Scalable Multi-Party Computation Protocols for Machine Learning in the **Honest-Majority Setting**

Fengrun Liu University of Science and Technology of China & Shanghai Qi Zhi Institute & PADO Labs Shanghai Qi Zhi Institute

Xiang Xie

Yu Yu Shanghai Jiao Tong University & State Key Laboratory of Cryptology, P. O. Box 5159, Beijing, 100878, China

Outline

- o Background
- o Our Results
- o Technique 1: 1-round Truncation with 1-bit Gap
 - o Efficient and well-defined operations in fields of Mersenne primes
 - o A large gap is involved in truncation
 - o Able to fix the positive overflow if we only allow positive overflow
 - o Seamlessly combined with DN protocol to obtain 1-round fixed-point mult
- o Technique 2: Improved Bitwise Primitives
 - o Efficient Prefix-OR and Bitwise Comparison

Background

MPC Protocols Tailored for Privacy-preserving Machine Learning (PPML)

Secure Multi-Party Computation (MPC):

enables a group of n parties to collaboratively compute a **function** on their private inputs while preserving the privacy of those inputs

Privacy-preserving Machine Learning (PPML):

allows multiple parties to collaborate on training or inference tasks on distributed datasets without exposing the individual data points and the ML model itself

- Small-scale scenarios for 2 4 parties only: rely on additive secrete sharing in the ring setting used in client-server model
- General-purpose MPC for n parties: lack of efficient protocols to realize non-linear functions, such as truncation and comparison.
- => Motivation: to develop the scalable and efficient MPC protocols tailored for PPML

Starting Point: Damgård-Nielsen [DN07] Protocol based on Shamir's Secret Sharing

Our Results

Scalable and Efficient MPC-baed PPML Framework in the Honest Majority Setting

In application: our protocols facilitate the efficient and scalable online oblivious inference.

We conduct experiments in various settings, ranging from 3PC to 63PC (simulated by 11 servers).

Setting	Online (s)	Offline (s)	
LAN	0.1	1.2	
WAN	4.6	10.7	

Network	3РС	7PC	11PC	21PC	31PC	63PC
3-layer DNN	0.37	0.37	0.38	0.39	0.40	0.47
3-layer CNN	0.39	0.40	0.41	0.44	0.48	0.68
4-layer CNN	1.2	1.3	1.3	1.7	2.0	4.6

Efficient:

runtime (s) for oblivious inference of 4-layer CNN with 63 parties

Scalable:

online runtime (s) for oblivious inference in the WAN setting from 3-63 PC

In theory: we optimize the following primitives leveraging the unique properties of Mersenne prime fields.

1. truncation (related to fixed-point multiplication)

This: 1-bit gap & 1 round

2. bitwise comparison (related to various non-linear functions)

This: no gap & 1 round

Mersenne Primes $p=2^\ell-1$ for prime ℓ

Efficient and well-defined operations in fields of Mersenne primes

In the ring setting \mathbb{Z}_{2^ℓ}

In the field setting $\mathbb{F}_{\!p}$ ($p=2^\ell-1$)

Representation of integer \bar{x}

$$x = \bar{x} \pmod{2^{\ell}}$$

$$x = \begin{cases} \bar{x}, & \bar{x} \ge 0 \\ 2^{\ell} - \bar{x}, & \bar{x} < 0 \end{cases} \quad \text{for } \bar{x} \in [-2^{\ell-1}, 2^{\ell-1})$$

 $x = \bar{x} \pmod{2^{\ell} - 1}$

$$x = \begin{cases} \bar{x}, & \bar{x} \ge 0 \\ 2^{\ell} - 1 - \bar{x}, & \bar{x} < 0 \end{cases} \quad \text{for } \bar{x} \in (-2^{\ell - 1}, 2^{\ell - 1})$$

1. MSB of x indicates the sign of \bar{x}

Representation of \bar{x} in \mathbb{Z}_{2^ℓ}

$\ell - d$	d	
------------	---	--

(2's complement)

 \approx

Representation of \bar{x} in $\mathbb{F}_{2^{\ell}-1}$

$\ell - d$	d

Efficient mod operation in practice

$$x + y = \bar{x} + \bar{y} \pmod{2^{\ell}}$$

$$x \cdot y = \bar{x} \cdot \bar{y} \pmod{2^{\ell}}$$

$$a \cdot 2^{\ell} + b = b \pmod{2^{\ell}}$$
shift bits

 $x + y = \bar{x} + \bar{y} \pmod{2^{\ell} - 1}$ $x \cdot y = \bar{x} \cdot \bar{y} \pmod{2^{\ell} - 1}$ $a \cdot 2^{\ell} + b = a + b \pmod{2^{\ell} - 1}$ shift bits + addition

well-defined truncation: $\bar{x}/2^d$

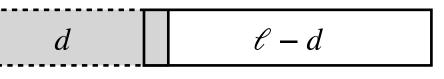
Representation of $\bar{x}/2^d$ in \mathbb{Z}_{2^ℓ}



(2's complement)

=

Representation of $\bar{x}/2^d$ $\mathbb{F}_{2^\ell-1}$



elaborated in Thm 3.1

2. Truncation on x: shift the bits down by d positions and fill the top d bits with the MSB of x

We can view the field element in $\mathbb{F}_{2^\ell-1}$ as the almost 2's complementation of some integer.

Fixed-point numbers represented in $\mathbb{F}_{2^\ell-1}$

Truncation is required when performing fixed-point multiplication

For a fixed-point number x

scale x by multiplying 2^d to obtain an integer $x' \in \mathbb{F}_{2^\ell-1}$

For two fixed-point numbers x and y, we can perform fixed-point operations with x' and y' $\in \mathbb{F}_{2^\ell-1}$

addition:
$$x' + y' = (x + y) \cdot 2^d$$

multiplication:
$$x' \cdot y' = (x \cdot y) \cdot 2^{2d}$$

multiplication with truncation:
$$\frac{(x' \cdot y')}{2^d} = (x \cdot y) \cdot 2^d$$

A large gap is involved when performing truncation on secret values

truncation on secret x

In the ring setting $\mathbb{Z}_{2\ell}$

1. MSB of x indicates the sign of \bar{x}

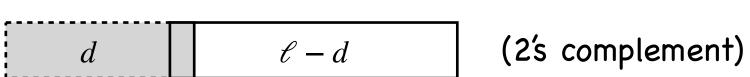
Representation of \bar{x} in \mathbb{Z}_{2^ℓ}

$\ell - d$	d

(2's complement)

2. Trunc(x): Truncation on x, performing $\bar{x}/2^d$

Representation of $\bar{x}/2^d$ in \mathbb{Z}_{2^ℓ}



Given (r, Trunc(r)), we can perform truncation on x in \mathbb{Z}_{2^k} as follows:

1.
$$a = x + r$$

(mod
$$2^{\ell}$$
)

(mod
$$2^{\ell}$$
) where $x \in \mathbb{Z}_{2^{\ell}}$ and $r \leftarrow \mathbb{Z}_{2^{\ell}}$

2. Reveal a

3. Trunc(x) = Trunc(a) - Trunc(r) (mod
$$2^{\ell}$$
) only holds iff $\bar{a} = \bar{x} + \bar{r}$ (in the view of integers)

where
$$\bar{x} \in [-2^{\ell-1}, 2^{\ell-1})$$
 and $\bar{r} \leftarrow [-2^{\ell-1}, 2^{\ell-1})$

When x + r is performed in Step 1, the **OVERFLOW** might happen, meaning $\bar{x} + \bar{r} \notin [-2^{\ell-1}, 2^{\ell-1})$.

A large gap is involved when performing truncation on secret values

truncation on secret x

In the ring setting \mathbb{Z}_{2^ℓ}

Given (r, Trunc(r)), we could perform truncation on x in \mathbb{Z}_{2^ℓ} as follows:

1.
$$a = x + r$$
 (mod 2^{ℓ}) where $x \in \mathbb{Z}_{2^{\ell}}$ and $r \leftarrow \mathbb{Z}_{2^{\ell}}$

- 2. Reveal a
- ?? 3. Trunc(x) = Trunc(a) Trunc(r) (mod 2^{ℓ}) only holds iff $\bar{a} = \bar{x} + \bar{r}$ (in the view of integers) where $\bar{x} \in [-2^{\ell-1}, 2^{\ell-1})$ and $\bar{r} \leftarrow [-2^{\ell-1}, 2^{\ell-1})$

When x + r is performed in Step 1, the OVERFLOW might happen, meaning $\bar{x} + \bar{r} \notin [-2^{\ell-1}, 2^{\ell-1})$.

For example, we have a = x + r in \mathbb{Z}_{2^8} .

positive overflow happens, i.e. $\bar{x} + \bar{r} = 323 \notin [-2^7, 2^7)$

 \therefore Trunc(a) \neq Trunc(x) + Trunc(r)

A large gap is involved when performing truncation on secret values

truncation on secret x

In the ring setting \mathbb{Z}_{2^ℓ}

In the field setting $\mathbb{F}_{\!p}$ ($p=2^\ell-1$)

- 1. MSB of x indicates the sign of \bar{x}
- 2. Trunc(x): Truncation on x, performing $\bar{x}/2^d$

Representation of \bar{x} in \mathbb{Z}_{2^ℓ}

$$\ell - d$$

d

(2's complement)

Representation of $\bar{x}/2^d$ in \mathbb{Z}_{2^ℓ}

 $\ell - d$

(2's complement)

Truncation on x in $\mathbb{F}_{2^{\ell}-1}$ as follows:

 $\ell - d$

Representation of \bar{x} in $\mathbb{F}_{2\ell-1}$

Representation of $\bar{x}/2^d$ $\mathbb{F}_{2^\ell-1}$

 $\ell - d$

1. a = x + r

(mod $2^{\ell}-1$)

- 2. Reveal a
- 3. Trunc(x) = Trunc(a) Trunc(r) (mod $2^{\ell} 1$) only holds iff $\bar{a} = \bar{x} + \bar{r} \in (-2^{\ell-1}, 2^{\ell-1})$

Truncation on x in \mathbb{Z}_{2^ℓ} as follows:

1.
$$a = x + r$$

(mod
$$2^{\ell}$$
)

2. Reveal a

3. Trunc(x) = Trunc(a) - Trunc(r) (mod
$$2^{\ell}$$
) only holds iff $\bar{a} = \bar{x} + \bar{r} \in [-2^{\ell-1}, 2^{\ell-1})$

Trunc(a) = Trunc(x) + Trunc(r) only holds w.h.p. for small x.

The above methods introduce A LARGE GAP between secrets and modulus.

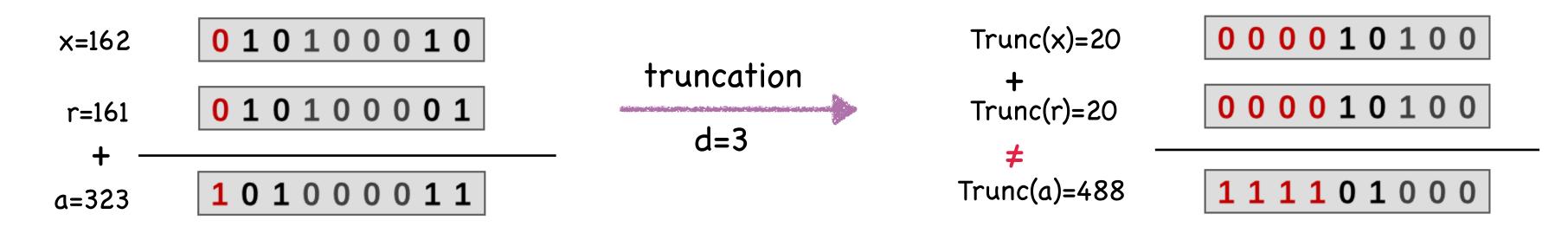
Given (r, Trunc(r)):

We are able to fix the positive overflow if we only allow positive overflow

truncation on secret x

In the field setting
$$\mathbb{F}_p$$
 ($p=2^\ell-1$)

Take the same example, we have a = x + r in \mathbb{Z}_{2^8-1} .

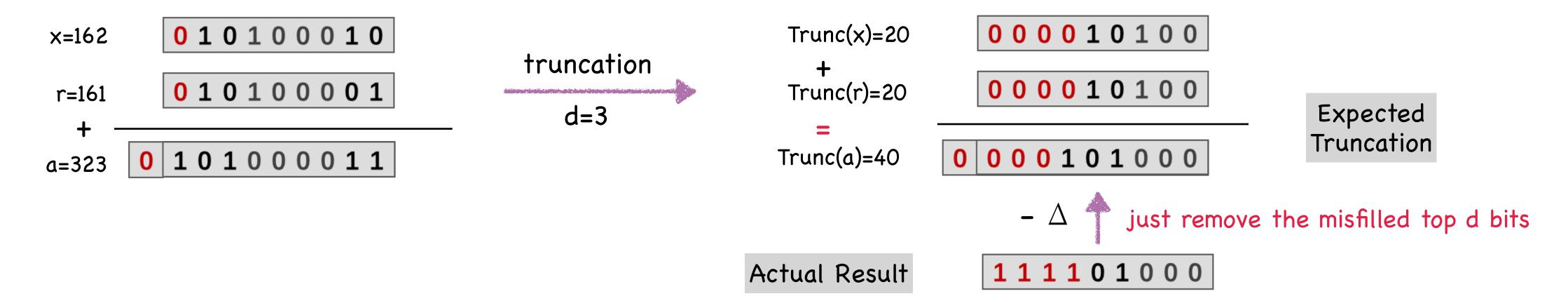


We are able to fix the positive overflow if we only allow positive overflow

truncation on secret x

In the field setting
$$\mathbb{F}_{\!p}$$
 ($p=2^\ell-1$)

Take the same example, we have a = x + r in \mathbb{Z}_{2^8-1} .



Q1: How to ensure only the positive overflow is allowed?

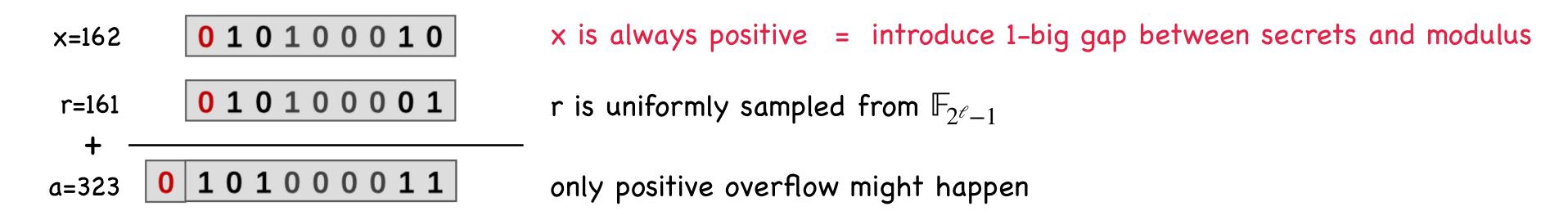
Q2: How to detect and correct the error introduced by positive overflow?

We are able to fix the positive overflow if we only allow positive overflow

truncation on secret x

In the field setting
$$\mathbb{F}_p$$
 ($p=2^\ell-1$)

Take the same example, we have a = x + r in \mathbb{Z}_{2^8-1} .



Q1: How to ensure only the positive overflow is allowed?

We can impose a constraint on the input x to ensure x is always positive.

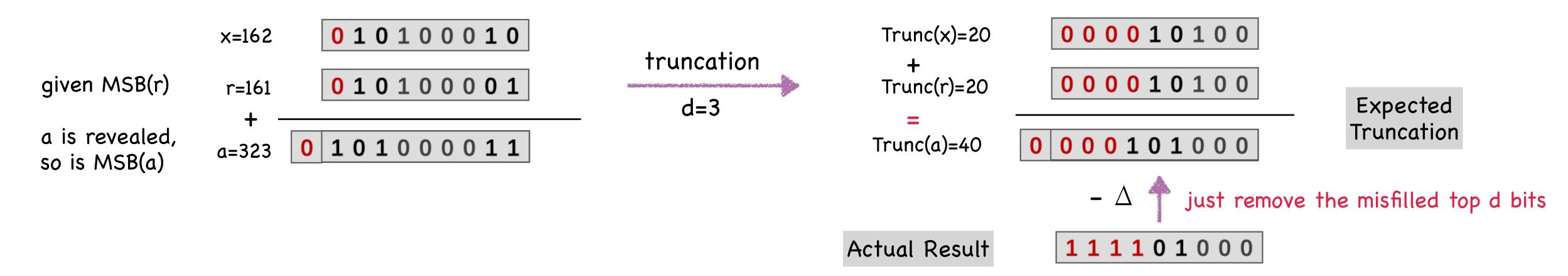
Q2: How to detect and correct the error caused by positive overflow?

We are able to fix the positive overflow if we only allow positive overflow

truncation on secret x

In the field setting
$$\mathbb{F}_{\!p}$$
 ($p=2^\ell-1$)

Take the same example, we have a = x + r in \mathbb{Z}_{2^8-1} .



Q1: How to ensure the only allowance of positive overflow?

We can impose a constraint on the input x to ensure x is always positive.

Q2: How to detect and correct the error caused by positive overflow?

elaborated in Thm 3.2

Positive overflow occurs iff MSB(r) = 0 and MSB(a) = 1, and we can then easily correct the error as above.

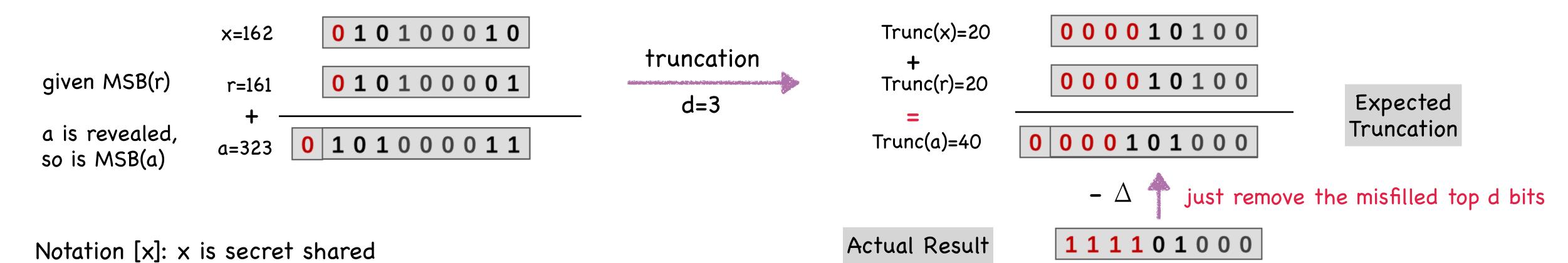
We are able to fix the positive overflow if we only allow positive overflow

truncation on secret x

In the field setting
$$\mathbb{F}_{\!p}$$
 ($p=2^\ell-1$)

Online Complexity: 1 round & 1-bit gap

Take the same example, we have a = x + r in \mathbb{Z}_{2^8-1} .



Given ([r], [Trunc(r)], [MSB(r)]), we can perform truncation on [x] in $\mathbb{F}_{2\ell-1}$ as follows:

- 1. [a] = [x] + [r]
- 2. Reveal a and MSB(a)
- 3. $[e] = (1 [MSB(r)]) \cdot MSB(a)$
- 4. [Trunc(x)] = Trunc(a) [Trunc(r)] + [e] $\cdot (2^{\ell-d} 1)$

e = 1 indicating positive overflow occurs

holds for any $x \in [0,2^{\ell-1})$ representing positive numbers

Seamlessly combined with DN protocol to obtain 1-round fixed-point mult

fixed-point mult on secret x and y

In the field setting $\mathbb{F}_{\!p}$ ($p=2^\ell-1$)

Online Complexity: 1 round & 1-bit gap

Notations [x]: degree-t sharing <x>: degree-2t sharing [x] · [y] = <xy>

Suppose x and y represent two fixed-point numbers:

DN Protocol: [xy]

- 1. $\langle a \rangle = [x] \cdot [y] + \langle r \rangle$
- 2. Reveal a
- 3. [xy] = a [r]

combined

Our Truncation Protocol: [Trunc(x)]

- 1. [a] = [x] + [r]
- 2. Reveal a and MSB(a)
- 3. $[e] = (1 [MSB(r)]) \cdot MSB(a)$
- 4. $[Trunc(x)] = Trunc(a) [Trunc(r)] + [e] \cdot (2^{\ell-d} 1)$

offline: random truncation triple: (<r>, [Trunc(r)], [MSB(r)])

Our Fixed-point Multiplication Protocol: [Trunc(xy)]

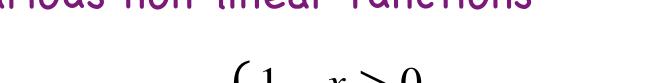
- 1. $[a] = [x] \cdot [y] + \langle r \rangle$
- 2. Reveal a and MSB(a)
- 3. $[e] = (1 [MSB(r)]) \cdot MSB(a)$
- 4. [Trunc(xy)] = Trunc(a) [Trunc(r)] + [e] $\cdot (2^{\ell-d} 1)$

Efficient Prefix-OR and Bitwise Comparison

Q: Why do we want to optimize bitwise comparison?

It underpins arithmetic comparison crucial for various non-linear operations.

various non-linear functions



$$DReLU(x) = \begin{cases} 1, & x \ge 0 \\ 0, & x < 0 \end{cases}$$

$$ReLU(x) = DReLU(x) \cdot x$$

$$Max(a, b) = ReLU(a - b) + b$$

$$MSB(a) = LSB (2a)$$
 holds in odd rings

1.
$$y = 2a + r$$

2. Reveal y

3. LSB(2a) = LSB(y)
$$\oplus$$
 LSB(r) \oplus (y_B < r_B) public secret

bitwise comparison (y_B < r_B)
look for the first different bit

* XOR

(secret)

0 0 0 0 0 0 1 0 1 0 0 1

* Prefix-OR

(secret)

0 0 0 0 0 0 1 1 1 1 1 1 1



(secret) e_B 0 0 0 0 0 1 0 0 0 0 0

$$(y_B < r_B) = \langle e_B, r_B \rangle$$

* XOR is free between secret bits and public bits: [a]
$$\oplus$$
 b = a + b - 2[a] \cdot b

* OR involves a multiplication between secrets: [a]
$$\vee$$
 [b] = a + b - [a] \cdot [b]

* Prefix-OR involves
$$\ell$$
 multiplications: compute $b_j = \bigvee_{i=1}^J a_i$ for $j=1,\ldots,\ell$

How to efficiently compute Prefix-OR?

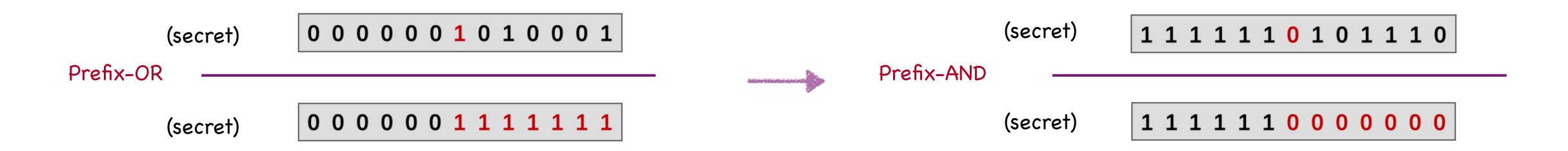
Efficient Prefix-OR and Bitwise Comparison

Prefix-OR on secrete bits

In the field setting $\mathbb{F}_{\!p}$ ($p=2^\ell-1$)

Online Complexity:

1 round



- 1. locate the first 1-bit's position starting from MSB
- 2. set all the following bits to 1 involved with OR operation [a] \vee [b] = a + b [a] \cdot [b]
 - * Prefix-OR: compute $b_j = \bigvee_{i=1}^j a_i$ for $j=1,\ldots,\mathcal{C}$

Online Complexity of [NO07]: 5 rounds

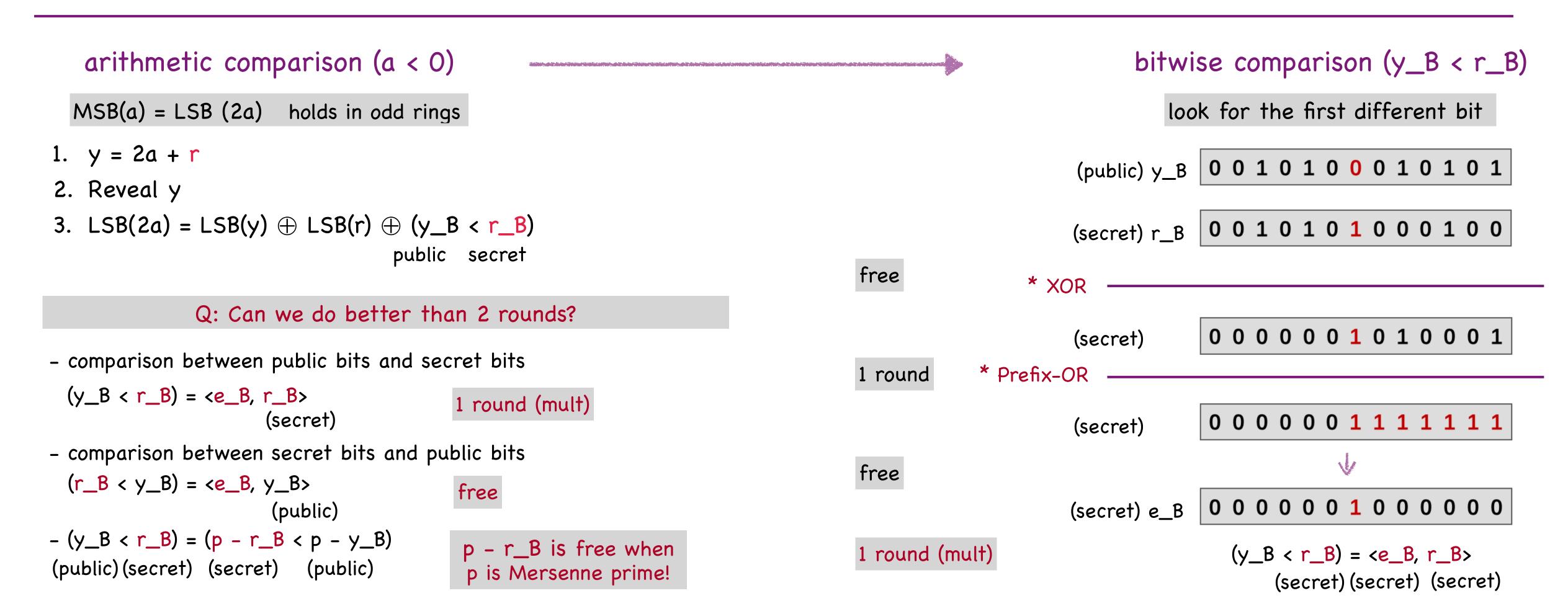
- 1. locate the first 0-bit's position starting from MSB
- 2. set all the following bits to 0 (zero out all the following bits) only involved with multiplication
 - * Prefix-MULT: compute $b_j = \Pi_{i=1}^j a_i$ for $j=1,\ldots,\mathscr{E}$

Online Complexity of [BB89]: 1 round

Efficient Prefix-OR and Bitwise Comparison

Bitwise Comparison between public bits and secret bits

In the field setting $\mathbb{F}_{\!p}$ ($p=2^\ell-1$)



Efficient Prefix-OR and Bitwise Comparison

 $(y_B < r_B) = (p - r_B < p - y_B)$

Bitwise Comparison between public bits and secret bits

In the field setting
$$\mathbb{F}_{\!p}$$
 ($p=2^\ell-1$)

Online Complexity: 1 round

arithmetic comparison (a < 0)

$$MSB(a) = LSB(2a)$$
 holds in odd rings

- 1. y = 2a + r
- 2. Reveal y
- 3. $LSB(2a) = LSB(y) \oplus LSB(r) \oplus (y_B < r_B)$ public secret

Q: Can we do better than 2 rounds?

- comparison between public bits and secret bits

$$(y_B < r_B) =$$
 (secret)

1 round (mult)

- comparison between secret bits and public bits

$$(r_B < y_B) = \langle e_B, y_B \rangle$$
 (public)

free

 $- (y_B < r_B) = (p - r_B < p - y_B)$ (public) (secret) (secret) (public)

p - r_B is free when p is Mersenne prime!

bitwise comparison (p - $r_B)$

look for the first different bit

* XOR

(secret)

0 0 0 0 0 0 1 0 1 0 0 0 1

* Prefix-OR — 1 round

(secret)

0 0 0 0 0 0 1 1 1 1 1 1 1

(secret) e_B 0 0 0 0 0 1 0 0 0 0 0

 $(p - r_B$ (public) (secret) (public)

free

free

free

Other Building Blocks

In the field setting
$$\mathbb{F}_{\!p}$$
 ($p=2^\ell-1$)

$$DReLU(x) = \begin{cases} 1, & x \ge 0 \\ 0, & x < 0 \end{cases}$$

$$ReLU(x) = DReLU(x) \cdot x$$

Protocols	Rounds Online Prep.		Communication		
Fiolocois			Online	Prep.	
$\overline{\Pi_{Fixed-Mult}}$	1	2	2	3ℓ	
$\overline{\Pi_{PreMult}}$	1	2	2ℓ	7ℓ	
$\overline{\Pi_{ ext{PreOR}}}$	1	2	2ℓ	7ℓ	
$\overline{\Pi_{Bitwise-LT}}$	1	2	2ℓ	7ℓ	
$\overline{\Pi_{\mathrm{DReLU}}}$	3	2	$4+2\ell$	$1+10\ell$	
$\overline{\Pi_{2L-DN}}$	1	1	2(m+1)	m+1	
$\overline{\Pi_{ m ReLU}}$	3	2	$6+2\ell$	$2 + 10\ell$	
$\overline{\Pi_{Maxpool}}$	$3\log m$	2	$(m-1)(6+2\ell)$	$(m-1)(2+10\ell)$	

This multiplication is saved by using the techniques of two-layer DN multiplication [ATLAS]