Middle East Technical University | Department of Computer Engineering

CENG 223

Discrete Computational Structures

Fall '2017-2018

Homework 4

Due date: 27 December 2017, 23:55

Question 1

Find the recurrence relation for the number of ternary strings that contain 3 consecutive 0s, 1s or 2s.
Note: *Solutions without explanations will get 0 even if the end result is correct.*

Question 2

a. Find the number of ways to cover a 3×2 board using 2×1 tiles.

b. Use the 3×2 boards you found in part **a.** as your tiles to cover a $3 \times n$ board. Two tilings are considered the same if they can be obtained from one another when mirrored along the side of length-n. (For example, one of the possible tilings should be dropped if this rule is applied for part **a.**) Find a recurrence relation a_n for the number of ways to cover a $3 \times n$ board.

c. Solve the recurrence relation you found in part **b.** using generating functions.

Question 3

Determine if the given relations are partial orderings of the given sets or not.

a. Set inclusion ($\subseteq$) on any set of sets

b. The relation "$|$" of divisibility on the set of integers $\mathbf{Z}$

c. Relation R defined as "$a\ R\ b$ if there is a positive integer r such that $b = a^r$" on $\mathbf{Z}$

Question 4

a. A partition of a positive integer n is a set of *positive* integers whose sum is n. Find the partitions of 5.

b. A partition P_1 precedes a partition P_2 if the integers in P_1 can be added to obtain the integers in P_2. For example, for 4's two partitions, $3-1$ and $2-1-1$, we have $2-1-1$ preceding $3-1$ as $2+1=3$. Give the Hasse diagram for partitions you found in part **a.** under this precedence relation.

1 Regulations

1. You have to write your answers to the provided sections of the template answer file given. Other than that, you cannot change the provided template answer file. If a latex structure you want to use cannot be compiled with the included packages in the template file, that means you should not use it.

2. Do not write any other stuff, e.g. question definitions, to answers' sections. Only write your answers. Otherwise, you will get 0 from that question.

3. **Late Submission: Not allowed**

4. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.

5. **Newsgroup:** You must follow the newsgroup (news.ceng.metu.edu.tr) for discussions and possible updates on a daily basis.

6. **Evaluation:** Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

2 Submission

Submission will be done via COW. Download the given template file, "the2.tex", when you finish your exam upload the .tex file with the same name to COW.
Note: You cannot submit any other files. Don't forget to make sure your .tex file is successfully compiled in Inek machines using the command below.

```
$ pdflatex hw4.tex
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