AIM:- To learn IIR filter design using Bilinear Transformation.

SOFTWARE:- Spyder (Python 3.8)

THEORY:-

Steps for designing IIR filter using BLT.

step 1: Prewarp analog frequency using the equation.

$$\Omega = \frac{2}{T_d} \tan\left(\frac{\omega}{2}\right)$$

step 2: Find the order based on approp~~riate~~ximation.
for butterworth filter

$$N \geq \frac{\log \sqrt{\frac{10^{0.1\times s}-1}{10^{0.1\times P}-1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)} = \frac{\log\left(\frac{\lambda}{\varepsilon}\right)}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

step 3: To find:

3. Poles of Bullerworth filter.

step 4: Find Normalized Transfer function. H(s) for given order.

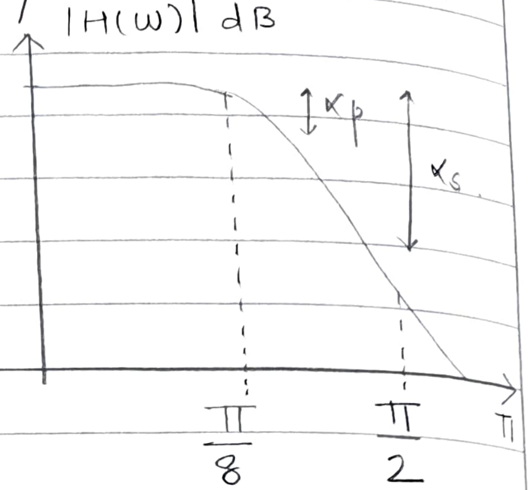
step 5: Analog to analog frequency transformation.

$$H(s) \Rightarrow H_a(s).$$

step 6: Compute digital filter Transfer Function

$$H_a(s)\Big|_{s \to \frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

Design an IIR filter with BLT and butterworth approximation using following specifications sampling rate - 8kHz
PB = 0 - 500 Hz   SB = 2 to 4 kHz   PassBand ripples - 3 dB.
SB-ripples $\geq$ 20 dB. Sampling frequency. - 8kHz.



$$\omega = \frac{2\pi f}{Fs}.$$

$$\omega_p = \frac{2\pi f_p}{Fs} = \frac{2\pi \times 500}{8000} = \frac{\pi}{8}$$

$$\omega_s = \frac{2\pi f_s}{Fs} = \frac{2\pi \times 2000}{8000} = \frac{\pi}{2}$$

step 1: Preworping the analog frequency.

$$\Omega_p' = \frac{2}{Td} \tan\left(\frac{\omega_p}{2}\right)$$

Let $Td = 2$

$$\tan\left(\frac{\pi/8}{2}\right) = \underline{0.198}$$

$$\Omega_s' = \frac{2}{Td} \tan\left(\frac{\omega_s}{2}\right)$$

$$= \tan\left(\frac{\pi}{4}\right)$$

$$\boxed{\Omega_s' = 1}$$

step 2: Find order N of Butterworth LPF

$$N \geq \frac{\log \sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log\left(\dfrac{\Omega_s'}{\Omega_p'}\right)} = \frac{\log \sqrt{\dfrac{10^{0.1(20)} - 1}{10^{0.1(3)} - 1}}}{\log\left(\dfrac{1}{0.198}\right)}$$

$N \geq 1.420$

$N = 2$.

step 3 :- $H(s) = \dfrac{1}{s^2 + \sqrt{2}s + 1}$

$$Ha(s) \Rightarrow H(s)\Big|_{s \to \frac{s}{\Omega c'}}$$

where $\Omega c' = \dfrac{\Omega p'}{\varepsilon^{1/N}} = \dfrac{\Omega p'}{\left(10^{0.1\alpha p} - 1\right)^{1/2N}}$

$$Ha(s) = \frac{1}{\left(\frac{s}{0.198}\right)^2 + \sqrt{2}\left(\frac{s}{0.198}\right) + 1}$$

$$= \frac{0.0392}{s^2 + \sqrt{2}(0.198)s + (0.198)^2}$$

$$Ha(s) = \frac{0.0392}{s^2 + 0.28s + 0.0392}$$

$$H(z) = Ha(s)\Big|_{s \to \frac{2}{Td}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)}$$

$Td = 2$ sec.

Put $s \to \left(\dfrac{1 - z^{-1}}{1 + z^{-1}}\right)$

$$H(z) = \frac{0.0392}{\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)^2 + 0.28\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) + 0.0392}$$

$$= \frac{0.0392(1 + z^{-1})^2}{(1 - z^{-1})^2 + 0.28(1 - z^{-1})(1 + z^{-1}) + 0.0392(1 + z^{-1})^2}$$

$$= \frac{0.0392(1 + 2z^{-1} + z^{-2})}{1.3272 - 1.92z^{-1} + 0.7135z^{-2}}$$

STEPS OF PROGRAM :

1. Import the required libraries.
2. Define all the filter specifications.
3. Convert the frequency into prewarped frequency.
4. Convert the ~~order~~ filter into Z-domain.
5. Print Numerator and denominator coefficients.
6. Plot the magnitude response of the filter.

CONCLUSION :

1. In this experiment, we have learned to design IIR filter using Bilinear transformation and Butterworth approximation.
2. The order N by program and analytically is same.