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Answer 1

Table 1: Truth Table for Question1.1

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg q \wedge (p \to q$	$(\neg q \wedge (p \to q) \to \neg p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Table 2: Truth Table for Question1.2

p	q	r	$\neg p$	$p \vee q$	$(\neg p \vee r)$	$(q \vee r)$	$(p \vee q) \wedge (\neg p \vee r)$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	T	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	T	T	T	F	T
F	F	F	T	F	T	F	F	T

Answer 2

$(p \to q) \vee (p \to r) \equiv (\neg q \wedge \neg r) \to \neg p$ *Given*

$(p \to q) \vee (p \to r) \equiv (\neg p \vee q) \vee (\neg p \vee r)$ **Table 7**

$(\neg p \vee q) \vee (\neg p \vee r) \equiv (\neg p \vee \neg p) \vee (q \vee r)$ **Associative law**

$(\neg p \vee \neg p) \vee (q \vee r) \equiv \neg p \vee (q \vee r)$ **Idempotent law**

$\neg p \vee (q \vee r) \equiv q \vee r \vee \neg p$ **Commutative law**

$(\neg q \wedge \neg r) \to \neg p \equiv \neg(\neg q \wedge \neg r) \vee \neg p$ **Table 7**

$\neg(\neg q \wedge \neg r) \vee \neg p \equiv q \vee r \vee \neg p$ **De Morgan law**

$q \vee r \vee \neg p \equiv q \vee r \vee \neg p$ *Result*

Answer 3

1.
a)Every cat has at least one friend which is dog.
b)There exists a cat that is friends with all dogs.
2.
a)$\forall_x \forall_y (Meal(y) \wedge Eats(x, y) \rightarrow Customer(x))$
b)$\neg \exists_x (Chef(x) \rightarrow \forall_y (Meal(y) \rightarrow Cooks(x, y))$
c)$\exists_x \forall_y \exists_z (Customer(x) \wedge Chef(z) \wedge Meal(y) \wedge Cooks(z, y) \wedge Eats(x, y) \wedge \forall_t (chef(t) \wedge Cooks(t, y) \rightarrow t = z))$
d)$\forall_x \exists_y (Chef(x) \rightarrow Chef(y) \wedge Knows(x, y) \wedge \forall_z (Meal(z) \wedge Cooks(y, z) \rightarrow \neg Cooks(x, z)))$

Answer 4

Table 3: Truth Table for Deduction rule

p	**q**	$\neg p$	$p \rightarrow q$	$\neg q$
T	T	F	T	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Natural deduction says that if we have premises that are true, then the right hand side proposition will also be true. However, when we build a truth table, in the cases where premises are true, we do not have true right hand side proposition. Therefore, this system cannot be deduction rule.

Answer 5

Table 4: Answer to the 5th question

1	$p \to q$	*premise*
2	$q \to r$	*premise*
3	$r \to p$	*premise*
4	p	*assumption*
5	q	$\to_e, 1$
6	r	$\to_e, 2$
7	$p \to r$	$\to_i, 4-6$
8	$p \leftrightarrow r$	$\leftrightarrow_i, 3, 7$
9	q	*assumption*
10	r	$\to_e, 2$
11	p	$\to_e, 3$
12	$q \to p$	$\to_i, 9-11$
13	$p \leftrightarrow q$	$\leftrightarrow_i, 1, 12$
14	$(p \leftrightarrow r) \wedge (p \leftrightarrow q)$	$\wedge_i, 5, 8$

Answer 6

Table 5: Answer to the 6th question

1	$\forall(Q(x) \to P(x))$	*premise*
2	$\exists(P(x) \to Q(x))$	*premise*
3	$\forall P(x)$	*premise*
4	$Q(x) \to P(x)$	$\forall_e, 1$
5	$P(c) \to Q(c)$	*assumption*
6	$P(c)$	$\forall_e, 3$
7	$Q(c)$	$\to_e, 5, 6$
8	$R(c)$	$\to_e, 4, 7$
9	$P(c) \wedge R(c)$	$\wedge_i, 5, 8$
10	$\exists(P(x) \wedge R(x))$	$\exists_i, 9$
11	$\exists(P(x) \wedge R(x))$	$\exists_e, 2, 6-10$