

CSE355/AMS345 Homework 3

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The following problems are due by October 19th, 9pm.

1. **Intersection of parabolas (20pts)** Let S be a set of n parabolas: the i th one is the graph of a function $y = a_i x^2 + b_i x + c_i$. Our goal is to find all k points where two of them cross. (You may assume no degeneracies: no three curves pass through a common point, and all n parabolas are distinct.) (a). What is the maximum possible value of k (as a function of n)? (Draw an example showing how the upper bound can be attained for n parabolas.) (b). How efficiently can you compute the k crossing points using a variant of the sweeping line algorithm for computing intersections of line segments? (Describe briefly, specifying the events, the sweeping line states, event handling.)
2. **Triangle enclosure (10pts)** Let S be a set of n triangles in the plane. The boundaries of the triangles are disjoint, but it is possible that a triangle lies completely inside another triangle. Let P be a set of n points in the plane. Give an $O(n \log n)$ algorithm that reports each point in P lying outside all triangles.
3. **Intersection of halfplanes (10pts)** Let H be a collection of n halfplanes, each of which contains a portion of the negative y axis (facing downwards). Let Q be the intersection of H . And take S to be the dual of the lines bounding D . Take P as the convex hull of S . Explain the relationship of P and Q . Give an algorithm to compute the intersection of halfplanes based on your observation.
4. **Stabber.(10pts)** Let S be a set of n segments in the plane. A line ℓ that intersects all segments of S is called a *traversal* or *stabber* of S . Give an $O(n^2)$ algorithm to decide if a stabber for S exists. Hint: Use duality.

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