Performance Characterization of Rectangular Waveguide for TE and TM Modes

Aim:

To analyze the performance of a rectangular waveguide for TE and TM modes over a specified frequency range in terms of wave impedance, phase velocity and group velocity.

Software: Scilab

Theory:

A rectangular waveguide has dimensions $a \times b$, where a is the larger dimension and b is the smaller dimension. In this experiment, a = 6 cm and b = 4 cm. The waveguide supports the operation of transverse electric (TE) and transverse magnetic (TM) modes.

TE and TM Modes:

The TE mode is also known as the H mode since it is associated with a non-zero value of

the axial magnetic field: Hz ≠ 0; Ez = 0.

The TM mode is also known as the E mode since it is associated with a non-zero value of

the axial electric field: Ez ≠ 0; Hz = 0.

Cut-off Frequency:

The cutoff frequency for TE and TM modes is given by:

$$f_{c_{mn}} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

where:
- m and n are mode numbers.
- a and b are the dimensions of the waveguide.
- μ is the permeability.
- ϵ is the permittivity of the medium inside the waveguide.

Wave Impedance:

The wave impedance for the TM and TE modes is given by:

$$Z_{TE} = \frac{Z_0}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$Z_{TM} = Z_0\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

where, Z_0 = 377 Ω is the impedance of free space.

Phase and Group Velocity

- Phase Velocity v_p:

$$v_p = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

- Group Velocity v_g:

$$v_g = c\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

where, *c* is the speed of light in free space.

Procedure:

1. Analytical Calculation:
   - Calculate the cutoff frequencies for the TE11 and TM11 modes using the waveguide dimensions and the given frequency range.
2. Scilab Simulation:
   - Write a SCILAB script to calculate and plot the wave impedance for TE11 and TM11 modes.
   - Plot the phase and group velocities for the waveguide over the frequency range.
3. Experimental Verification:
   - Compare the analytical results with the experimental results.

Numerical Example:

A rectangular waveguide with dimensions 6 cm × 4 cm operates from 5 GHz to 15 GHz. Write a SCILAB code to plot -(i) variation of wave impedance w.r.t. frequency for TM11 and TE11 modes and (ii) variation of phase and group velocity w.r.t. frequency. Verify experimental results with analytical computations.

Observation Table:

Sr. No.	Frequency (GHz)	Zg_TM11 (Ω)	Zg_TE11 (Ω)	Vp (m/sec.)	Vg (m/sec.)
1	5				
2	6				
3	7				
4	8				
5	9				
6	10				
7	11				
8	12				
9	13				
10	14				
11	15				

Conclusion:

After performing this experiment, we can conclude that -

1. The wave impedance for a TM wave is always less than the free space impedance whereas the wave impedance for a TE wave is always greater than the free space impedance.
2. The value of phase velocity is observed to be greater than c at all frequencies whereas the value of group velocity is observed to be lesser than c over entire operating frequency range.