**Question 4.**

Algorithms Assignment 1

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# 1 Part a

True, for all $k > 5$ and $n \geq 0$, $k \cdot n^3$ is greater than $5n^3$. Hence $5n^3 \epsilon O(n^3)$

# 2 Part b

True
This condition is true as $100n^2 < n^4$ for some $n > 10$
e.g. $100 \times 11^2 < k \times 11^4$ for all $k \geq 1$
Since there exists a k, for which $100n^2 < k \cdot n^4$ is true for all $n \geq 0$, $100n^2 \epsilon O(n^4)$

# 3 Part c

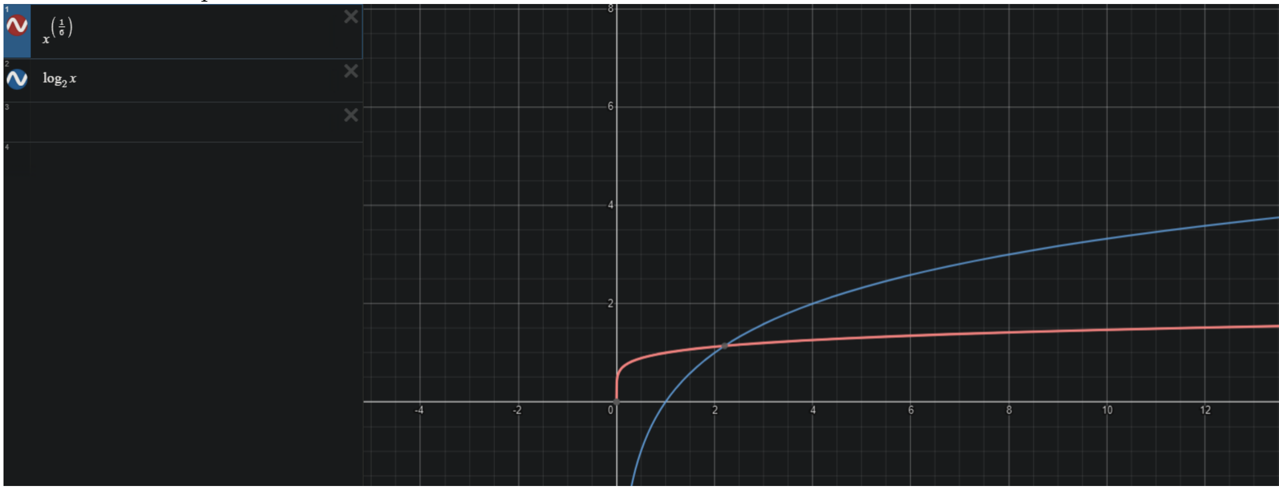
True
$\log n^2 = 2 \log n$
$2 \log n < k \cdot \log n$ for any $k > 2$
Since there exists a k, for which $2 \log n < k$ is true for all $n \geq 0$, $\log n^2 \epsilon O(\log n)$

# 4 Part d

True
The largest term in the above expression (once expanded) will be $(n^2)^3 = n^6$. All other terms will be smaller. Hence, there exists a constant k for which $k \cdot n^6$ is greater than the expression given, and the function will be $O(n^6)$

# 5 Part e

True In this part, it will be helpful to consider the bounds of the two functions. Before that, we can do some simplification.
$\sqrt{n} \le k \cdot (\log n)^3$
$\sqrt[6]{n} \le k_2 \cdot \log n$
We can now compare the two functions.




The point of intersection is at about a value of $n = 2.205$. The functions then diverge permanently, and $logn$ stays larger than $\sqrt[6]{n}$. This means that $\sqrt[6]{n} \le k \cdot \log n$ for values of k, and all values of n larger than 2.205.