Progetto - Fondamenti di informatica

Francesco Andreuzzi
IN0500630

Anno 2018-2019

1 Calcolo della funzione

Ricavo i valori assunti dalla funzione $f(x, y, z, k)$ dal resto della divisione del numero di matricola (0500630) per 2^{16}:

$(500630 \mod 65536) = 41878$
$41878_{10} = 1010001110010110_2$ $\longrightarrow$

x	**y**	**z**	**k**	**f(x,y,z,k)**
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

Minterm

Per ottenere la prima forma canonica della funzione, riscrivo le combinazioni (x, y, z, k) in cui la funzione assume valore 1:

x	**y**	**z**	**k**	**f(x,y,z,k)**
0	0	0	0	1
0	0	1	0	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	1

La funzione $f(x, y, z, k)$ si può esprimere come somma di prodotti nel seguente modo:

$$\boldsymbol{f(x,y,z,k)} = (\overline{x} \cdot \overline{y} \cdot \overline{z} \cdot \overline{k}) + (\overline{x} \cdot \overline{y} \cdot z \cdot \overline{k}) + (\overline{x} \cdot y \cdot z \cdot \overline{k}) + (\overline{x} \cdot y \cdot z \cdot k) + (x \cdot \overline{y} \cdot \overline{z} \cdot \overline{k}) + (x \cdot \overline{y} \cdot z \cdot k) + \\ + (x \cdot y \cdot \overline{z} \cdot k) + (x \cdot y \cdot z \cdot \overline{k})$$

Maxterm

Per ottenere la seconda forma canonica della funzione, riscrivo le combinazioni (x, y, z, k) in cui la funzione assume valore 0:

x	**y**	**z**	**k**	**f(x,y,z,k)**
0	0	0	1	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	0

La funzione $f(x, y, z, k)$ si può esprimere come prodotto di somme nel seguente modo:

$$\boldsymbol{f(x,y,z,k)} = (x + y + z + \overline{k}) \cdot (x + y + \overline{z} + \overline{k}) \cdot (x + \overline{y} + z + k) \cdot (x + \overline{y} + z + \overline{k}) \cdot (\overline{x} + y + z + \overline{k}) \cdot \\ \cdot (\overline{x} + y + \overline{z} + k) \cdot (\overline{x} + \overline{y} + z + k) \cdot (\overline{x} + \overline{y} + \overline{z} + \overline{k})$$

2 Semplificazione

Semplificazione algebrica

Minterm

$$
\begin{aligned}
\boldsymbol{f(x,y,z,k)} &= \underline{(\bar{x}\cdot\bar{y}\cdot\bar{z}\cdot\bar{k})} + (\bar{x}\cdot\bar{y}\cdot z\cdot\bar{k}) + (\bar{x}\cdot y\cdot z\cdot\bar{k}) + (\bar{x}\cdot y\cdot z\cdot k) + \underline{(x\cdot\bar{y}\cdot\bar{z}\cdot\bar{k})} + (x\cdot\bar{y}\cdot z\cdot k)+ \\
&\qquad + (x\cdot y\cdot\bar{z}\cdot k) + (x\cdot y\cdot z\cdot\bar{k}) \\
&\overset{T9}{=} \boldsymbol{(\bar{y}\cdot\bar{z}\cdot\bar{k})} + (\bar{x}\cdot\bar{y}\cdot z\cdot\bar{k}) + \underline{(\bar{x}\cdot y\cdot z\cdot\bar{k})} + (\bar{x}\cdot y\cdot z\cdot k) + (x\cdot\bar{y}\cdot z\cdot k) + (x\cdot y\cdot\bar{z}\cdot k) \\
&\qquad + (x\cdot y\cdot z\cdot\bar{k}) \\
&\overset{T1}{=} (\bar{y}\cdot\bar{z}\cdot\bar{k}) + \underline{(\bar{x}\cdot\bar{y}\cdot z\cdot\bar{k})} + \underline{\boldsymbol{(\bar{x}\cdot y\cdot z\cdot\bar{k})}} \boldsymbol{+ (\bar{x}\cdot y\cdot z\cdot\bar{k}) + (\bar{x}\cdot y\cdot z\cdot\bar{k})} + (\bar{x}\cdot y\cdot z\cdot k)+ \\
&\qquad + (x\cdot\bar{y}\cdot z\cdot k) + (x\cdot y\cdot\bar{z}\cdot k) + (x\cdot y\cdot z\cdot\bar{k}) \\
&\overset{T9}{=} (\bar{y}\cdot\bar{z}\cdot\bar{k}) + \boldsymbol{(\bar{x}\cdot z\cdot\bar{k})} + \underline{(\bar{x}\cdot y\cdot z\cdot\bar{k})} + (\bar{x}\cdot y\cdot z\cdot\bar{k}) + \underline{(\bar{x}\cdot y\cdot z\cdot k)} + (x\cdot\bar{y}\cdot z\cdot k)+ \\
&\qquad + (x\cdot y\cdot\bar{z}\cdot k) + (x\cdot y\cdot z\cdot\bar{k}) \\
&\overset{T9}{=} (\bar{y}\cdot\bar{z}\cdot\bar{k}) + (\bar{x}\cdot z\cdot\bar{k}) + \boldsymbol{(\bar{x}\cdot y\cdot z)} + \underline{(\bar{x}\cdot y\cdot z\cdot\bar{k})} + (x\cdot\bar{y}\cdot z\cdot k) + (x\cdot y\cdot\bar{z}\cdot k)+ \\
&\qquad + \underline{(x\cdot y\cdot z\cdot\bar{k})} \\
&\overset{T9}{=} (\bar{y}\cdot\bar{z}\cdot\bar{k}) + (\bar{x}\cdot z\cdot\bar{k}) + (\bar{x}\cdot y\cdot z) + \boldsymbol{(y\cdot z\cdot\bar{k})} + (x\cdot\bar{y}\cdot z\cdot k) + (x\cdot y\cdot\bar{z}\cdot k)
\end{aligned}
$$

Maxterm

$$
\begin{aligned}
\boldsymbol{f(x,y,z,k)} &= \underline{(x+y+z+\bar{k}}) \cdot (x+y+\bar{z}+\bar{k}) \cdot (x+\bar{y}+z+k) \cdot (x+\bar{y}+z+\bar{k}) \cdot (\bar{x}+y+z+\bar{k})\cdot \\
&\qquad \cdot (\bar{x}+y+\bar{z}+k) \cdot (\bar{x}+\bar{y}+z+k) \cdot (\bar{x}+\bar{y}+\bar{z}+\bar{k}) \\
&\overset{T1}{=} \underline{\boldsymbol{(x+y+z+\bar{k})}} \boldsymbol{\cdot (x+y+z+\bar{k}) \cdot (x+y+z+\bar{k})} \cdot \underline{(x+y+\bar{z}+\bar{k})} \cdot (x+\bar{y}+z+k)\cdot \\
&\qquad \cdot (x+\bar{y}+z+\bar{k}) \cdot (\bar{x}+y+z+\bar{k}) \cdot (\bar{x}+y+\bar{z}+k) \cdot (\bar{x}+\bar{y}+z+k) \cdot (\bar{x}+\bar{y}+\bar{z}+\bar{k}) \\
&\overset{T9}{=} \boldsymbol{(x+y+\bar{k})} \cdot \underline{(x+y+z+\bar{k})} \cdot (x+y+z+\bar{k}) \cdot (x+\bar{y}+z+k) \cdot \underline{(x+\bar{y}+z+\bar{k})}\cdot \\
&\qquad \cdot (\bar{x}+y+z+\bar{k}) \cdot (\bar{x}+y+\bar{z}+k) \cdot (\bar{x}+\bar{y}+z+k) \cdot (\bar{x}+\bar{y}+\bar{z}+\bar{k}) \\
&\overset{T9}{=} (x+y+\bar{k}) \cdot \boldsymbol{(x+z+\bar{k})} \cdot \underline{(x+y+z+\bar{k})} \cdot (x+\bar{y}+z+k) \cdot \underline{(\bar{x}+y+z+\bar{k})}\cdot \\
&\qquad \cdot (\bar{x}+y+\bar{z}+k) \cdot (\bar{x}+\bar{y}+z+k) \cdot (\bar{x}+\bar{y}+\bar{z}+\bar{k}) \\
&\overset{T9}{=} (x+y+\bar{k}) \cdot (x+z+\bar{k}) \cdot \boldsymbol{(y+z+\bar{k})} \cdot \underline{(x+\bar{y}+z+k)} \cdot (\bar{x}+y+\bar{z}+k) \cdot \underline{(\bar{x}+\bar{y}+z+k)}\cdot \\
&\qquad \cdot (\bar{x}+\bar{y}+\bar{z}+\bar{k}) \\
&\overset{T9}{=} \underline{(x+y+\bar{k})} \cdot \underline{(x+z+\bar{k})} \cdot (y+z+\bar{k}) \cdot \boldsymbol{(\bar{y}+z+k)} \cdot (\bar{x}+y+\bar{z}+k) \cdot (\bar{x}+\bar{y}+\bar{z}+\bar{k}) \\
&\overset{A6}{=} [\underline{\boldsymbol{x\cdot(x+z+\bar{k})}} \boldsymbol{+ y\cdot(x+z+\bar{k}) +} \underline{\boldsymbol{\bar{k}\cdot(x+z+\bar{k})}}] \cdot (y+z+\bar{k}) \cdot (\bar{y}+z+k)\cdot \\
&\qquad \cdot (\bar{x}+y+\bar{z}+k) \cdot (\bar{x}+\bar{y}+\bar{z}+\bar{k})
\end{aligned}
$$

$$\boldsymbol{f(x,y,z,k)} = \ldots$$

$$\overset{T8}{=} [\boldsymbol{x} + \underline{y\cdot(x+z+\overline{k})} + \boldsymbol{\overline{k}}]\cdot(y+z+\overline{k})\cdot(\overline{y}+z+k)\cdot(\overline{x}+y+\overline{z}+k)\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})$$

$$\overset{A6}{=} [\underline{x+\boldsymbol{x\cdot y}}\boldsymbol{+y\cdot z+}\underline{\boldsymbol{y\cdot\overline{k}}+\overline{k}}]\cdot(y+z+\overline{k})\cdot(\overline{y}+z+k)\cdot(\overline{x}+y+\overline{z}+k)\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})$$

$$\overset{T4}{=} (\boldsymbol{x}+y\cdot z+\boldsymbol{\overline{k}})\cdot\underline{(y+z+\overline{k})}\cdot\underline{(\overline{y}+z+k)}\cdot(\overline{x}+y+\overline{z}+k)\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})$$

$$\begin{aligned}\overset{A6}{=} &(x+y\cdot z+\overline{k})\cdot[\boldsymbol{y\cdot(\overline{y}+z+k)+}\underline{\boldsymbol{z\cdot(\overline{y}+z+k)}}\boldsymbol{+\overline{k}\cdot(\overline{y}+z+k)}]\cdot(\overline{x}+y+\overline{z}+k)\cdot\\ &\quad\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})\end{aligned}$$

$$\overset{T8}{=} (x+y\cdot z+\overline{k})\cdot[\underline{y\cdot(\overline{y}+z+k)}+\boldsymbol{z}+\underline{\overline{k}\cdot(\overline{y}+z+k)}]\cdot(\overline{x}+y+\overline{z}+k)\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})$$

$$\begin{aligned}\overset{A6}{=} &(x+y\cdot z+\overline{k})\cdot[\underline{\boldsymbol{y\cdot\overline{y}}}\boldsymbol{+y\cdot z+y\cdot k}+z+\boldsymbol{\overline{y}\cdot\overline{k}+z\cdot\overline{k}+}\underline{\boldsymbol{\overline{k}\cdot k}}]\cdot(\overline{x}+y+\overline{z}+k)\cdot\\ &\quad\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})\end{aligned}$$

$$\overset{A7}{=} (x+y\cdot z+\overline{k})\cdot[\mathbf{0}+\underline{z+y\cdot z+z\cdot\overline{k}}+y\cdot k+\overline{y}\cdot\overline{k}+\mathbf{0}]\cdot(\overline{x}+y+\overline{z}+k)\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})$$

$$\overset{T4}{=} \underline{(x+y\cdot z+\overline{k})}\cdot\underline{(y\cdot k+\boldsymbol{z}+\overline{y}\cdot\overline{k})}\cdot(\overline{x}+y+\overline{z}+k)\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})$$

$$\begin{aligned}\overset{A6}{=} &[\underline{\boldsymbol{x\cdot(y\cdot k+z+\overline{y}\cdot\overline{k})}}\boldsymbol{+}\underline{\boldsymbol{(y\cdot z)\cdot(y\cdot k+z+\overline{y}\cdot\overline{k})}}\boldsymbol{+}\underline{\boldsymbol{\overline{k}\cdot(y\cdot k+z+\overline{y}\cdot\overline{k})}}]\cdot\\ &\quad\cdot(\overline{x}+y+\overline{z}+k)\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})\end{aligned}$$

$$\begin{aligned}\overset{A6}{=} &(\boldsymbol{x\cdot y\cdot k+x\cdot z+x\cdot\overline{y}\cdot\overline{k}+y\cdot z\cdot k+y\cdot z+}\underline{\boldsymbol{y\cdot z\cdot\overline{y}\cdot\overline{k}}}\boldsymbol{+}\underline{\boldsymbol{y\cdot\overline{k}\cdot k}}\boldsymbol{+z\cdot\overline{k}+\overline{y}\cdot\overline{k}})\cdot\\ &\quad\cdot(\overline{x}+y+\overline{z}+k)\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})\end{aligned}$$

$$\begin{aligned}\overset{A7}{=} &(x\cdot y\cdot k+x\cdot z+x\cdot\overline{y}\cdot\overline{k}+\underline{y\cdot z\cdot k+y\cdot z}+\mathbf{0}+\mathbf{0}+z\cdot\overline{k}+\overline{y}\cdot\overline{k})\cdot(\overline{x}+y+\overline{z}+k)\cdot\\ &\quad\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})\end{aligned}$$

$$\overset{T4}{=} \underline{(x\cdot y\cdot k+x\cdot z+x\cdot\overline{y}\cdot\overline{k}+\boldsymbol{y\cdot z}+z\cdot\overline{k}+\overline{y}\cdot\overline{k})}\cdot\underline{(\overline{x}+y+\overline{z}+k)}\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})$$

$$\begin{aligned}\overset{A6}{=} &[\underline{\boldsymbol{x\cdot y\cdot k\cdot(\overline{x}+y+\overline{z}+k)}}\boldsymbol{+}\underline{\boldsymbol{x\cdot z\cdot(\overline{x}+y+\overline{z}+k)}}\boldsymbol{+}\underline{\boldsymbol{x\cdot\overline{y}\cdot\overline{k}\cdot(\overline{x}+y+\overline{z}+k)}}\boldsymbol{+}\\ &\quad\boldsymbol{+}\underline{\boldsymbol{y\cdot z\cdot(\overline{x}+y+\overline{z}+k)}}\boldsymbol{+}\underline{\boldsymbol{z\cdot\overline{k}\cdot(\overline{x}+y+\overline{z}+k)}}\boldsymbol{+}\underline{\boldsymbol{\overline{y}\cdot\overline{k}\cdot(\overline{x}+y+\overline{z}+k)}}]\cdot\\ &\quad\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})\end{aligned}$$

1 $$\begin{aligned}\overset{A6}{=} &[\boldsymbol{x\cdot y\cdot k+x\cdot y\cdot\overline{z}\cdot k+x\cdot y\cdot k+x\cdot y\cdot z+x\cdot z\cdot k+x\cdot\overline{y}\cdot\overline{z}\cdot\overline{k}+\overline{x}\cdot y\cdot z+y\cdot z+}\\ &\quad\boldsymbol{+y\cdot z\cdot k+\overline{x}\cdot z\cdot\overline{k}+y\cdot z\cdot\overline{k}+\overline{x}\cdot\overline{y}\cdot\overline{k}+\overline{y}\cdot\overline{z}\cdot\overline{k}}]\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})\end{aligned}$$

$$\begin{aligned}\overset{A4}{=} &[\underline{x\cdot y\cdot k+x\cdot y\cdot\overline{z}\cdot k+x\cdot y\cdot k}+x\cdot z\cdot k+\underline{y\cdot z+x\cdot y\cdot z+y\cdot z\cdot k+\overline{x}\cdot y\cdot z+y\cdot z\cdot\overline{k}}+\\ &\quad+\underline{\overline{y}\cdot\overline{z}\cdot\overline{k}+x\cdot\overline{y}\cdot\overline{z}\cdot\overline{k}}+\overline{x}\cdot z\cdot\overline{k}+\overline{x}\cdot\overline{y}\cdot\overline{k}]\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})\end{aligned}$$

$$\overset{T4}{=} \underline{(\boldsymbol{x\cdot y\cdot k}+x\cdot z\cdot k+\boldsymbol{y\cdot z}+\overline{x}\cdot z\cdot\overline{k}+\overline{x}\cdot\overline{y}\cdot\overline{k}+\boldsymbol{\overline{y}\cdot\overline{z}\cdot\overline{k}})}\cdot\underline{(\overline{x}+\overline{y}+\overline{z}+\overline{k})}$$

$$\begin{aligned}\overset{A6}{=} &[\boldsymbol{x\cdot y\cdot k\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})+x\cdot z\cdot k\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})+y\cdot z\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})+}\\ &\quad\boldsymbol{+\overline{x}\cdot z\cdot\overline{k}\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})+\overline{x}\cdot\overline{y}\cdot\overline{k}\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})+}\\ &\quad\boldsymbol{+\overline{y}\cdot\overline{z}\cdot\overline{k}\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})}]\end{aligned}$$

$$
\begin{aligned}
\boldsymbol{f(x,y,z,k)} &= \ldots \\
&\overset{A6}{=}{}^{1} \boldsymbol{(x\cdot y\cdot \bar{z}\cdot k + x\cdot \bar{y}\cdot z\cdot k + \bar{x}\cdot y\cdot z + y\cdot z\cdot \bar{k} + \underline{\bar{x}\cdot z\cdot \bar{k} + \bar{x}\cdot \bar{y}\cdot z\cdot \bar{k} + \bar{x}\cdot z\cdot \bar{k}}+} \\
&\qquad \boldsymbol{\underline{+\bar{x}\cdot \bar{y}\cdot \bar{k} + \bar{x}\cdot \bar{y}\cdot \bar{k} + \bar{x}\cdot \bar{y}\cdot \bar{z}\cdot \bar{k} + \bar{x}\cdot \bar{y}\cdot \bar{k} + \bar{x}\cdot \bar{y}\cdot \bar{z}\cdot \bar{k}} + \underline{\bar{y}\cdot \bar{z}\cdot \bar{k}+}} \\
&\qquad \boldsymbol{\underline{+\bar{y}\cdot \bar{z}\cdot \bar{k} + \bar{y}\cdot \bar{z}\cdot \bar{k}})} \\
&\overset{T4}{=} (x\cdot y\cdot \bar{z}\cdot k + x\cdot \bar{y}\cdot z\cdot k + \underline{\bar{x}\cdot y\cdot z} + \underline{y\cdot z\cdot \bar{k}} + \underline{\boldsymbol{\bar{x}\cdot z\cdot \bar{k}}} + \underline{\boldsymbol{\bar{x}\cdot \bar{y}\cdot \bar{k}}} + \underline{\boldsymbol{\bar{y}\cdot \bar{z}\cdot \bar{k}}}) \\
&\overset{T9}{=} (x\cdot y\cdot \bar{z}\cdot k + x\cdot \bar{y}\cdot z\cdot k + \boldsymbol{\bar{x}\cdot y\cdot z\cdot k + \bar{x}\cdot y\cdot z\cdot \bar{k}} + \boldsymbol{x\cdot y\cdot z\cdot \bar{k} + \bar{x}\cdot y\cdot z\cdot \bar{k}}+ \\
&\qquad + \boldsymbol{\bar{x}\cdot y\cdot z\cdot \bar{k} + \bar{x}\cdot \bar{y}\cdot z\cdot \bar{k}} + \boldsymbol{\bar{x}\cdot \bar{y}\cdot z\cdot \bar{k} + \bar{x}\cdot \bar{y}\cdot \bar{z}\cdot \bar{k}} + \boldsymbol{x\cdot \bar{y}\cdot \bar{z}\cdot \bar{k} + \bar{x}\cdot \bar{y}\cdot \bar{z}\cdot \bar{k}}) \\
&\overset{A4}{=} (x\cdot y\cdot \bar{z}\cdot k + x\cdot \bar{y}\cdot z\cdot k + \underline{\bar{x}\cdot y\cdot z\cdot \bar{k} + \bar{x}\cdot y\cdot z\cdot \bar{k} + \bar{x}\cdot y\cdot z\cdot \bar{k}} + \underline{\bar{x}\cdot \bar{y}\cdot \bar{z}\cdot \bar{k} + \bar{x}\cdot \bar{y}\cdot \bar{z}\cdot \bar{k}} \\
&\qquad + \bar{x}\cdot y\cdot z\cdot k + x\cdot y\cdot z\cdot \bar{k} + \underline{\bar{x}\cdot \bar{y}\cdot z\cdot \bar{k} + \bar{x}\cdot \bar{y}\cdot z\cdot \bar{k}} + x\cdot \bar{y}\cdot \bar{z}\cdot \bar{k}) \\
&\overset{A7}{=} (x\cdot y\cdot \bar{z}\cdot k + x\cdot \bar{y}\cdot z\cdot k + \bar{x}\cdot y\cdot z\cdot k + \boldsymbol{\bar{x}\cdot y\cdot z\cdot \bar{k}} + x\cdot y\cdot z\cdot \bar{k} + \boldsymbol{\bar{x}\cdot \bar{y}\cdot z\cdot \bar{k}} + \boldsymbol{\bar{x}\cdot \bar{y}\cdot \bar{z}\cdot \bar{k}}+ \\
&\qquad + x\cdot \bar{y}\cdot \bar{z}\cdot \bar{k}) \\
&\overset{A4}{=} (\bar{x}\cdot \bar{y}\cdot \bar{z}\cdot \bar{k} + \bar{x}\cdot \bar{y}\cdot z\cdot \bar{k} + \bar{x}\cdot y\cdot z\cdot \bar{k} + \bar{x}\cdot y\cdot z\cdot k + x\cdot \bar{y}\cdot \bar{z}\cdot \bar{k} + x\cdot \bar{y}\cdot z\cdot k + x\cdot y\cdot \bar{z}\cdot k+ \\
&\qquad + x\cdot y\cdot z\cdot \bar{k}) \\
&= \ldots^{2}
\end{aligned}
$$

1. Sono state omesse le combinazioni che si annullano per l'assioma $A7$
2. La semplificazione può procedere come si è fatto sopra per i minterm

Mappa di Karnaugh

x	y	z	k	f(x,y,z,k)
0	0	0	0	1
0	0	1	0	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	1

$zk \backslash xy$	00	01	11	10
00	1	0	0	1
01	0	0	1	0
11	0	1	0	1
10	1	1	1	0

$\bar{x}\bar{y}\bar{z}\bar{k} + x\bar{y}\bar{z}\bar{k} = (x+\bar{x})\bar{y}\bar{z}\bar{k}$

$xy\bar{z}k$

$\bar{x}yzk + \bar{x}yz\bar{k} = \bar{x}yz(k+\bar{k})$

$x\bar{y}zk$

$\bar{x}\bar{y}z\bar{k} + \bar{x}yz\bar{k} = \bar{x}z\bar{k}(\bar{y}+y)$

$xyz\bar{k} + \bar{x}yz\bar{k} = (x+\bar{x})yz\bar{k}$

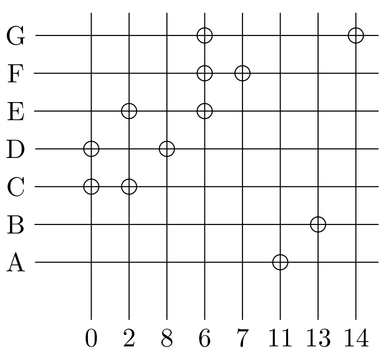
$$\Longrightarrow \boldsymbol{f(x,y,z,k)} = (\bar{y}\cdot \bar{z}\cdot \bar{k}) + (x\cdot y\cdot \bar{z}\cdot k) + (\bar{x}\cdot z\cdot \bar{k}) + (\bar{x}\cdot y\cdot z) + (y\cdot z\cdot \bar{k}) + (x\cdot \bar{y}\cdot z\cdot k)$$

Metodo tabellare di Quine - Mc Cluskey

x	y	z	k	f(x,y,z,k)
0	0	0	0	1
0	0	1	0	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	1

	Livello		$\boldsymbol{xyzk}$
	0	0	0000
	1	2	0010
		8	1000
	2	6	0110
	3	7	0111
A		11	1011
B		13	1101
		14	1110

		$\boldsymbol{xyzk}$	
C	0,2	00-0	$\overline{x}\overline{y}\overline{k}(z+\overline{z})$
D	0,8	-000	$(x+\overline{x})\overline{y}\overline{z}\overline{k}$
E	2,6	0-10	$\overline{x}z\overline{k}(y+\overline{y})$
F	6,7	011-	$\overline{x}yz(k+\overline{k})$
G	6,14	-110	$(x+\overline{x})yz\overline{k}$



Implicante	**Implicati**	**Espressione**
A	11	$x \cdot \overline{y} \cdot z \cdot k$
B	13	$x \cdot y \cdot \overline{z} \cdot k$
G	6,14	$y \cdot z \cdot \overline{k}$
F	6,7	$\overline{x} \cdot y \cdot z$
D	0,8	$\overline{y} \cdot \overline{z} \cdot \overline{k}$
E	0,2	$\overline{x} \cdot z \cdot \overline{k}$

Implicanti primi necessari

Per coprire il termine 2 posso scegliere l'implicante C oppure l'implicante E. Scegliendo l'implicante E mi riconduco all'espressione della funzione trovata con la mappa di Karnaugh.

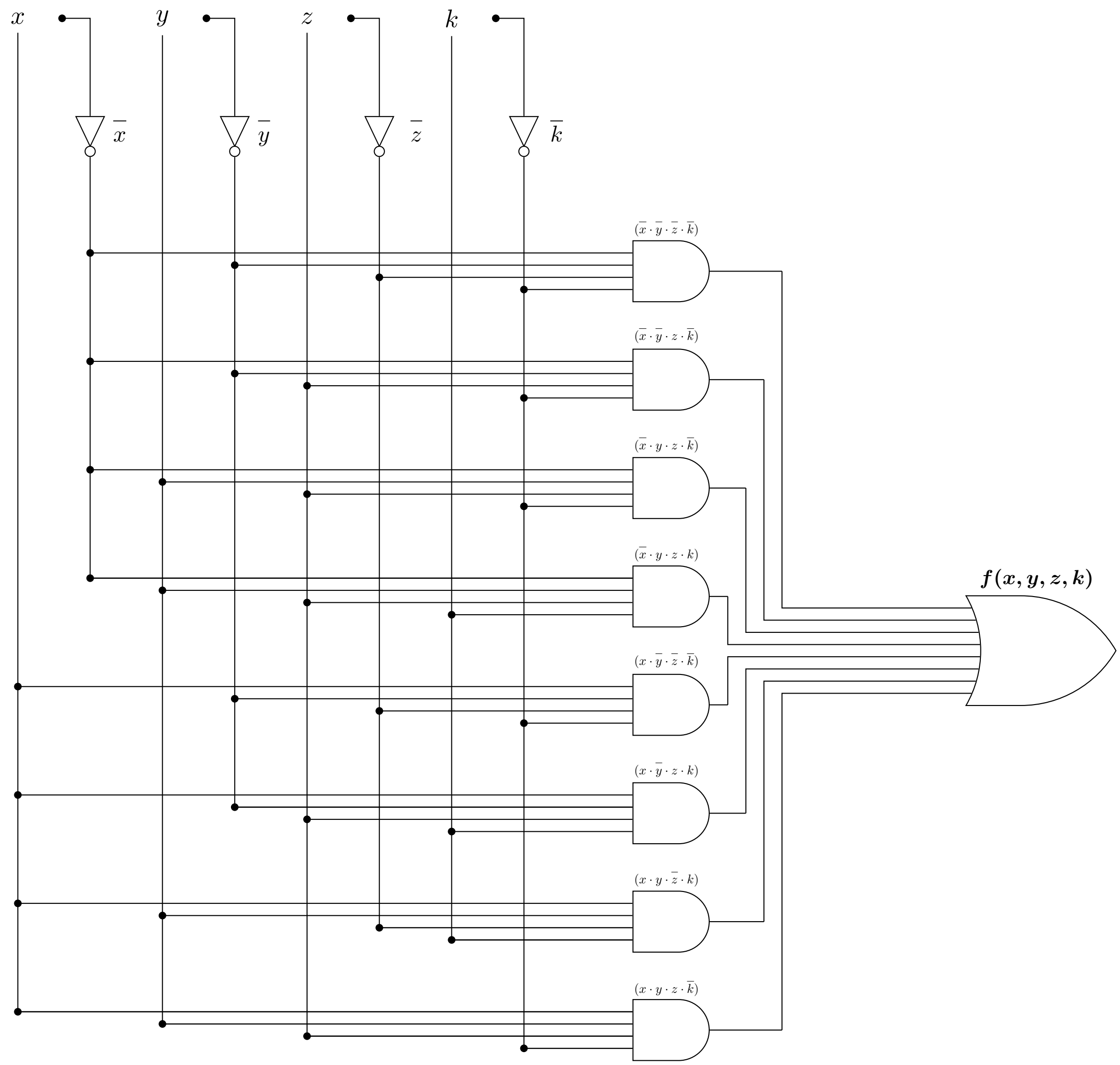
La funzione ottenuta è la seguente:

$$\boldsymbol{f(x,y,z,k)} = \underbrace{(\overline{y} \cdot \overline{z} \cdot \overline{k})}_{\text{D}} + \underbrace{(x \cdot y \cdot \overline{z} \cdot k)}_{\text{B}} + \underbrace{(x \cdot \overline{y} \cdot z \cdot k)}_{\text{A}} + \underbrace{(\overline{x} \cdot y \cdot z)}_{\text{F}} + \underbrace{(\overline{x} \cdot z \cdot \overline{k})}_{\text{E}} + \underbrace{(y \cdot z \cdot \overline{k})}_{\text{G}}$$

3 Schema logico

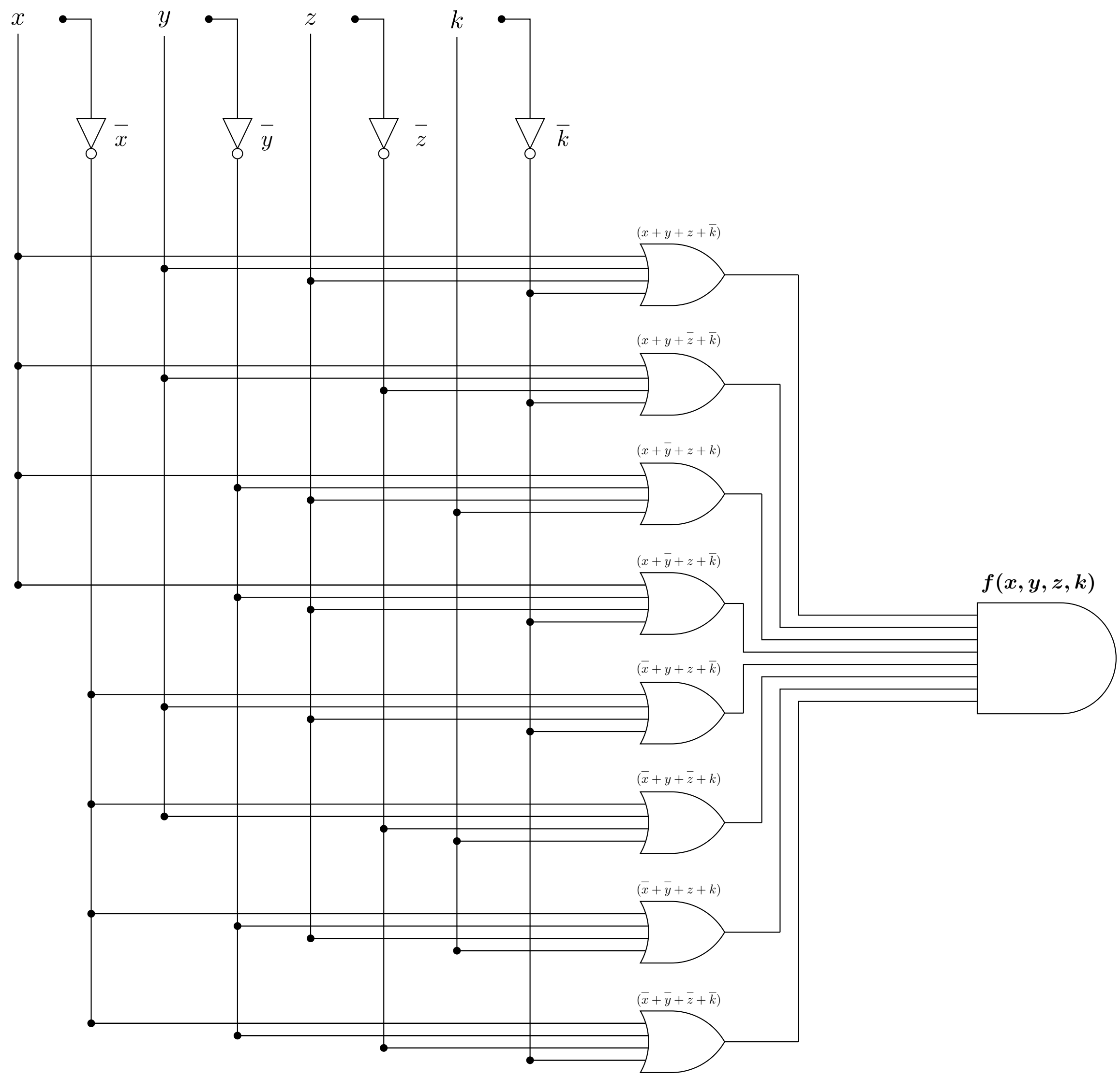
Minterm:

$$\boldsymbol{f(x,y,z,k)} = (\overline{x}\cdot\overline{y}\cdot\overline{z}\cdot\overline{k}) + (\overline{x}\cdot\overline{y}\cdot z\cdot\overline{k}) + (\overline{x}\cdot y\cdot z\cdot\overline{k}) + (\overline{x}\cdot y\cdot z\cdot k) + (x\cdot\overline{y}\cdot\overline{z}\cdot\overline{k}) + (x\cdot\overline{y}\cdot z\cdot k) + \\ + (x\cdot y\cdot\overline{z}\cdot k) + (x\cdot y\cdot z\cdot\overline{k})$$



Maxterm:

$$\boldsymbol{f(x,y,z,k)} = (x+y+z+\overline{k})\cdot(x+y+\overline{z}+\overline{k})\cdot(x+\overline{y}+z+k)\cdot(x+\overline{y}+z+\overline{k})\cdot(\overline{x}+y+z+\overline{k})\cdot$$
$$\cdot(\overline{x}+y+\overline{z}+k)\cdot(\overline{x}+\overline{y}+z+k)\cdot(\overline{x}+\overline{y}+\overline{z}+\overline{k})$$



Funzione semplificata:

$\boldsymbol{f(x,y,z,k)} = (\overline{y}\cdot\overline{z}\cdot\overline{k}) + (x\cdot y\cdot\overline{z}\cdot k) + (x\cdot\overline{y}\cdot z\cdot k) + (\overline{x}\cdot y\cdot z) + (\overline{x}\cdot z\cdot\overline{k}) + (y\cdot z\cdot\overline{k})$