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CS323

Writ #2: Sorting and Trees

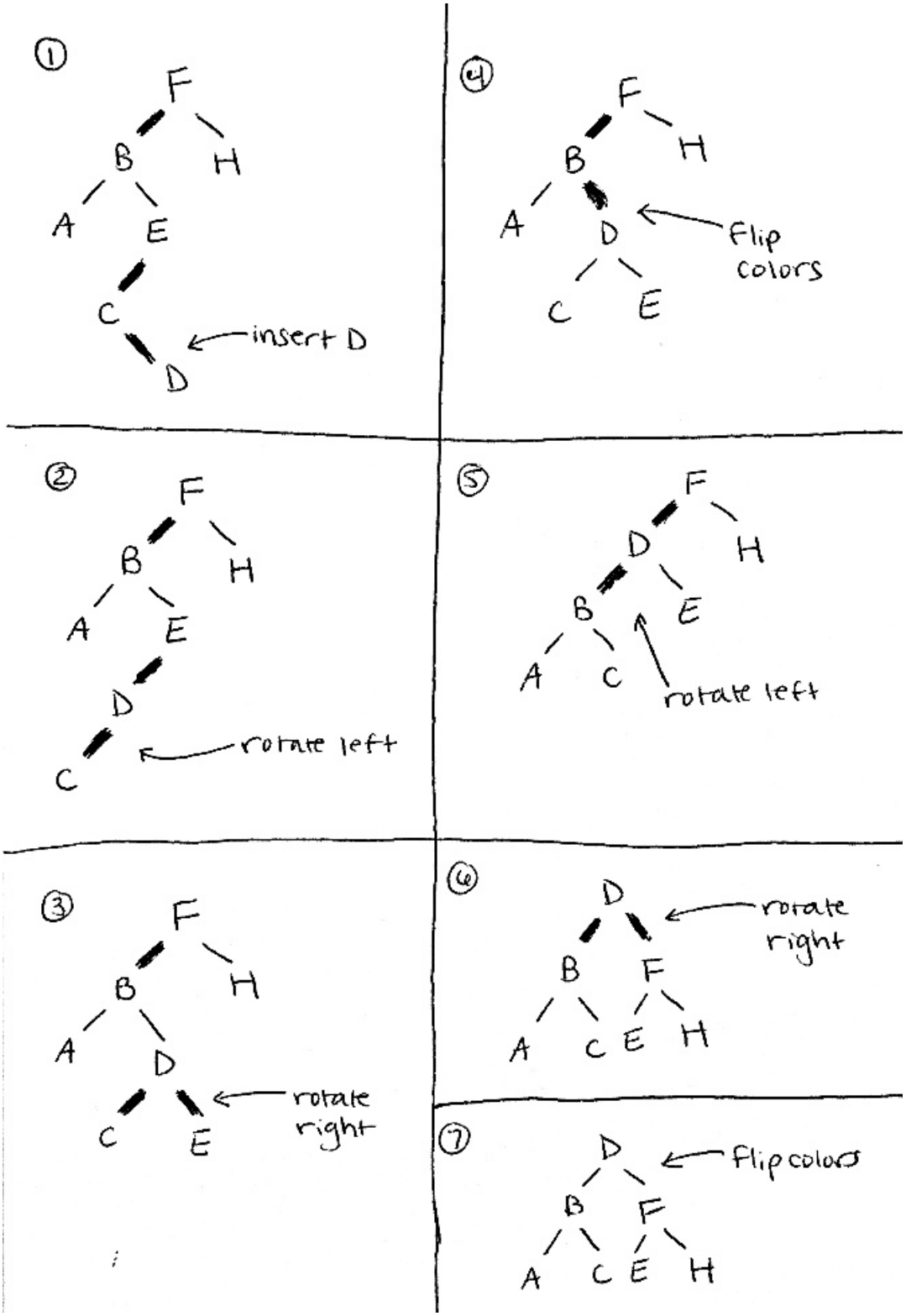
1. Quicksort Space
    a. This modified version of Quicksort can still use O(N) extra space in its worst case, because it uses recursion to sort the left subproblem. Since the array is randomly shuffled, it is possible that each pivot chosen is the greatest and next greatest numbers. This means that when partitioning, there would be no items greater than the pivot so the right subarray after the first partition would be of length zero, and the left subarray after the first partition would be of length N-1. Then, if the next pivot chosen is the greatest item in the left subarray of length N-1, the recursive right subarray would again be of length zero. If the pivot chosen continues to be the greatest number in the subarray, then iteration is never used and the recursive calls on the left subarray can still have depth up to N.

b.
```
private static void sort (Comparable[] a, int lo, int hi)
    while lo < hi
        int j = partition(a, lo, hi)
        int left = j
        int right = a.length – j
        if (left < right)
            sort(a, lo, j-1)
            lo = j + 1          //iterate on right
        else
            sort(a, j+1, hi)
            hi = j-1            //iterate on left
    end while
end sort
```

c. This revised version of Quicksort only needs O(lgN) extra space. This is because the recursion is only called on the smaller portion of the array. The size of this subarray is always less than or equal to half the size of the whole array (subarr.length <= arr.length/2). Therefore, at most, recursion will ever only be called on half the array, and Quicksort will only need O(lgN) extra space.

2. Cuckoo UF (see java code)
    a. java Cuckoo 10 1000 17
        i. E[M]/N = 0.858800
    b. java Cuckoo 1000 1000 17
        i. E[M]/N = 0.547906
    c. java Cuckoo 100000 1000 17
        i. E[M]/N = 0.508899

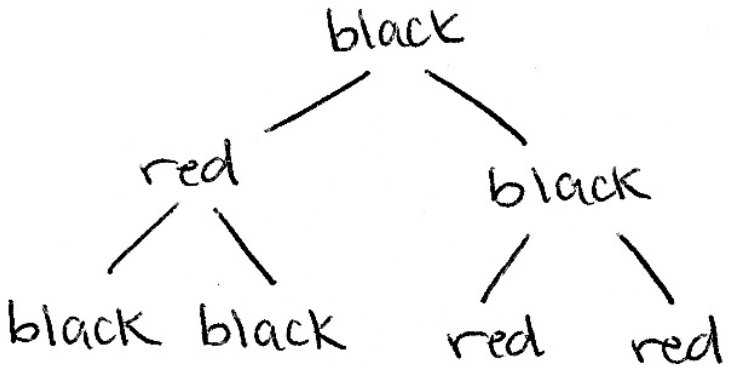
3. LLRB Tree insertion



4. Red-Black Tree

    a. In the Wikipedia article, a red-black tree stores the color in each node, rather than in the link. While the LLRB tree from class technically stores the color in each node, conceptually, we consider the color to be applied to the link. Additionally, a node can have a red child node on the left or on the right. Therefore, in a red-black tree, both children of a black node can be red, meaning that 4-nodes are valid in these trees.

    b.




    c. To insert into a red-black tree, a maximum of two rotations are needed as in Case 4 on Wikipedia.