Experiment No. 2 USE OF TRANSMISSION LINE AS A CIRCUIT ELEMENT

Aim:

To understand and verify the use of transmission line as a circuit element.

Software requirements:

Software- QUCS (Quite Universal Circuit Simulator)

Theory:

The input impedance of a transmission line is the impedance offered by it at the input terminals. As the source is connected at the input terminals, this quantity has some special significance while selecting the source. During computations, input impedance is quite useful parameter to find the power flowing into the line when a generator is connected to it. To push maximum power over to the line, the source impedance and input impedance of line must have a complex conjugate relationship.

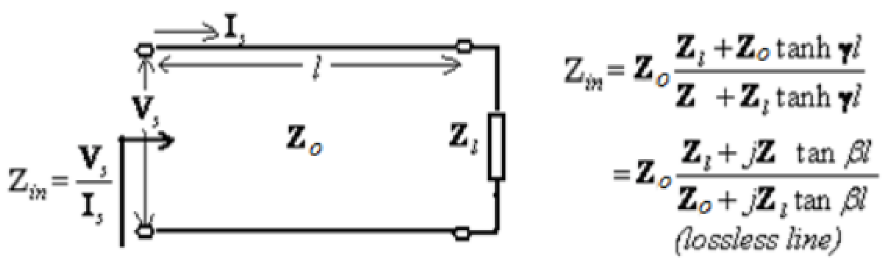


Figure Input impedance of a line terminated over an impedance.

Formally, the input impedance of a line can be defined as the ratio of complex phasor voltage to complex phasor current at its input terminals. Mathematically,

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_{in}}{\mathbf{I}_{in}} = \frac{\text{Input voltage}}{\text{Input current}}$$

It is complex quantity, value being dependent upon the configuration, length and termination of the line.

For a general (lossy) line:

$$\mathbf{Z}_{in} = \mathbf{Z}_o \frac{\mathbf{Z}_l + \mathbf{Z}_o \tanh \gamma l}{\mathbf{Z}_o + \mathbf{Z}_l \tanh \gamma l}$$

For an ideal (lossless) line:

$$\mathbf{Z}_{in} = \mathbf{Z}_o \frac{\mathbf{Z}_l + j\mathbf{Z}_o \tan\beta l}{\mathbf{Z}_o + j\mathbf{Z}_l \tan\beta l}$$

So, for a short-circuited transmission line, an input impedance is given by –

$$\mathbf{Z}_{in}\Big|_{\mathbf{Z}_l=0} = \mathbf{Z}_{sc} = \mathbf{Z}_o \frac{\mathbf{Z}_l + \mathbf{Z}_o \tanh\gamma l}{\mathbf{Z}_o + \mathbf{Z}_l \tanh\gamma l}\Bigg|_{\mathbf{Z}_l=0} = \mathbf{Z}_o \tanh\gamma l$$

With open circuit termination, its input impedance becomes,

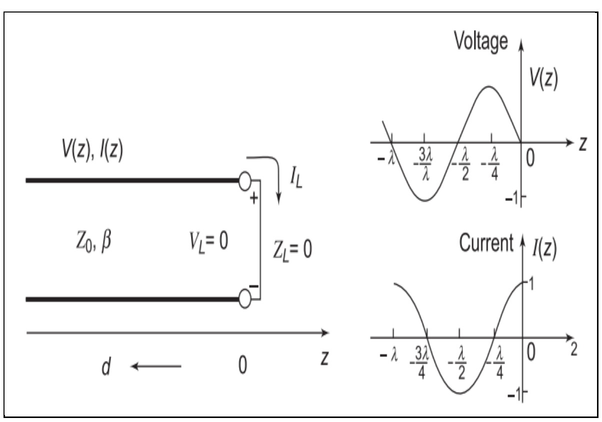
$$\mathbf{Z}_{in}\Big|_{\mathbf{Z}_l\to\infty} = \mathbf{Z}_{oc} = \mathbf{Z}_o \frac{\mathbf{Z}_l + \mathbf{Z}_o \tanh\gamma l}{\mathbf{Z}_o + \mathbf{Z}_l \tanh\gamma l}\Bigg|_{\mathbf{Z}_l\to\infty} = \mathbf{Z}_o \coth\gamma l$$

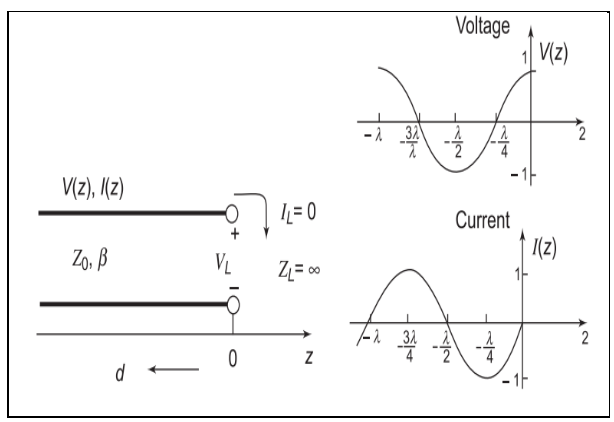
In case of loss-less line, $\gamma = j\beta$ resulting in $Z_{sc} = jZ_o\tan(\beta l)$ and $Z_{oc} = -jZ_o\cot(\beta l)$. So, overall equations for an input impedance and input admittance are as summarized below:

Line input Immitance

- **Input impedance**
  - General line $\boldsymbol{\gamma} = \alpha + j\beta$:
    $$\mathbf{Z}_{in} = \mathbf{Z}_o \frac{\mathbf{Z}_l + \mathbf{Z}_o \tanh\gamma l}{\mathbf{Z}_o + \mathbf{Z}_l \tanh\gamma l} \qquad \mathbf{z}_{in} = \frac{\mathbf{z}_l + \tanh\gamma l}{1 + \mathbf{z}_l \tanh\gamma l}$$
  - Loss-less line $\boldsymbol{\gamma} = j\beta$:
    $$\mathbf{Z}_{in} = \mathbf{Z}_o \frac{\mathbf{Z}_l + j\mathbf{Z}_o \tan\beta l}{\mathbf{Z}_o + j\mathbf{Z}_l \tan\beta l} \qquad \mathbf{z}_{in} = \frac{\mathbf{z}_l + j\tan\beta l}{1 + j\mathbf{z}_l \tan\beta l}$$
- **Input admittance**
  - General line $\boldsymbol{\gamma} = \alpha + j\beta$:
    $$\mathbf{Y}_{in} = \mathbf{Y}_o \frac{\mathbf{Y}_l + \mathbf{Y}_o \tanh\gamma l}{\mathbf{Y}_o + \mathbf{Y}_l \tanh\gamma l} \qquad \mathbf{y}_{in} = \frac{\mathbf{y}_l + \tanh\gamma l}{1 + \mathbf{y}_l \tanh\gamma l}$$
  - Loss-less line $\boldsymbol{\gamma} = j\beta$:
    $$\mathbf{Y}_{in} = \mathbf{Y}_o \frac{\mathbf{Y}_l + j\mathbf{Y}_o \tan\beta l}{\mathbf{Y}_o + j\mathbf{Y}_l \tan\beta l} \qquad \mathbf{y}_{in} = \frac{\mathbf{y}_l + j\tan\beta l}{1 + j\mathbf{y}_l \tan\beta l}$$

The voltage and current waveforms for short-circuited and open-circuited transmission lines are as given below:





In case of short circuit termination, it is a current anti-node and a voltage node that exists right over the load. In this case, as the total voltage is required to be zero over the load, the voltage must get reflected with 180^0 phase shift whereas the current need not under go any phase shift. It results in voltage node and current anti-node over the short circuit termination.

When the termination is open circuit, the current gets reflected with 180^0 phase shift, since the total current has to be zero on an open circuit and the reflected current has to cancel the incident current which can happen only if they are out of phase. However, the voltage gets reflected without any phase shift, as the direction of travel of the wave and phase of current being reversed, reflected voltage cannot have a phase shift. Thus, it is a current node and consequently, a voltage anti-node that exists right over the open circuit load.

So, an input impedance of short-circuited and open-circuited transmission lines is purely reactive in nature. Hence, these can behave like an inductor, capacitor and also as a resonant circuit as summarized below:

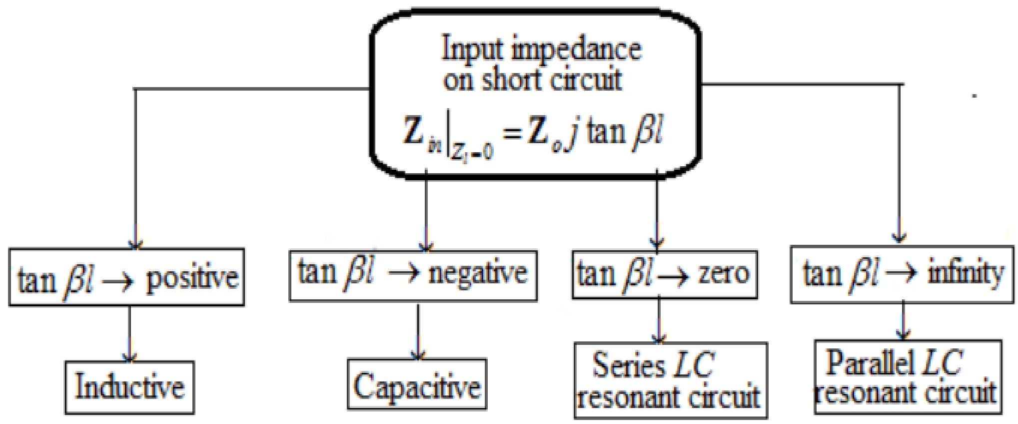


Fig.: Use of Short-circuit T.L. as a Circuit Element

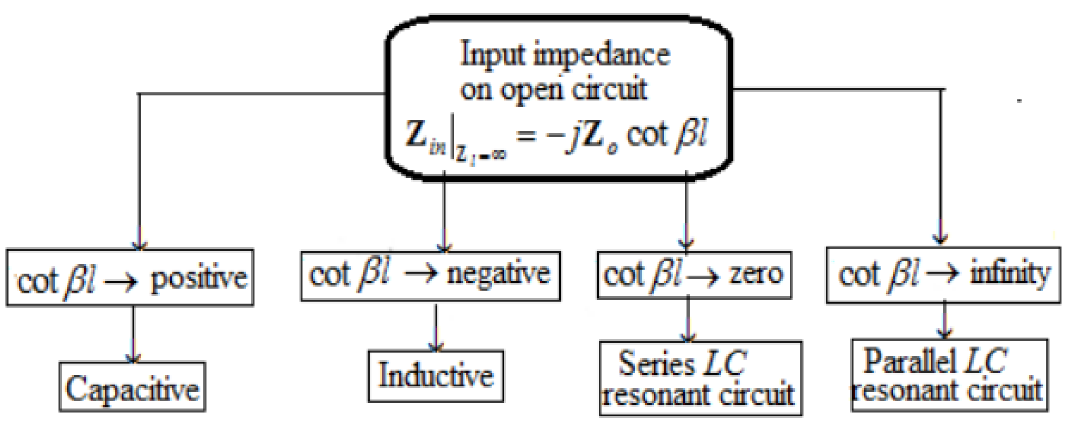


Fig.: Use of Open-circuit T.L. as a Circuit Element

The variation of an input impedance of a lossless line with line length is given below:

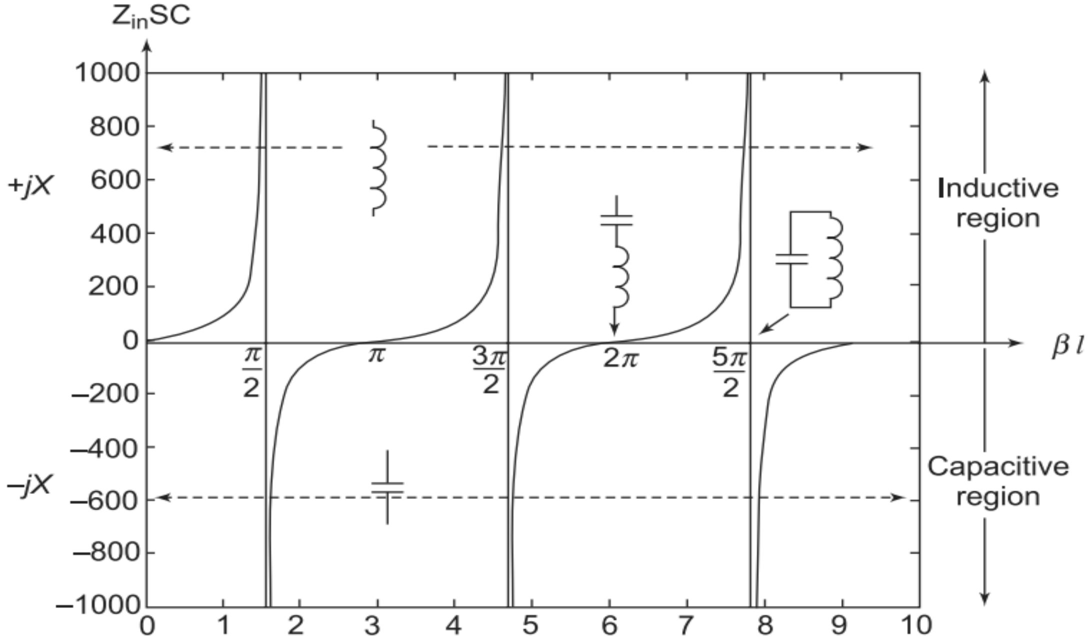


Fig.: Variation of input impedance of lossless short-circuited line with the line length

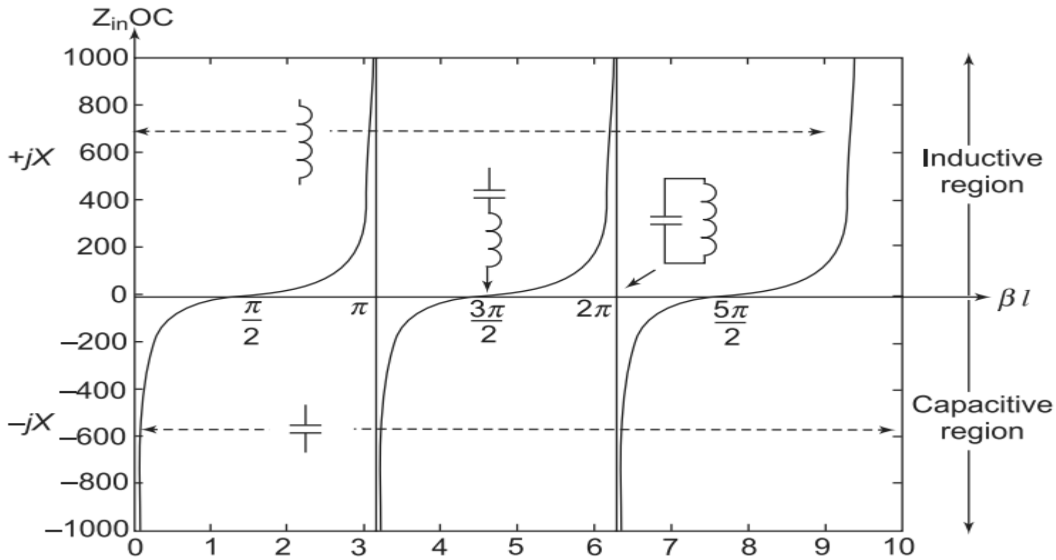
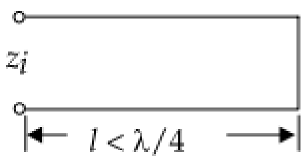
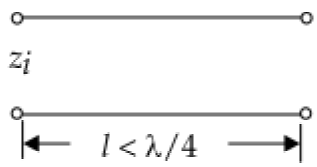
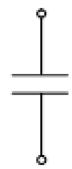
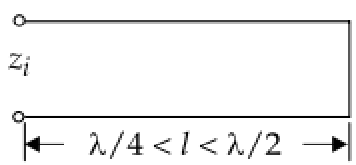
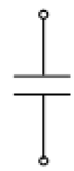
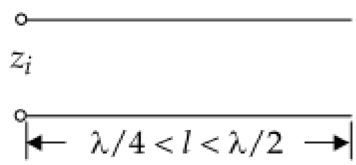
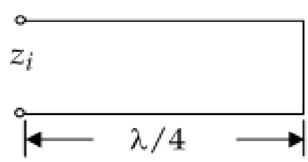
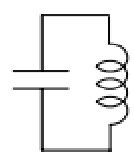
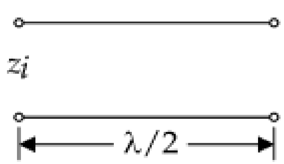
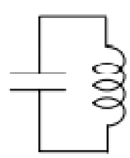


Fig.: Variation of input impedance of lossless open-circuited line with the line length

lines of different length with short and open ends along with their equivalent circuits are as given below:

Transmission line	*Equivalent circuit*	*Input impedance*
	$\equiv$ 	$z_i = +jz_0 \tan \beta l$
	$\equiv$ 	$z_i = -jz_0 \cot \beta l$
	$\equiv$ 	$z_i = +jz_0 \tan \beta l$
	$\equiv$ 	$z_i = -jz_0 \cot \beta l$
	$\equiv$ 	$z_i \approx \frac{2z_0^2}{Rl} = \frac{z_0^2}{\tanh \alpha l}$
	$\equiv$ 	$z_i \approx \frac{2z_0^2}{Rl} = \frac{z_0^2}{\tanh \alpha l}$

Conclusion:

After performing this experiment, we can conclude that by selecting a terminated line of suitable length, it is possible to produce the equivalent of a pure inductance and capacitance or any desired combination thereof.