CSE355/AMS345 Fall 2014 Computational Geometry

Midterm

- This is a **OPEN book, OPEN notes** exam. You may bring a calculator or laptop. But no Internet is allowed.
- For all problems, unless specified otherwise, assume that there is no degeneracy (i.e., no three points co-linear, no four points co-circular, no three lines having a common intersection).
- There are 5 problems and total of 33 points.
- Exam starts at 11:30am and ends at 12:50pm sharp on October 28th, 2014.
- Be brief. But if you wish, you may attach additional pages if you need.

By signing below I declare that I follow the rule of academic integrity and finish the exam on my own, without the help of others.

Name ___________________________

ID ___________________________

Signature ___________________________

1	2	3	4	5

1. **Art Gallery Problem.** (6pts) For the simple polygon P below, find an art gallery solution.

   (a) Draw a triangulation by iteratively cutting off an 'ears' (a vertex whose two neighboring vertices on the polygon boundary are visible to each other). (2pt)

   (b) Use your triangulation to obtain a 3-coloring of the vertices of the triangulation. (2pts)

   (c) Use your 3-colored triangulation a set of vertex guards for P such that all points of P are guarded (visible to at least one guard). Follow the algorithm we discussed in class to obtain at most $\lfloor n/3 \rfloor$ guards for any polygon of n vertices. Highlight (e.g., circle) those vertices. (2pt)

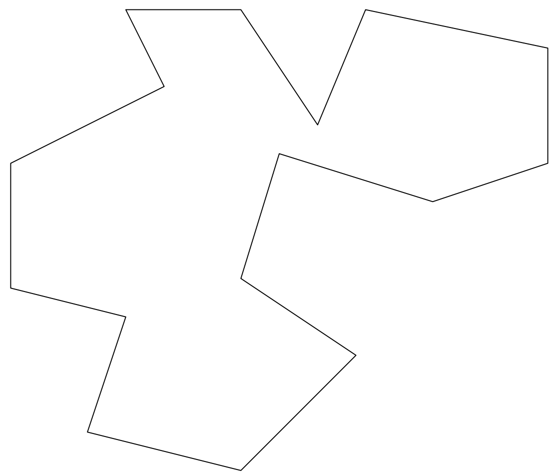

Figure 1: Art Gallery Solution for this polygon.

2. **Point Line Duality** (5pts) In point line duality, a point (p_x, p_y) maps to a line $p^* : y = p_x x - p_y$ in the dual space. Consider the points in the shaded region below. They will map to a collection of lines. Draw an illustration to show the collection of lines in the dual space.

   (a) Highlight what is a^*, b^*, c^*, the dual of the three vertices of the triangle $\triangle abc$.(1pt)

   (b) Highlight what is $\ell^*_{ab}, \ell^*_{bc}, \ell^*_{ac}$, the dual of the three lines ℓ_{ab}, ℓ_{bc}, ℓ_{ac} defined by the three vertices of the triangle $\triangle abc$.(1pt)

   (c) Highlight the dual of the given point $p \in \triangle abc$. And color the region that is covered by the lines that are dual of points in $\triangle abc$. (3pts)

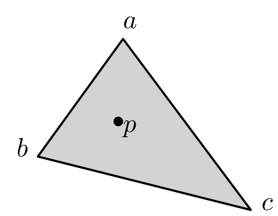

Figure 2: Points inside the triangle $\triangle abc$.

3. **Convex Hull.** (8pts)

(a) Please execute the Graham scan algorithm to compute the convex hull for the following examples. The figure below shows the current state of the upper hull (a chain p_1, p_2, p_3, p_4). They are stored in a stack S with p_4 on top. The points to be processed are p_5, p_6, p_7, p_8 in that order. Show what is the sequence of operations including both orientation test and stack operation (push or pop). For example, to process p_5, the following operations are used:

i. Orientation(p_3, p_4, p_5)

ii. Pop p_4

iii. Orientation(p_2, p_3, p_5)

iv. Push p_5

The upper hull at this moment becomes a chain p_1, p_2, p_3, p_5. Your task is to run the algorithm on the points p_6, p_7, p_8. Please show the sequence of operations as well as the upper hull *after* each point is processed. (5pts)

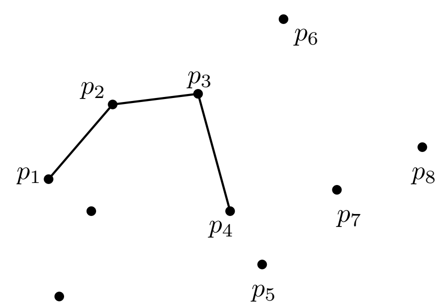

Figure 3: Run Graham Scan algorithm on the above point set.

(b) Suppose you are given a collection of n points that are known to be placed on a $\sqrt{n}$ by $\sqrt{n}$ grid with very tiny perturbation. What is your best guess of the running time for each of the following algorithm for this specific data set? Use function of n and big-O notation. For gift wrapping please try your best to guess what h, the number of points on the convex hull, is. (3pts)

i. Graham Scan

ii. Gift wrapping

iii. Quickhull

4. **Manhattan skyline** (9pts) Given a set of n rectangles, each rectangle i represented by its left and right x-coordinates a_i and b_i and its height h_i. The bottom of each rectangle is on the x-axis. Use a sweeping line algorithm to compute the vertices of this skyline, defined as the upper envelope. Please explain the algorithm steps (2pts), how the sweeping line state is defined (2pts), what are the events (2pts), how to process the events (2pts), and what is the running time of the algorithm (1pt).

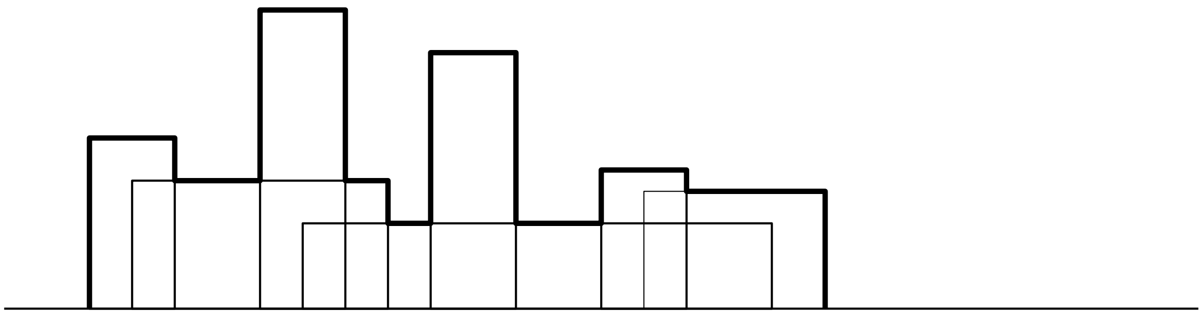

Figure 4: Manhattan skyline.

5. **Voronoi Diagram.** (5pts) Given a set of points P in the plane, for each point $p \in P$, define p's closest point to be the point in $P \setminus \{p\}$ with smallest distance to p.

   Claim: for *any* point $p \in P$, denote its closest point as q. Then in the Voronoi diagram of P, the Voronoi cell of p, q share a Voronoi edge.

   Do you think this claim is true? If so give a proof. If not give a counter example.