

CSE355/AMS345 Homework 4

Jie Gao*

The following problems are due by November 20th, 9pm.

1. **Voronoi diagram (20pts)** Given a set of sites $P = \{p_1, p_2, \dots, p_n\}$ in the plane, prove the following statements about Voronoi diagram or Delaunay triangulations.
 - (a) Prove that the Voronoi cell $V(p_i)$ is unbounded if p_i is on the convex hull of the point set (5pts).
 - (b) Prove that if p_j is a nearest neighbor of p_i , then p_i, p_j is an edge in the Delaunay triangulation (5pts).
 - (c) Describe the Voronoi diagram and Delaunay triangulation of the vertices of a n -regular polygon (5pts).
 - (d) Design a set of n points with no four points co-circular, such that one vertex of the Delaunay triangulation has degree $n - 1$. (5pts)
2. **Relative Neighborhood Graph (20pts)** The relative neighborhood graph (RNG) of a set of points p_1, p_2, \dots, p_n is a graph whose nodes correspond to the points, and with two nodes p_i and p_j connected by an arc if and only if they are at least as close to each other as to any other point, that is, if

$$|p_i - p_j| \leq \max_{m \neq i, j} \{|p_i - p_m|, |p_j - p_m|\}$$

This equation determines a forbidden region, called the lune (p_i, p_j) , within which no point p_m may lie if p_i and p_j are adjacent in the RNG. This region is the intersection of two open disks centered on p_i and p_j , both of radius $|p_i - p_j|$.

- (a) Design a brute-force algorithm to construct RNG, do not worry about running time. What is the time complexity? (5pts)
- (b) Prove that $RNG \subseteq D(P)$, where $D(P)$ is the Delaunay triangulation. (10pts)
- (c) Use the above for a faster algorithm. (5pts)

*Department of Computer Science, Stony Brook University, Stony Brook, NY 11794. Email: jgao@cs.sunysb.edu