CSCE 629 - 602 Analysis of Algorithms

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1A. 29.1-4 Solution

Given

Objective Function: Minimize
$$2x_1 + 7x_2 + x_3 \tag{1}$$
Subject To
$$x_1 - x_3 = 7 \tag{2}$$
$$3x_1 + x_2 \geq 24 \tag{3}$$
$$x_2 \geq 0 \tag{4}$$
$$x_3 \leq 0 \tag{5}$$

Converting to Standard Form

Taking negation of the objective function (1) Objective Function: Maximize
$$-2x_1 - 7x_2 - x_3 \tag{6}$$

since we do not have non-negative condition for x_1 we do the following.
$$x_1 = x'_1 - x''_1 \tag{7}$$
$$x_1 = x'_1 - x''_1 \tag{8}$$
with
$$x'_1 \geq 0 \tag{9}$$
$$x''_1 \geq 0 \tag{10}$$
Replacing $x'_1 \rightarrow x_0$ and $x''_1 \rightarrow x_1$
Substituting this in (1), (2), (3), (4) we get,

Objective Function: Maximize

$$-2x_0 + 2x_1 - 7x_2 - x_3 \tag{11}$$

Subject To

$$x_0 - x_1 - x_3 = 7 \tag{12}$$

$$3x_0 - 3x_1 + x_2 \geq 24 \tag{13}$$

$$x_0, x_1, x_2 \geq 0 \tag{14}$$

$$x_3 \leq 0 \tag{15}$$

Next we change the greater than inequality in (14) to less than equal to by multiplying by -1.

Objective Function: Maximize

$$-2x_0 + 2x_1 - 7x_2 - x_3 \tag{16}$$

Subject To

$$x_0 - x_1 - x_3 = 7 \tag{17}$$

$$-3x_0 + 3x_1 - x_2 \leq -24 \tag{18}$$

$$x_0, x_1, x_2 \geq 0 \tag{19}$$

$$x_3 \leq 0m \tag{20}$$

Since we have $x_3 \leq 0$ we have to change this to a non-negative constraint. So we multiply with -1 and change $-x_3$ with x_3'. And we finally represent x_3' with x_3 But this is not same as the one given in the initial Linear programming.

Now we split the equality (17) in two inequalities.

$$x_0 - x_1 + x_3 = 7$$

to

$$x_0 - x_1 + x_3 \leq 7$$

$$x_0 - x_1 + x_3 \geq 7$$

Negating the greater than inequality by multiplying with -1.

$$-x_0 + x_1 - x_3 \leq -7$$

Standard Form

Objective Function, Maximize

$$-2x_0 + 2x_1 - 7x_2 + x_3 \tag{21}$$

Subject To

$$x_0 - x_1 + x_3 \leq 7 \tag{22}$$

$$-x_0 + x_1 - x_3 \leq -7 \tag{23}$$

$$-3x_0 + 3x_1 - x_2 \leq -24 \tag{24}$$

$$x_0, x_1, x_2, x_3 \geq 0 \tag{25}$$

2A. 29.1-5 Solution

Given

Maximize :

$$2x_1 - 6x_3$$

Subject to:

$$\begin{aligned} x_1 + x_2 - x_3 &\leq 7 \\ 3x_1 - x_2 &\geq 8 \\ -x_1 + 2x_2 + 2x_3 &\geq 0 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Convert to Slack form

Making greater than equal to to less than equal to

Maximize :

$$2x_1 - 6x_3$$

Subject to:

$$\begin{aligned} x_1 + x_2 - x_3 &\leq 7 \\ -3x_1 + x_2 &\leq -8 \\ x_1 - 2x_2 - 2x_3 &\leq 0 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Introducing basic variables x_4, x_5, x_6

Maximize :

$$Z = 2x_1 - 6x_3$$

Subject to:

$$\begin{aligned} x_4 &= 7 - x_1 - x_2 + x_3 \\ x_5 &= -8 + 3x_1 - x_2 \\ x_6 &= -x_1 + 2x_2 + 2x_3 \end{aligned}$$

In the form above x_1, x_2, x_3 are non-basic variable and x_4, x_5, x_6 are basic variables.

3A. 29.2-1 Solution

Given

Maximize :

$$d_t$$

Subject to:

$$\begin{aligned}
&d_v \le d_u + w(u,v), \text{for every edge } (u,v) \in E \\
&d_s = 0
\end{aligned}$$

Convert to Standard form

Objective function is in standard form. Let us convert the constraints to standard form.

Maximize :

$$d_t$$

Subject to:

$$\begin{aligned}
d_v - d_u &\le w(u,v), && \text{for every edge } (u,v) \in E \\
d_s &\le 0 \\
-d_s &\le 0 \\
d_v &\ge 0 && \text{, for every } v \in V
\end{aligned}$$

Note: the equation $d_v \ge 0$, for every $v \in V$ *is under the assumption that all the weights of the edges are non-negative*

If we are not given weights to be Non negative then we will replace every d_v with $d'_v - d''_v$. Hence our LP is as follows

Maximize :

$$d'_t - d''_t$$

Subject to:

$$\begin{aligned}
d'_v - d''_v - d'_u + d''_u &\le w(u,v), && \text{for every edge } (u,v) \in E \\
d'_s - d''_s &\le 0 && \text{, where } s \text{ is the source vertex} \\
-d'_s + d''_s &\le 0 && \text{, where } s \text{ is the source vertex} \\
d'_v, d''_v &\ge 0 && \text{, for every } v \in V
\end{aligned}$$

4A. 29.2-7 Solution

Linear Programming Formulation

Minimize :

$$\sum_{(u,v)\in E} a(u,v) f_{uv}$$

Subject to:

$$\sum_{i=1}^{k} f_{iuv} \leq c(u,v) \qquad \text{for each } u, v \in V$$

$$\sum_{v\in V} f_{iuv} - \sum_{v\in V} f_{ivu} = 0 \qquad \text{for } i = 1, 2, 3, ...k, \text{ for each } u \in V - \{s_i, t_i\}$$

$$\sum_{v\in V} f_{is_iv} - \sum_{v\in V} f_{ivs_i} = d_i \qquad \text{for } i = 1, 2, 3, ...k$$

$$f_{iuv} \geq 0 \qquad \text{for } i = 1, 2, 3, ...k, \text{ for each } u, v \in V$$

References

Introduction to Algorithms by T. Cormen, C. Leiserson, R. Rivest, C. Stein