EXPERIMENT NO.: 6 FILTER DESIGN USING POLE-ZERO PLACEMENT METHOD.

AIM:— To design simple FIR/IIR filters using pole-zero placement method.

SOFTWARE:— Spyder Python 3.8.

THEORY:—

In signal processing, a digital filter is a system that performs mathematical operations on a sampled, discrete-time signal to reduce or enhance certain aspects of that signal. The primary types of digital filters are Low Pass Filter, High Pass Filter, Band Pass Filter, Band Stop Filter and notch Filter.

Frequency Response of Filters:

The frequency response of a system is the quantitative measure of the magnitude and phase of the output as a function of input frequency. The frequency domain characteristic of a filter can be analysed by obtaining the transfer function of the filter.
The Frequency Response is divided into two parts. Magnitude and Phase Response.
The equation of Magnitude Response is

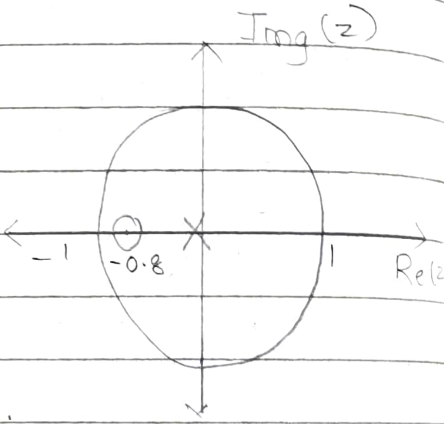
$$|H(\omega)| = \sqrt{(X_{RE})^2(\omega) + X_{IMG}^2(\omega)}$$

Properties of poles and zeros to decide filter characteristic.

1. For FIR filters, all poles are present at the origin.
2. The frequency locations where the poles are present, emphasize the frequency.
3. The frequency location where zeros are present are, attenuated and zeros present on the unit circle reject the frequency completely.

Expt : 6A.

(1) FIR LPF



$$H(z) = \frac{z + 0.8}{z}$$

$$= 1 + 0.8z^{-1}$$

$$H(z) = \sum_{n=0}^{N} h(n) z^{-n}$$

$$= h(0)z^{0} + h(1)z^{-1}$$

$$h(n) = [1, 0.8]$$

$$\cancel{H(\omega)} = H(z) = 1 + 0.8z^{-1}$$

$$= 1 + 0.8\,e^{-j\omega}$$

$$= 1 + 0.8(\cos\omega - j\sin\omega)$$

$$= 1 + 0.8\cos\omega - j\,0.8\sin\omega.$$

$$|H(\omega)| = \sqrt{H_R^2(\omega) + H_I^2(\omega)}$$

$$= \sqrt{(1+0.8\cos\omega)^2 + (-0.8\sin\omega)^2}$$

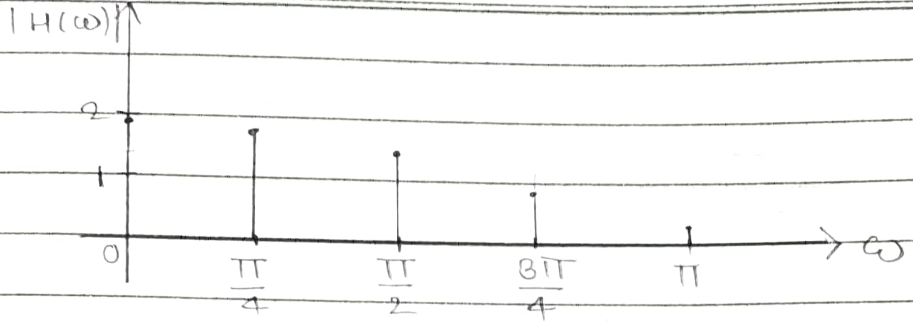
$$= \sqrt{\cancel{1 + 16 +} 1.6\cos\omega + 0.64\cos^2\omega + 0.64\sin^2\omega}$$

$$= \sqrt{1 + 1.6\cos\omega + 0.64}.$$

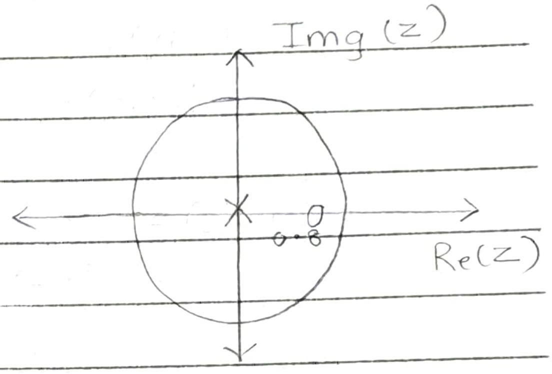
$$|H(\omega)| = \sqrt{1.64 + 1.6\cos\omega}$$

Magnitude response :

ω =	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$\lvert H(\omega)\rvert$ =	1.8	1.66	1.28	0.71	0.2



(2) <u>FIR HPF</u>



$$H(z) = \frac{z-0.8}{z}$$

$$= \frac{1-0.8z^{-1}}{1}$$

$$= \sum_{n=0}^{N} h(n) z^{-n}.$$

$$= h(0)z^{0} + h(1)z^{-1}$$

$$= 1z^{0} + 0.8z^{-1} \qquad = 1-(-0.8)z^{-1}$$

$$h(n) = [1, -0.8]$$

$$H(z) = 1-0.8z^{-1}$$

$$= 1-0.8e^{-j\omega}.$$

$$= 1-0.8\cos\omega + j\,0.8\sin\omega.$$

$$|H(\omega)| = \sqrt{H_R^2(\omega) + H_I^2(\omega)}.$$

$$= \sqrt{(1-0.8\cos\omega)^2 + (0.8\sin\omega)^2}$$

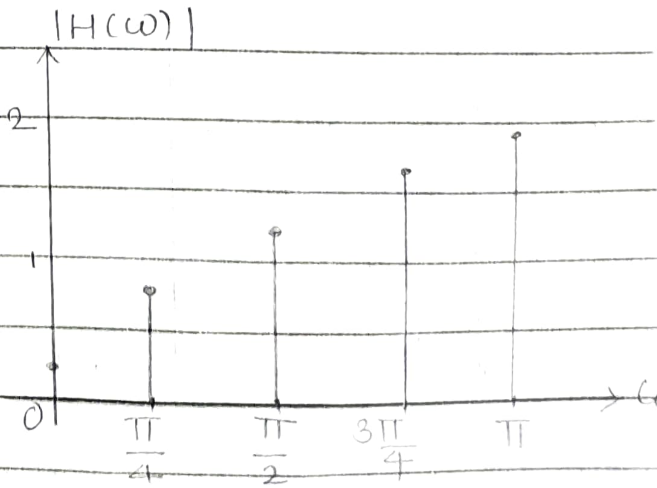
$$= \sqrt{1-1.6\cos\omega + 0.64\cos^2\omega + 0.64\sin^2\omega}$$

$$= \sqrt{1-1.6\cos\omega + 0.64}$$

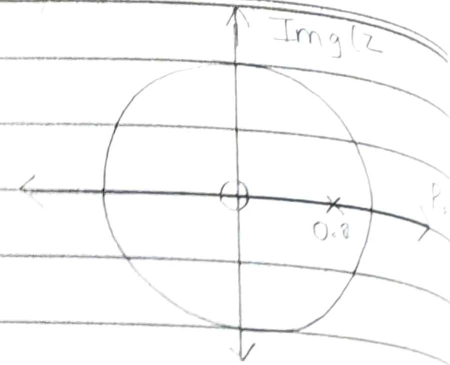
$$\therefore |H(\omega)| = \sqrt{1.64 - 1.6\cos\omega}.$$

Magnitude Response:

ω	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$\lvert H(\omega)\rvert$	0.2	0.71	1.28	1.66	1.8



(3) IIR LPF :—



$$H(z) = \frac{z}{z-0.8}$$

$$= \frac{1}{1-0.8z^{-1}}$$

$$\therefore h(n) = (0.8)^n u(n).$$

$$h(n) = 0.8^n \quad ; n \geq 0$$
$$= 0 \quad ; n < 0$$

$$H(z) = \frac{1}{1-0.8z^{-1}} = \frac{1}{1-0.8e^{-j\omega}}$$

$$= \frac{1}{1-0.8\cos\omega + j0.8\cancel{\cos}\sin\omega}$$

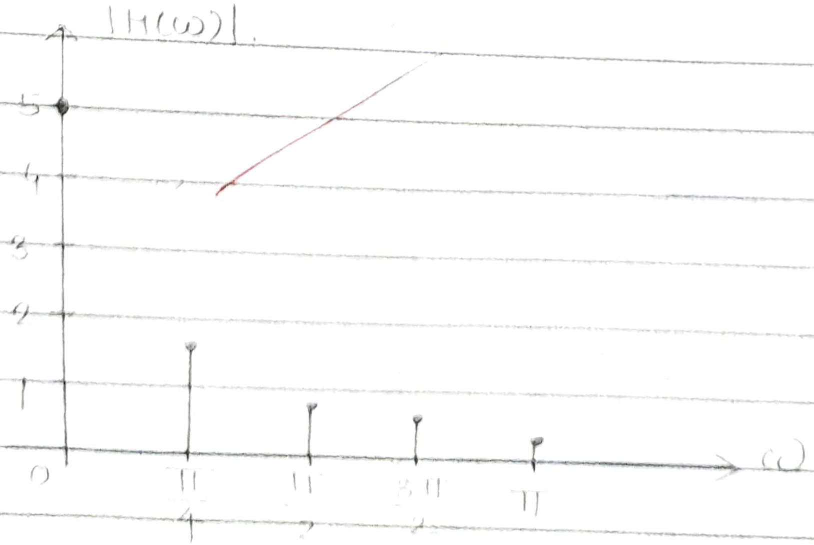
$$\cancel{A}\,|H(\omega)| = \frac{1}{\sqrt{(1-0.8\cos\omega)^2 + (0.8\sin\omega)^2}}$$

$$= \frac{1}{\sqrt{1-1.6\cos\omega + 0.64\cos^2\omega + 0.64\sin^2\omega}}$$

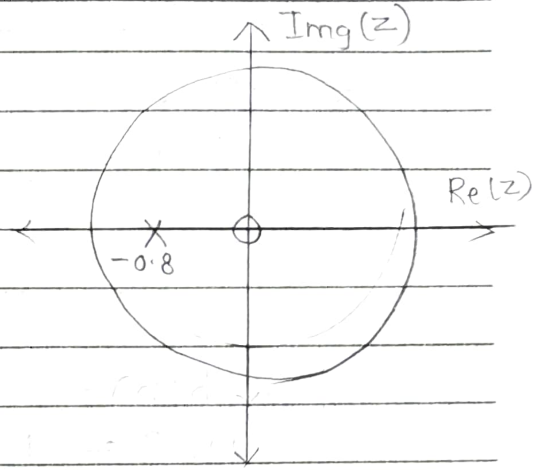
$$|H(\omega)| = \frac{1}{\sqrt{1.64 - 1.6\cos\omega}}$$

Magnitude Response:

ω	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$\lvert H(\omega)\rvert$	5.	1.4	0.7	0.6	0.5



(4) IIR HPF:



$$H(z) = \frac{z}{z+0.8}$$

$$= \frac{1}{1+0.8z^{-1}}$$

$$h(n) = (-0.8)^n u(n).$$

$$H(z) = \frac{1}{1+0.8z^{-1}}$$

$$= \frac{1}{1+0.8e^{-j\omega}} = \frac{1}{1+0.8\cos\omega - j0.8\sin\omega.}$$

$$|H(\omega)| = \frac{1}{\sqrt{(1+0.8\cos\omega)^2+(0.8\sin\omega)^2}}$$

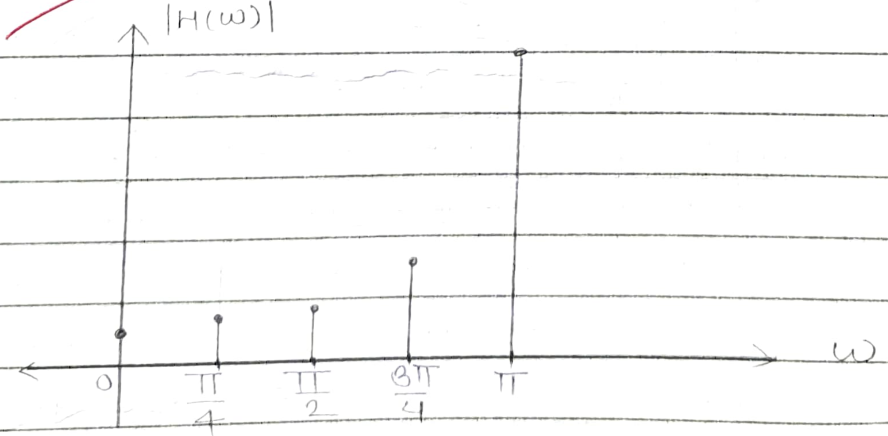
$$= \frac{1}{\sqrt{1+1.6\cos\omega+0.64\cos^2\omega+0.64\sin^2\omega}}$$

$$= \frac{1}{\sqrt{1+1.6\cos\omega+0.64}}.$$

$$|H(\omega)| = \frac{1}{\sqrt{1.64+1.6\cos\omega}}$$

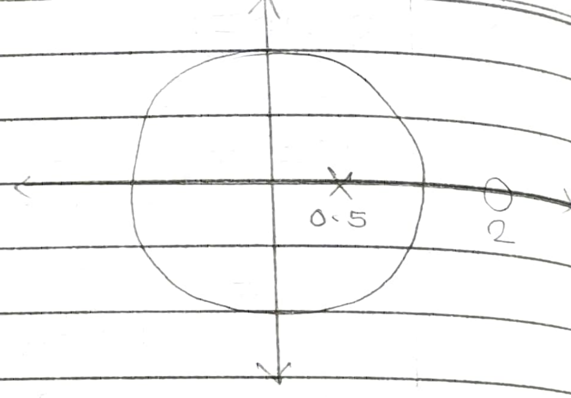
Magnitude response.

ω	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$\|H(\omega)\|$	0.5	0.6	0.7	1.4	5



5. **All pass filter :**

$$H(z) = \frac{z-2}{z-0.5}$$



$$= \frac{1-2z^{-1}}{1-0.5z^{-1}}$$

$h(n) =$

$$H(z) = \frac{1-2z^{-1}}{1-0.5z^{-1}} = \frac{1-2e^{-j\omega}}{1-0.5e^{-j\omega}} = \frac{1-2\cos\omega + j2\sin\omega}{1-0.5\cos\omega + j\,0.5\sin\omega}$$

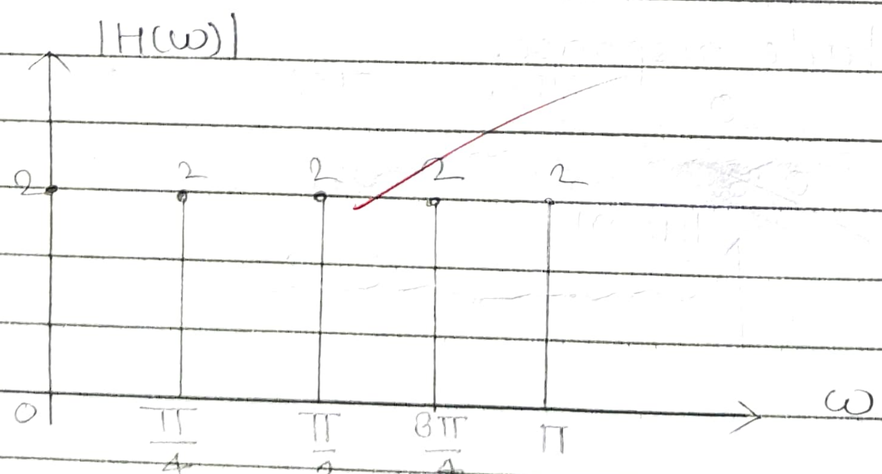
$$|H(\omega)| = \frac{\sqrt{(1-2\cos\omega)^2 + (2\sin\omega)^2}}{\sqrt{(1-0.5\cos\omega)^2 + (0.5\sin\omega)^2}}$$

$$= \frac{\sqrt{1-4\cos\omega + 4\cos^2\omega + 4\sin^2\omega}}{\sqrt{1-\cos\omega + 0.25\cos^2\omega + 0.25\sin^2\omega}}$$

$$|H(\omega)| = \frac{\sqrt{1-4\cos\omega+4}}{\sqrt{1-\cos\omega+0.25}} = \frac{\sqrt{5-4\cos\omega}}{\sqrt{\cancel{0.75}\ 1.25-\cos\omega}}$$

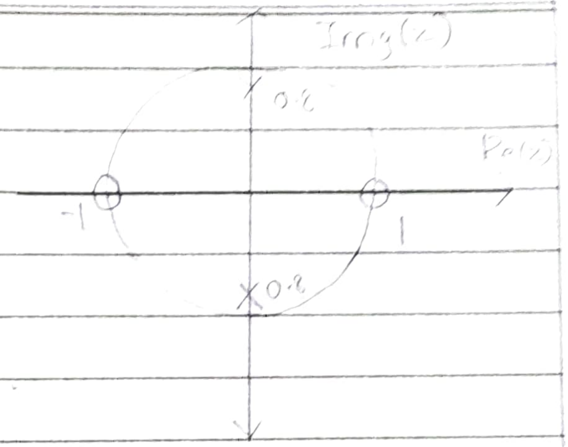
Magnitude Response.

ω	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$\lvert H(\omega)\rvert$	2	2	2	2	2



EXP 6B :

1. Band Pass Filter :-



$$H(z) = \frac{(z-1)(z+1)}{(z-0.8e^{j\pi/2})(z-0.8e^{-j\pi/2})}$$

$$= \frac{(z-1)(z+1)}{z^2 - 2.6\cos\frac{\pi}{2} z + 0.64}$$

$$= \frac{z^2-1}{z^2 - 1.6\cos \pi/2 \, z + 0.64}$$

$$= \frac{z^2-1}{z^2+0.64} = \frac{1-z^{-2}}{1+0.64z^{-2}}$$

$$\cancel{h(n)} \quad H(z) = \frac{1-e^{-j\omega}}{1+0.64e^{-2j\omega}}$$

$$H(z) = \frac{1-\cos 2\omega + j\sin 2\omega}{1+0.64\cos 2\omega - j0.64\sin 2\omega}$$

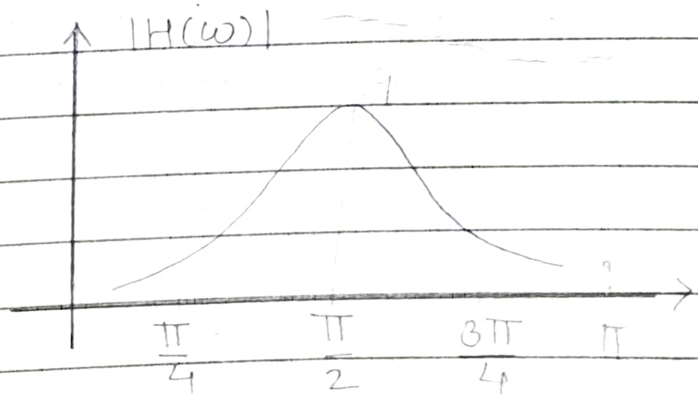
$$|H(\omega)| = \frac{\sqrt{(1-\cos 2\omega)^2 + (\sin 2\omega)^2}}{\sqrt{(1+0.64\cos 2\omega) + (0.64\sin 2\omega)}}$$

$$= \frac{\sqrt{1-2\cos 2\omega + \cos 2^2\omega + \sin^2 \omega}}{\sqrt{1+1.28\cos 2\omega + 0.4096\cos 2^{\omega} + 0.4096\sin 2^2\omega}}$$

$$= \frac{\sqrt{1-2\cos 2\omega + 1}}{\sqrt{1+1.28\cos 2\omega + 0.4096}} = \frac{\sqrt{2\cos 2\omega}}{\sqrt{1.4096 + 1.28\cos 2\omega}}$$

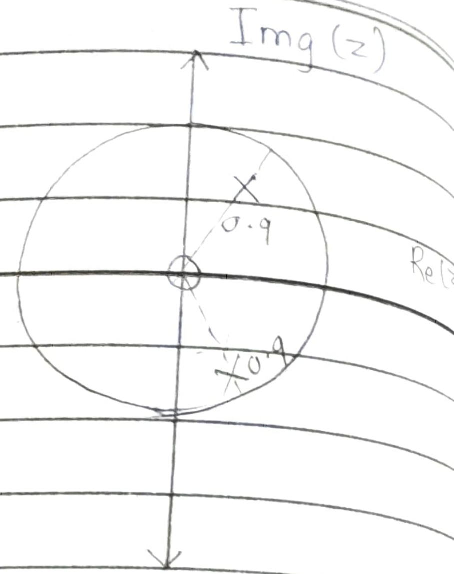
Magnitude Response:

ω	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$\lvert H(\omega)\rvert$	0.86	0	4.	0	0.86



2. Notch Filter :

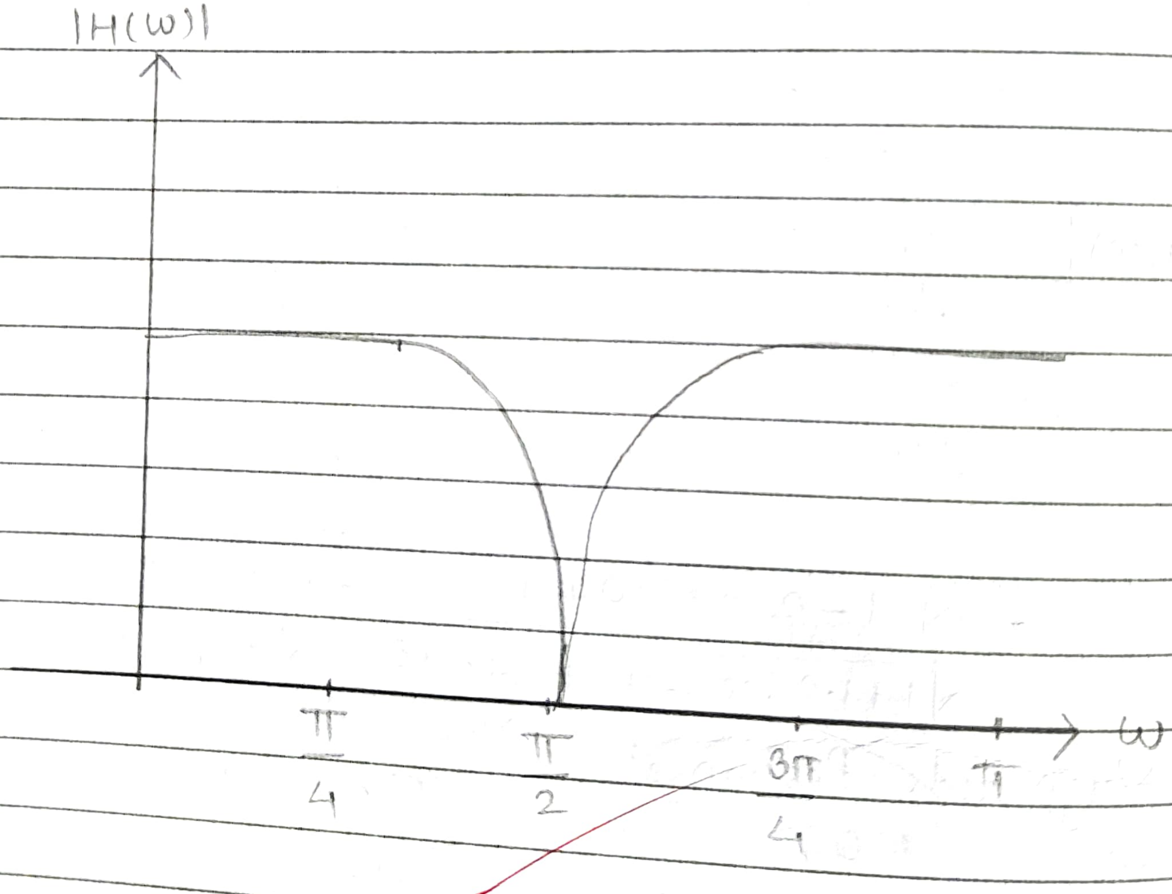
$$H(z) = \frac{z}{(z-0.9e^{j\pi/4})(z-0.9e^{-j\pi/4})}$$



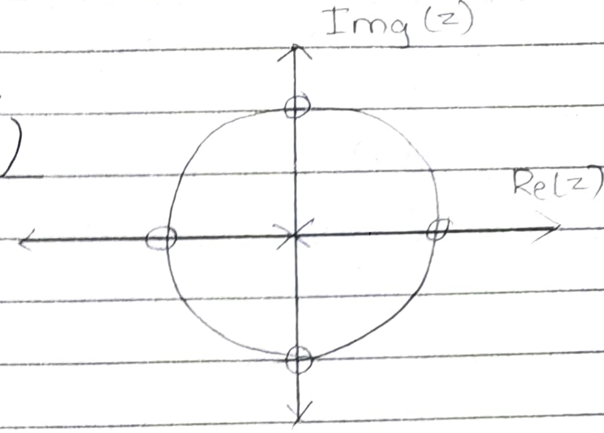
$$= \frac{z}{z^2 - 1.8z\cos\frac{\pi}{4} + 0.81}$$

$$= \frac{z}{z^2 - 1.27z + 0.81}$$

=



3. ## Comb filter:



$$H(z) = \frac{(z-1)(z+1)(z-e^{j\frac{\pi}{2}})(z-e^{-j\frac{\pi}{2}})}{z^4}$$

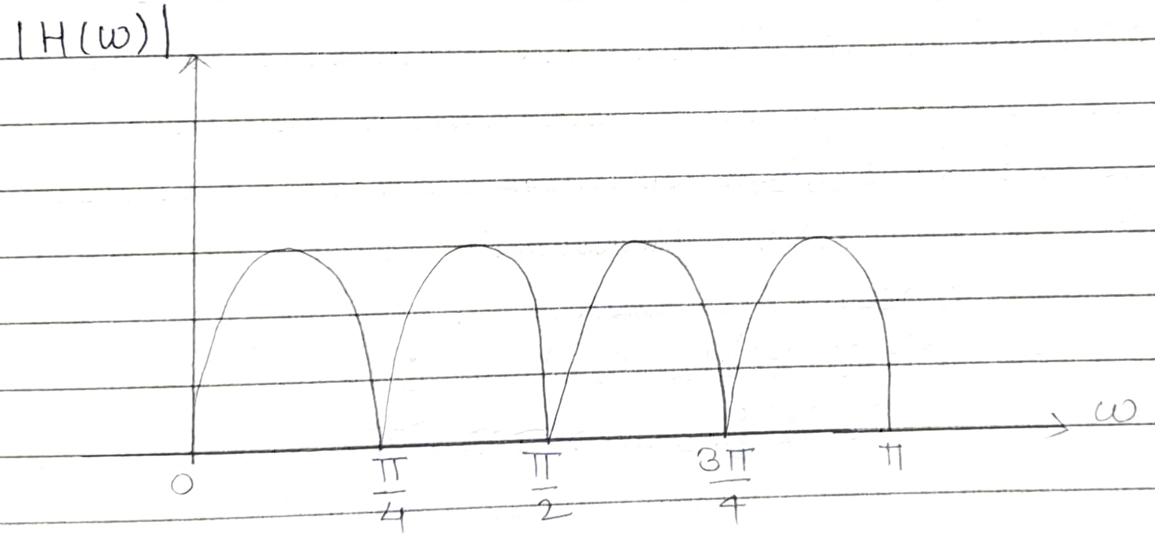
$$= \frac{(z^2-1)(z^2+1)}{z^4}$$

$$H(z) = \frac{z^4-1}{z^4}$$

for a comb filter

$$H(z) = 1 - z^{-N} = 1 - \left(\frac{1}{z}\right)^N = \frac{z^N - 1}{z^N}$$

for $N = 4$



STEPS OF PROGRAM:

step1: Import required libraries.
step2: define function for pole zero diagram in z plane.
step3: set figure parameters.
step4: Define ~~equation~~ of various filters, plot pole zero diagram by calling its function and plot magnitude response.
~~step 5:~~

CONCLUSION :

(1) In this experiment, we learned the design of simple FIR and IIR filters using pole zero placement method.

(2) Pole enhances the frequencies (provide gain) to frequencies at which it is placed in z-plane.

(3) Zero deemphasizes or attenuates the frequencies at which it is placed in z-plane.

(4) We have also understood the classification of filter as FIR or IIR based on location of poles.