Middle East Technical University Department of Computer Engineering

CENG 223

Discrete Computational Structures

Fall 2017-2018

Homework 1

Due date: November 3 2017, Friday, 23:55

Question 1

Construct a truth table for the following propositions:

1. $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$
2. $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$

Question 2

Show that $(p \rightarrow q) \vee (p \rightarrow r)$ and $(\neg q \wedge \neg r) \rightarrow \neg p$ are logically equivalent. You should use tables 6, 7, and 8 given in pages 27 and 28 of your textbook.
In each step give the reference to the law **OR** the table.

Question 3

1. Let $D(x)$ be "x is a dog", $C(x)$ be "x is a cat", and $Friends(x, y)$ be "x and y are friends", where x and y represent animals.

   Translate the following into English statements.

   (a) $\forall x(C(x) \rightarrow \exists y(D(y) \wedge Friends(x, y)))$
   (b) $\exists x(C(x) \wedge \forall y(D(y) \rightarrow Friends(x, y)))$

2. Let $Chef(x)$ be "x is a chef", $Customer(x)$ be "x is a customer", $Meal(x)$ be "x is a meal", $Knows(x, y)$ be "x knows y", $Cooks(x, y)$ be "Chef x can cook meal y", $Eats(x, y)$ be "x can eat meal y".

   Use these predicates to express the following statements using quantifiers $\forall$ and $\exists$.

   (a) Only customers can eat meals.
   (b) Not all chefs can cook every meal.
   (c) There are some customers who can eat every meal cooked by a certain chef.
   (d) Every chef knows a chef who can cook the meals he/she cannot cook.

Question 4

Show that

$$\frac{p \to q\,, \quad \neg p}{\neg q}$$

cannot be a deduction rule in a sound deductive system.
(**Hint**: Show a counterexample)

Question 5

Prove the following by using only the natural deduction rules for $\vee, \wedge, \to$, and $\neg$ introduction and elimination along with the *biconditional*-introduction rule,

$$\frac{p \to q\,, \quad q \to p}{p \longleftrightarrow q} \quad \leftrightarrow i.$$

Any other rules/lemmas used should be proven by natural deduction as well.

$$p \to q, q \to r, r \to p \vdash (p \longleftrightarrow q) \wedge (p \longleftrightarrow r)$$

Question 6

Prove the following by using only the natural deduction rules for $\vee, \wedge, \to, \neg, \forall$, and $\exists$ introduction and elimination. Any other rules/lemmas used should be proven by natural deduction as well.

$$\forall x(Q(x) \to R(x)), \exists x(P(x) \to Q(x)), \forall x P(x) \vdash \exists x(P(x) \wedge R(x))$$

1 Regulations

1. You have to write your answers to the provided sections of the template answer file given.
2. **Late Submission:** Not allowed.
3. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.
4. **Updates & Announces:** You must follow the newsgroup (news.ceng.metu.edu.tr) for discussions and possible updates.
5. **Evaluation:**Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

2 Submission

Submission will be done via COW. Download the given template answer file "the1.tex". When you finish your exam upload the .tex file with the same name to COW.
Note: You cannot submit any other files. Don't forget to make sure your .tex file is successfully compiled in Inek machines using the command below.

```
$ pdflatex the1.tex
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