EXPERIMENT NO: 2 FREQUENCY

ANALYISIS OF DISCRETE TIME SIGNALS USING DFT.

AIM:— To plot magnitude and phase spectrum and perform frequency analysis of Discrete Time signals.

SOFTWARE:— Spyder 3: Python 3.8.

THEORY:— The frequency spectrum of an signal is the distribution of the amplitudes and phases of each frequency component against frequency.

A signal can be converted between the time and frequency domains with a pair of mathematical operators called as transform. An example is the Fourier transform, which decomposes a function into the sum of a (potentially ifinite) number of sine wave frequency components. The 'spectrum' of frequency components is the frequency domain representation of the signal. The inverse Fourier transform converts the frequency domain function back to time function.

Important frequency characteristic of a signal $x(t)$ with Fourier transform $X(\omega)$ are displayed by plots of the magnitude spectrum, $|X(\omega)|$ versus 'ω, and phase spectrum $\angle X(\omega)$ versus ω.

Definition:- The discrete-time Fourier transform (DTFT). $X(e^{j\omega})$ of sequence $x[n]$ is given by.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

- In general, $X(e^{j\omega})$ is a complex function of ω as follows.

$$X(e^{j\omega}) = X_{re}(e^{j\omega}) + j X_{im}(e^{j\omega}).$$

- $X_{re}(e^{j\omega})$ and $X_{im}(e^{j\omega})$ are, respectively, the real and imaginary parts of $X(e^{j\omega})$, and are real function of ω

- $X(e^{j\omega})$ can alternatively be expressed as.

$$X(e^{j\omega}) = |X(e^{j\omega})| e^{j\theta(\omega)}$$

where,

$$\theta(\omega) = \arg\{X(e^{j\omega})\}.$$

Here.

- $|X(e^{j\omega})|$ is called the magnitude function.
- $\theta(\omega)$ is called the phase function.
- In many applications, the DTFT is called the Fourier spectrum
- Likewise, $|X(e^{j\omega})|$ and $\theta(\omega)$ are called the magnitude and phase spectra.
- For a real sequence $x(n)$, $|X(e^{j\omega})|$ and $X_{re}(e^{j\omega})$ are even functions of ω, whereas $\theta(\omega)$ and $X_{im}(e^{j\omega})$ are odd function of ω.
- Note: $X(e^{j\omega}) = |X(e^{j\omega})| e^{j(\theta(\omega)+2\pi k)} = |X(e^{j\omega})| e^{j\theta(\omega)}$

  The phase function $\theta(\omega)$ cannot be uniquely specified for any DTFT.

DFT is not suitable for DSP applications because.

- In DSP, we are able to compute the spectrum only at specific discrete values of ω.
- Any signal in any DSP application can be measured only in a finite number of points.
- Therefore, DFT is suitable tool for frequency analysis, using finite number of samples of frequency spectrum

obtained from DTFT.

$$X(k) = X(k\Delta\omega), \quad \Delta\omega = \frac{2\pi}{N} \Rightarrow$$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N} \quad \text{DFT}$$

Representing the signal into its "N" number of frequency components is N points DFT or analysis and reconstruction of input sequence from the knowledge of frequency component is called IDFT or synthesis.

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi \frac{kn}{N}} \quad \text{analysis.}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j2\pi \frac{kn}{N}} \quad \text{Synthesis.}$$

STEPS OF PROGRAM:—

step 1: Declare an input sequence and calculate its length using len () function.

step 2: Declare a varialbe for user to input ~~a~~ no. of DFT points Required.

step 3: Calculate number of trailing zeros and them to the sequence using zeropadding.

step 4: Compute N-point DFT using fft () function.

step 5: Compute N point IDFT using ifft() function and round up the values to 1 decimal point using np. round() function.

step 6: import matplotlib library

step 7: To plot magnitude and phase response, declare two variables for discrete time range (n) and discrete frequency index (k).

step 8: Plot input sequence $x_1(n)$, magintude plot using np.abs() and phase plot using np.angle(). seperate the plots using subplot function. Display it using plot.show().

step 9: Assign X and Y label to it and assign the title.

PRACTICAL PROBLEM:—

$x(n) = [4, 2, 1; 3]$

$X(k) = W_N \cdot x_N$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 4+2+1+3 \\ 4-2j-1+3j \\ 4-2+-3+1 \\ 4+2j-1-3j \end{bmatrix} = \begin{bmatrix} 10 \\ 3+j \\ 0 \\ 3-j \end{bmatrix}$$

$X(k) = [10, 3+j, 0, 3-j]$

$x_N = \frac{1}{N} W_N^* \cdot X(k)$

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 10 \\ 3+j \\ 0 \\ 3-j \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 10+3+j+3-j \\ 10-3j+3j-1\cancel{+}1 \\ 10-3+j-3+j \\ 10-3j+1+3j+1 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 16 \\ 8 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 1 \\ 3 \end{bmatrix}$$

$x(n) = [4, 2, 1, 3]$

CONCLUSION :—

(1) Through this experiment, we have learned to plot magnitude and phase spectrum of discrete time sequences.

(2) For a discrete time sequence of L point, idelly N=L point DFT is required to compute.

(3) For number of DFT points N>L, the frequency spectrum (magnitude and phase) become denser and better represented in frequency domain.

(4) Resulting IDFT for N>L is having (N-L) trailing zeros.