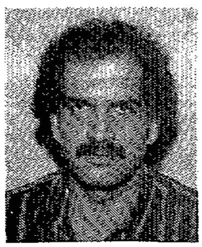

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