Gabriel Chen, 50895168
Assignment 2, ELEC 481 – Summer 2020

Question 1:

$$\mathbf{1}\boldsymbol{a})\ Use\ uniform\ series\ capital\ recovery\ factor\ = \left(\frac{A}{P}, i\%, n\right) = \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

$$A = P * \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]; where\ A = amount\ paid\ per\ period; where\ P = amount\ loaned$$

$$i(per\ month) = \frac{\frac{9\%(annum)}{12}}{100} = 0.0075$$

$$n = \#\ periods\ in\ 30\ years = 12 * 30 = 360\ periods$$

$$P = 500{,}000 - 100{,}000 = 400{,}000$$

$$A = 400000 * \left[\frac{0.0075(1+0.0075)^{360}}{(1+0.0075)^{360} - 1}\right] = \boxed{\$\mathbf{3218.4905}} \rightarrow \boldsymbol{monthly\ loan\ payment}$$

$$\mathbf{1}\boldsymbol{b})\ Use\ uniform\ series\ present\ worth\ formula\ = \left(\frac{P}{A}, i\%, n\right) = \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$P = A * \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$to\ solve\ for\ the\ remaining\ value\ past\ 10\ years\ equivalent\ to\ the\ present\ worth$
$Present\ period\ when\ n_1 = 120\ with\ reference\ to\ the\ final\ \ n_2 = 360;$
$n = n_2 - n_1 = 240$

$$P = 3218.4905 * \left[\frac{(1+0.0075)^{240} - 1}{0.0075(1+0.0075)^{240}}\right] = \boxed{\$\mathbf{357{,}718.975}} \rightarrow \boldsymbol{remaining\ amount\ owed}$$

Question 2:

$$F = A\left[\frac{(1+i)^n - 1}{i}\right];$$

$A = \$200\ per\ year; n = 15\ ; i = 0.07$

$$\$200\left[\frac{(1+0.07)^{15} - 1}{0.07}\right] = \$5025.8044$$

$However, since\ the\ deposit\ is\ made\ at$
$\ the\ beginning\ of\ the\ year$
$an\ additional\ rate\ of\ 0.07\ interest\ will\ be\ added\ \dots\dots$

$$\$5025.8044 * (1.07) = \boxed{\$\mathbf{5377.61}}$$

interest rate		0.07
Initial deposit/Next annual deposit		200.00
Year	Beginning of year acct balance	interest earned
1	200.00	14.00
2	414.00	28.98
3	642.98	45.01
4	887.99	62.16
5	1150.15	80.51
6	1430.66	100.15
7	1730.80	121.16
8	2051.96	143.64
9	2395.60	167.69
10	2763.29	193.43
11	3156.72	220.97
12	3577.69	250.44
13	4028.13	281.97
14	4510.10	315.71
15	5025.80	351.81
16th Year Mark (After 15 years)/ No $200	5377.61	

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Question 3:

$$P_{cashflow1} = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$P_{cashflow1} = A\left[\frac{(1+0.12)^4 - 1}{0.12(1+0.12)^4}\right] = 3.073A \; units\ of\ Cash\ flow$$

$$P_{cashflow2} = P' + P''$$

$$P_{cashflow2} = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] + G\left[\frac{(1+i)^n - in - 1}{i^2(1+i)^n}\right]$$

$$P_{cashflow2} = 150\left[\frac{(1+0.12)^5 - 1}{0.12(1+0.12)^5}\right] + 150\left[\frac{(1+0.12)^5 - (0.12*5) - 1}{0.12^2(1+0.12)^5}\right] = \$540.716 + \$959.5524$$

$$= \$1500.2688$$

Solve for A in term of (\$):

$$\$1500.2688 \;=\; 3.073A \;\rightarrow \boxed{A = \$493.94}$$

Question 3	
Cash Flow 1	
i	12%
Cash flow (units of A)	$1
n	4
For reference:	
P'(Sum) (A units)	**3.0373493466**
$P' = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$	
A units of Cash Flow	3.037349347
Equivalent Amount:	$1,500.27
How much is 1 $A cash flow	$493.94

Cash Flow 2		
i	12%	
Cash flow (units of $)	150	$P'' = G\left[\frac{(1+i)^n - in - 1}{i^2(1+i)^n}\right]$
G	150	
n	5	
		$P' = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$
A'	150	
A/G	1.775	
A''	177.46	
A total	327.46	
For reference:		
P'	540.72	
P''	959.55	
Sum	**1500.268835**	

Question 3)

$$P = F\frac{1}{(1+i)^n}$$

	interest rate	12%

year	**Cash Flow option 2** **costs at the year itself**	present worth
1	150	$134
2	300	$239
3	450	$320
4	600	$381
5	$750.00	$426
Sum of PW	**Cash Flow option 2**	$1,500.27

year	**Cash Flow option 1** **costs at the year itself(Units of A)**	present worth
1	1	$1
2	1	$1
3	1	$1
4	1	$1
5		
Sum of PW	**Cash Flow option 1 (Units of A)**	3.037349347

Cash Flow option 1 (Units of A)	3.037349347
Cash Flow option 2 (equivalent v	$ 1,500.27
A value ($)	$ 493.94

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Question 4:

$\mathbf{4a}) \therefore \boldsymbol{Nominal\ annual\ interest} = \boldsymbol{r_{percent}} = \mathbf{1.25\% \times 12} = \boxed{\mathbf{15\ \%}}$

$\mathbf{4b}) \therefore \boldsymbol{Effective\ annual\ interest\ rate}: \boldsymbol{i_a} = \left(\mathbf{1} + \frac{\boldsymbol{r}}{\boldsymbol{m}}\right)^{\boldsymbol{m}} - \mathbf{1} = \left(\mathbf{1} + \frac{\mathbf{0.15}}{\mathbf{12}}\right)^{\mathbf{12}} - \mathbf{1}$

$= \boxed{\mathbf{0.1607545}}\ \boldsymbol{or}\ \boxed{\mathbf{16.07545\%}}$

$\mathbf{4c}) A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = P\left(\frac{A}{P}, i, n\right)$

$n = 12 * 4 years = 48\ periods;\ \ P = 10{,}000\ ; i = interest\ per\ period = 0.0125 = 1.25\%$

$\therefore\ 10{,}000 * \left[\frac{0.0125(1+0.0125)^{48}}{(1+0.0125)^{48} - 1}\right] = \boxed{\mathbf{\$278.3075}}$

Question 5:

Q5a)

$Option\ A \rightarrow \frac{\$300}{2\ years}\ ; require\ to\ buy\ (\times 2) to\ fit\ 3yr\ minimum\ lifespan$

$Option\ B \rightarrow \$400\ for\ inifinite\ time; require\ (\times 1) to\ fit\ 3yr\ minimum\ lifespan$

$We\ need\ to\ compare\ the\ present\ worth\ (PW) values\ of\ both\ options\ to\ evaluate\ whether$
$which\ muffler\ is\ the\ most\ economical. Assuming\ Option\ B\ is\ able\ to\ fit\ the\ 3\ yr\ life\ (minimum\ life\ span)$
$, we\ only\ need\ a\ PW\ for\ Option\ B\ to\ be\ simply\ \$400.$
$Use\ geometric\ series\ present\ worth\ factor\ where\ i = g$

$P = [\frac{F}{(1+i)^n}]$

$PW\ of\ Option\ A \rightarrow 300 + 300\left(\frac{P}{F}, 20\%, 2\right) = 300 + 300(\frac{1}{1.2^2}) = \$508.33$

$\boxed{Option\ A = \$400}$ $\boxed{Option\ B\ (PW) = \$508.33}$

$\therefore \boldsymbol{Option\ A\ (muffler\ that\ is\ \$400\ upfront\ cost\ ) is\ obviously\ the\ better\ choice}$
$\boldsymbol{since\ it\ has\ a\ lower\ present\ value\ worth}$

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Question 6:

6a)

$For\ Infinite\ analysis, we\ use\ ...$

$\boxed{A = F\left(\frac{A}{F}, i, n\right)}$ $\boxed{P\ = \frac{A}{i}} \rightarrow \boxed{P_{maintenance_1}\ or\ P_{friction} = \frac{F\left[\frac{i}{(1+i)^n - 1}\right]}{i}}$

$\boxed{P_V = \frac{(\$n_i)}{(1+i)^n} \rightarrow\ used\ for\ computing\ the\ PW\ of\ P_{maintenance_1}\ equivalent\ when\ n = 20}$

$Full\ Capacity\ Cost: (initial\ installation) + (P_{maintenance})$ $-(1)$

$\frac{1}{2}\ Capacity\ Cost\ equation_{first\ half}\ or\ \$n_{10}: (initial\ installation) + (P_{maintenance_1}) + (P_{friction})$ $-(2)$

$\frac{1}{2}\ Capacity\ Cost\ equation_{second\ half}: (P_V)\ or\ \frac{(\$n_{10})}{(1+i)^{20}}$ $-(3)$

$$Full\ Capacity\ Capitalized\ Cost: \$556{,}000 + \frac{40{,}000\left[\frac{0.07}{(1+0.07)^{10} - 1}\right]}{0.07} = \$597{,}358.573$$

$$\approx \boxed{\$597{,}360} \rightarrow PW\ of\ full\ \ tunnel$$

$$\frac{1}{2}\ Capacity\ Capitalized\ Cost\ (10\ year\ mark): \$402{,}000 + \frac{32{,}000\left[\frac{0.07}{(1+0.07)^{10} - 1}\right]}{0.07} + \frac{2000}{0.07}$$

$$= \$463{,}658.287$$

$$\frac{1}{2}\ Capacity\ Capitalized\ Cost\ (beyond\ 10\ year\ mark): \frac{\$463{,}658.287}{(1+0.07)^{20}} = \$119{,}818.112$$

$$\$119{,}818.112 + \ \$463{,}658.287 = \$583{,}476.4\ \approx \boxed{\$583{,}480} \rightarrow PW\ of\ half\ tunnel$$

$\therefore$ ***The 2 − half tunnels cost should be chosen as the PW worth of cost is lower***

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6b $\boxed{\textit{We chose a period of } n = 30 \textit{ years}}$

Question6b)

	Annual Friction Cost	10 year Maintenance
Half Tunnel	2000	32000
Full Tunnel	0	40000

$$P = F\frac{1}{(1+i)^n}$$

interest rate	7%

year	Half Tunnel costs	Half Tunnel present worth	Full Tunnel costs	Full Tunnel present worth
0	$402,000.00	$402,000.00	$556,000.00000	$556,000.00000
1	$2,000.00	$1,869.16	$0.00000	$0.00000
2	$2,000.00	$1,746.88	$0.00000	$0.00000
3	$2,000.00	$1,632.60	$0.00000	$0.00000
4	$2,000.00	$1,525.79	$0.00000	$0.00000
5	$2,000.00	$1,425.97	$0.00000	$0.00000
6	$2,000.00	$1,332.68	$0.00000	$0.00000
7	$2,000.00	$1,245.50	$0.00000	$0.00000
8	$2,000.00	$1,164.02	$0.00000	$0.00000
9	$2,000.00	$1,087.87	$0.00000	$0.00000
10	$34,000.00	$17,283.88	$40,000.00000	$20,333.97169
11	$2,000.00	$950.19	$0.00000	$0.00000
12	$2,000.00	$888.02	$0.00000	$0.00000
13	$2,000.00	$829.93	$0.00000	$0.00000
14	$2,000.00	$775.63	$0.00000	$0.00000
15	$2,000.00	$724.89	$0.00000	$0.00000
16	$2,000.00	$677.47	$0.00000	$0.00000
17	$2,000.00	$633.15	$0.00000	$0.00000
18	$2,000.00	$591.73	$0.00000	$0.00000
19	$2,000.00	$553.02	$0.00000	$0.00000
20	$436,000.00	$112,670.69	$40,000.00000	$10,336.76011
21	$4,000.00	$966.05	$0.00000	$0.00000
22	$4,000.00	$902.85	$0.00000	$0.00000
23	$4,000.00	$843.79	$0.00000	$0.00000
24	$4,000.00	$788.59	$0.00000	$0.00000
25	$4,000.00	$737.00	$0.00000	$0.00000
26	$4,000.00	$688.78	$0.00000	$0.00000
27	$4,000.00	$643.72	$0.00000	$0.00000
28	$4,000.00	$601.61	$0.00000	$0.00000
29	$4,000.00	$562.25	$0.00000	$0.00000
30	$68,000.00	$8,932.96	$40,000.00000	$5,254.68469
	Half Tunnel (Total)	**$567,276.66**	**Full Tunnel (Total)**	**$591,925.42**

∴ ***The half tunnel option will still be a cheaper and better choice***
regardless of the time frame.
This proves that when both infrustructure plans converges to infinite years, both plans will coverge to a finite cost, and the half tunnel cost will always be cheaper than the full tunnel cost

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Question 7:

The excel spreadsheet below note values in terms of cost, a positive net value means that the option would give the user a net loss in money. Whereas a negative value means that there is a net gain. Utimately we want to get the largest negative value in terms of cost to find which option is the most worth

Question 7a)

	Uniform Annual benefi	EOL Salavage
Alternative A	135	0
Alternative B	100	250
Alternative C	100	180
interest rate	8%	

$$P = F\frac{1}{(1+i)^n}$$

Alternative D	0

	Alternative A		Alternative B		Alternative C	
year	costs	present worth	costs	present worth	costs	present worth
0	$500	$500	$600.00	$600.00	$700.00	$700.00
1	-135	-$125	-$100.00	-$92.59	-$100.00	-$92.59
2	-135	-$116	-$100.00	-$85.73	-$100.00	-$85.73
3	-135	-$107	-$100.00	-$79.38	-$100.00	-$79.38
4	-135	-$99	-$100.00	-$73.50	-$100.00	-$73.50
5	$365.00	$248	$250.00	$170.15	-$100.00	-$68.06
6	-135	-$85	-$100.00	-$63.02	-$100.00	-$63.02
7	-135	-$79	-$100.00	-$58.35	-$100.00	-$58.35
8	-135	-$73	-$100.00	-$54.03	-$100.00	-$54.03
9	-135	-$68	-$100.00	-$50.02	-$100.00	-$50.02
10	-$135.00	-$63	-$350.00	-$162.12	-$280.00	-$129.69
Sum of PW	Option A	-$65.57	Option B	$ 51.40	Option C	-$54.38

∴ Alternative A is the most viable as there will be a net gain of $65.57 of income after the 10 year period, when interest is 8%

Question 7b)

	Uniform Annual benefi	EOL Salavage
Alternative A	135	0
Alternative B	100	250
Alternative C	100	180
interest rate	12%	

$$P = F\frac{1}{(1+i)^n}$$

Alternative D	0

	Alternative A		Alternative B		Alternative C	
year	costs	present worth	costs	present worth	costs	present worth
0	$500	$500	$600.00	$600.00	$700.00	$700.00
1	-135	-$121	-$100.00	-$89.29	-$100.00	-$89.29
2	-135	-$108	-$100.00	-$79.72	-$100.00	-$79.72
3	-135	-$96	-$100.00	-$71.18	-$100.00	-$71.18
4	-135	-$86	-$100.00	-$63.55	-$100.00	-$63.55
5	$365.00	$207	$250.00	$141.86	-$100.00	-$56.74
6	-135	-$68	-$100.00	-$50.66	-$100.00	-$50.66
7	-135	-$61	-$100.00	-$45.23	-$100.00	-$45.23
8	-135	-$55	-$100.00	-$40.39	-$100.00	-$40.39
9	-135	-$49	-$100.00	-$36.06	-$100.00	-$36.06
10	-$135.00	-$43	-$350.00	-$112.69	-$280.00	-$90.15
Sum of PW	Option A	$20.93	Option B	$ 153.08	Option C	$77.02

∴ Alternative D is the most viable as the option wouldn't lose money compared to the other options, when the interest is 12%

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Question 8:

$$Calculating\ \ present\ worth\ of\ compounded\ regular\ payment = A * \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$Let\ A\ be\ the\ reference\ or\ the\ annual\ payment\ for\ fuel\ usage$
$n = 20\ ,\ \ i = 0.08$

$$\boldsymbol{PW_{gas}} = \$\mathbf{30,000} + (\boldsymbol{A} + \mathbf{7500}) * \left[\frac{\mathbf{(1+0.08)^{20}-1}}{\mathbf{0.08(1+0.08)^{20}}}\right] = \$\mathbf{103,636.11} + \$\cancel{\mathbf{9.818815}\boldsymbol{A}}$$

$$\boldsymbol{PW_{fuel}} = \$\mathbf{55,000} + \boldsymbol{A} * \left[\frac{\mathbf{(1+0.08)^{20}-1}}{\mathbf{0.08(1+0.08)^{20}}}\right] = \$\mathbf{55,000} + \ \$\cancel{\mathbf{9.818815}\boldsymbol{A}}$$

$$\boldsymbol{PW_{coal}} = \$\mathbf{180,000} + (\boldsymbol{A} - \mathbf{15000}) * \left[\frac{\mathbf{(1+0.08)^{20}-1}}{\mathbf{0.08(1+0.08)^{20}}}\right] = \boxed{\$\mathbf{32,727.788}} + \ \$\cancel{\mathbf{9.818815}\boldsymbol{A}}$$

$\therefore$ ***<u>By calculating the present value worth of energy resources, we have determine coal resource to use the least in expenditure</u>*** $\rightarrow \$\mathbf{32,727.788}$