

EXPERIMENT No: 5 LINEAR PHASE FIR SYSTEMS

AIM: To learn compulsory zero locations of types of linear phase system and classify system as linear phase FIR system.

SOFTWARE USED: Spyder Python 3.8.

THEORY:

Linear Phase is a property of a filter where the phase response of filter is linear function of frequency. The result is that all frequency components of the input signal are shifted in time by the same constant amount which is referred to as group delay.

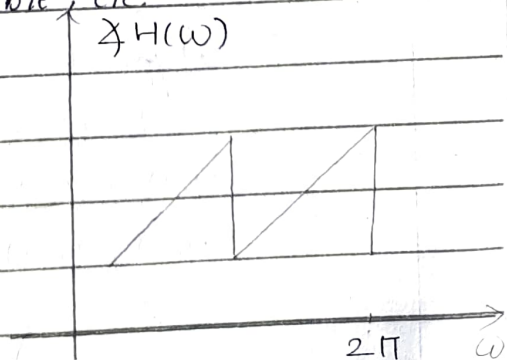
Phase Response of filter: In signal processing, phase response is the relationship between the phase of a sinusoidal input and the output signal passing through any device that accepts input and the output signal passing through any device that accepts input and produces an output signal.

Group Delay: Group delay is the delay time experienced by a signal's various frequency components when the signal passes through a system that is linear time-invariant (LTI) such as a microphone, coaxial cable, etc.

$$\text{grd}(\omega) = \frac{-d}{d\omega} \times H(\omega)$$

$$= \frac{-d}{d\omega} (-\omega\alpha)$$

$$\text{grd}(\omega) = \alpha = \frac{M}{2}$$



where $\frac{M}{2}$ is a constant

Necessary conditions for systems to have linear phase in terms of impulse response.

(1) The impulse response of the system should be symmetric
ie.
$$h(n) = h(M-n) \quad ; \quad 0 \leq n \leq M$$

$$0 \quad ; \quad \text{otherwise}$$

(2) The impulse response of the system should be antisymmetric
ie
$$h(n) = -h(M-n) \quad ; \quad 0 \leq n \leq M$$

$$= 0 \quad ; \quad \text{otherwise}$$

Types of Linear Phase FIR systems

- 1) Type 1:- $h(n)$ is symmetric and M is even.
No. zeros at on the unit circle at $z=1, z=-1$.
- 2) Type 2:- $h(n)$ is symmetric and M is odd.
Zero is present at $z=-1$.
- 3) Type 3:- $h(n)$ is antisymmetric and M is even.
Zero is present at $z=-1$ or $z=1$.
- 4) Type 4:- $h(n)$ is antisymmetric and M is odd.
zero is present at $z=1$.

Equation for Linear Phase FIR systems

For type 1 and type 2

$$H(z_0) = z^{-M} H(z_0^{-1}) \quad (1)$$

For type 1 system

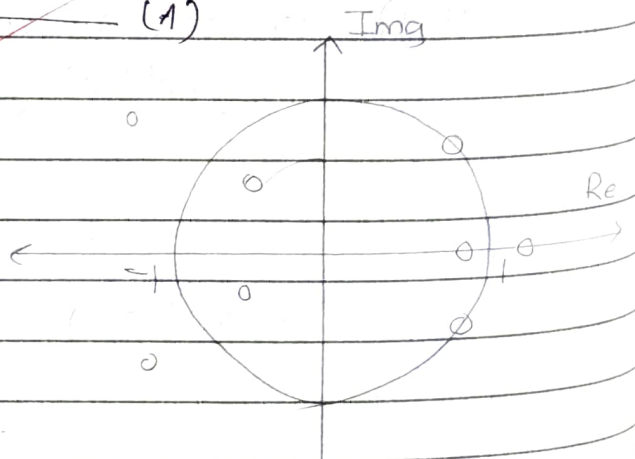
$z_0 = 1$ in equation A

$$H(1) = (1)^{-M} H(1^{-1})$$

$$H(1) = H(1)$$

Identity system

\therefore No 0 at $z=1$.



For Put $z_0 = -1$ in equation (A)

$$H(-1) = (-1)^{-M} H(-1)^{-1}$$

$$H(-1) = H(-1) \quad \text{--- NOT}$$

Identity system.

\therefore No zero at $z = -1$.

For type 2:

$$H(z_0) = z_0^{-M} H(z_0^{-1}) \quad \text{--- (B)}$$

put $z_0 = 1$.

$$H(1) = 1^{-M} H(1^{-1})$$

$$H(1) = H(1)$$

It is identity system.

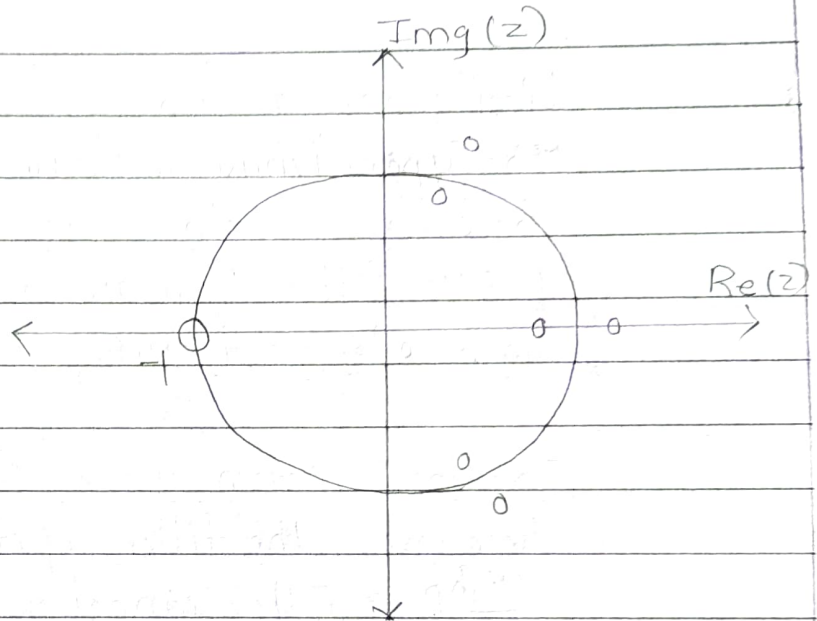
\therefore No zero at $z = 1$.

put $z_0 = -1$.

$$H(-1) = (-1)^{-M} H(-1)^{-1}$$

$$H(-1) = -H(-1)$$

Zero is present at $z = 1$.



For type 3:

$$H(z_0) = -z_0^{-M} H(z_0^{-1}) \quad \text{--- (C)}$$

put $z = 1$.

$$H(1) = -(1)^{-M} H(1^{-1})$$

$$H(1) = -H(1)$$

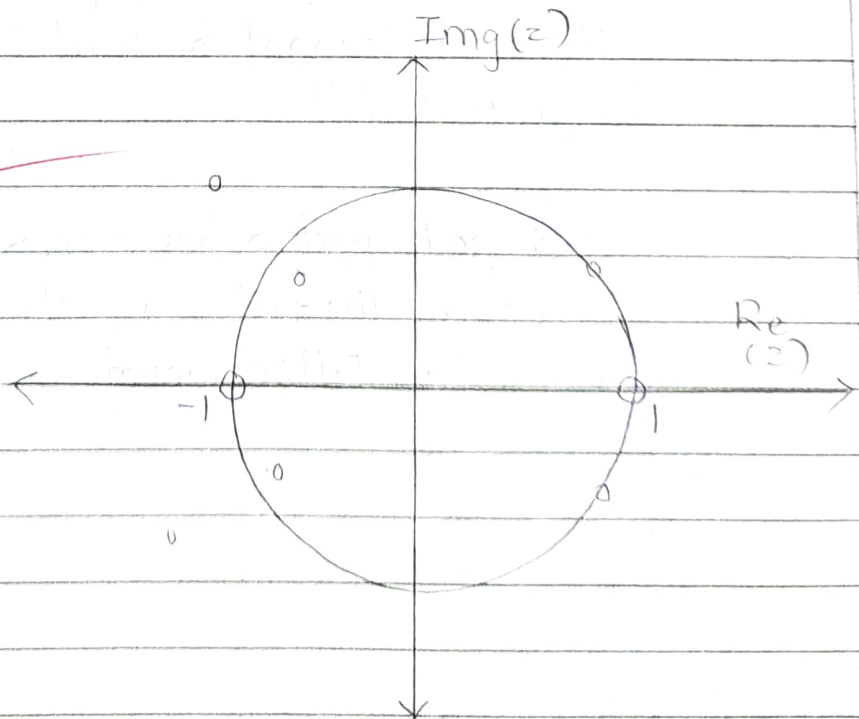
\therefore Zero is present at $z = 1$.

put $z = -1$

$$H(-1) = -(-1)^{-M} H(-1^{-1})$$

$$H(-1) = -H(-1)$$

Zero is present at $z = -1$.



For type 4:

$$H(z_0) = -z_0^{-M} H(z_0^{-1})$$

put $z = 1$ in c

$$H(1) = - (1)^{-M} H(1^{-1})$$

$$H(1) = -H(1)$$

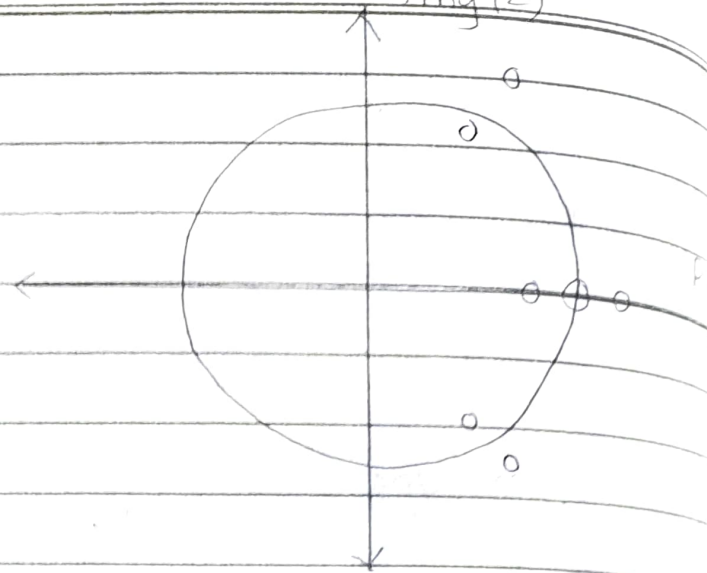
\therefore Zero is present at $z = 1$

put $z = -1$ in (c)

$$\begin{aligned} H(-1) &= -(-1)^{-M} H(-1^{-1}) \\ &= H(-1) \end{aligned}$$

Identity system.

\therefore No zero at $z = -1$.



Limitations of each type because of compulsory zero location. The property of zero in FIR system is to attenuate frequency. Therefore, as there are no zeros on the unit circle in type 1 FIR filter there are no limitations to it. Any type of filter can be designed using Type 1 system.

In type 2 system a zero is present at $z = -1$ i.e. $\omega = \pi$. Therefore, the filter rejects frequency at $\omega = \pi$. Hence, a ~~low~~ ^{High} Pass Filter cannot be designed using this type.

In type 3 system a zero is present at $z = 1, z = -1$ i.e. $\omega = 0, \omega = \pi$. Therefore, the filter rejects frequencies at $\omega = 0$ and $\omega = \pi$. Hence, it cannot be used to design a low pass filter as well as high pass filter.

In type 4 system has a zero at $z = 1$ i.e. $\omega = \pi$. Therefore, the filter rejects frequency at $\omega = \pi$. Hence, a low Pass Filter cannot be designed using this type.

STEPS OF PROGRAM :

- (1) To plot linear phase FIR system.
- (2) Import packages from numpy, scipy for importing signal and matplotlib.
- (3) Define function for plotting pole zero plot in z -plane
- (4) Define input sequence for type I, system impulse response and call function.
- (5) Plot Phase Response. for type II system and call the function.
- (6) Similarly follow the above steps for type III and type IV.

CONCLUSION :— In this experiment we have learned the classification of linear phase system based on location of zeros of system transfer function.

- (1) For a linear phase FIR system for every real zero at $z = x$ its inverse is at $z = 1/x$ is always present.
- (2) For every complex zero, a pair of complex conjugate zeros at reciprocal locations are present.
- (3) For type I, system, no zero is present at $z = 1$, $z = -1$.
For type II, system a compulsory zero is present at $z = -1$.
For type III system, compulsory zero is present at $z = 1$ and $z = -1$.

For type IV system, compulsory zero is present at $z = 1$.

- (4) Type I linear phase system is most suitable for the design of all types FIR filters, as there is no compulsory zero at $z = 1$ and $z = -1$.