

# Building Linear Regression Models Using TensorFlow

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# Implementing Regression in TensorFlow

## Baseline

Non-TensorFlow implementation  
Regular python code

## Cost Function

Mean Square Error (MSE)  
Quantifying goodness-of-fit

## Training

Invoke optimizer in epochs  
Batch size for each epoch

## Computation Graph

Neural network of 1 neuron  
Affine transformation suffices

## Optimizer

Gradient Descent optimizers  
Improving goodness-of-fit

## Converged Model

Values of  $W$  and  $b$   
Compare to baseline

# Simple Regression



**Cause**

Changes in S&P 500 equity index



**Effect**

Changes in price of Google stock

# Implementing Regression in TensorFlow

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# Regression in Python

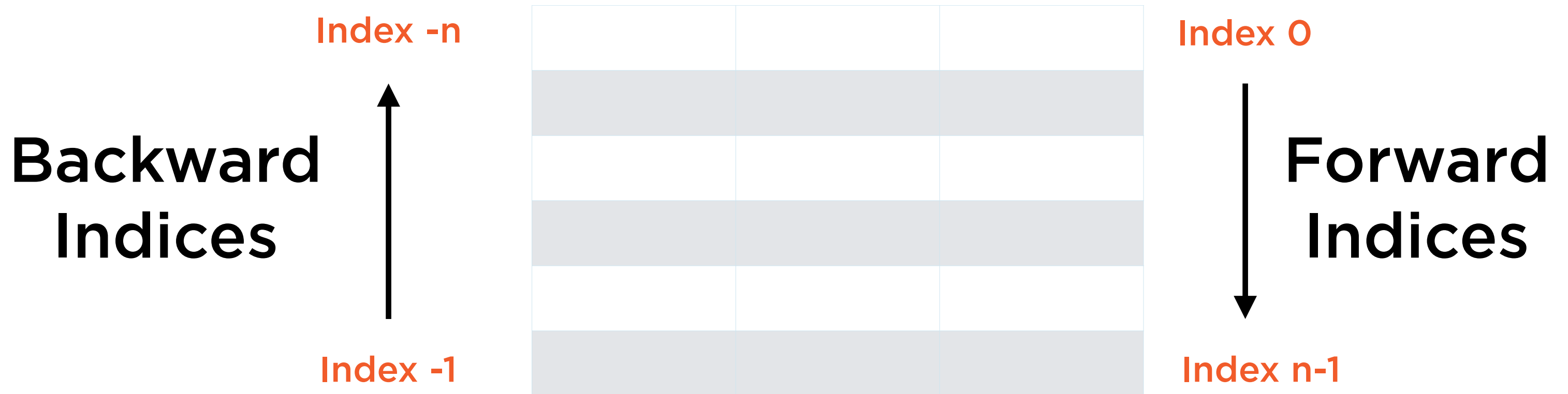
**Pandas for dataframes**

**NumPy for arrays**

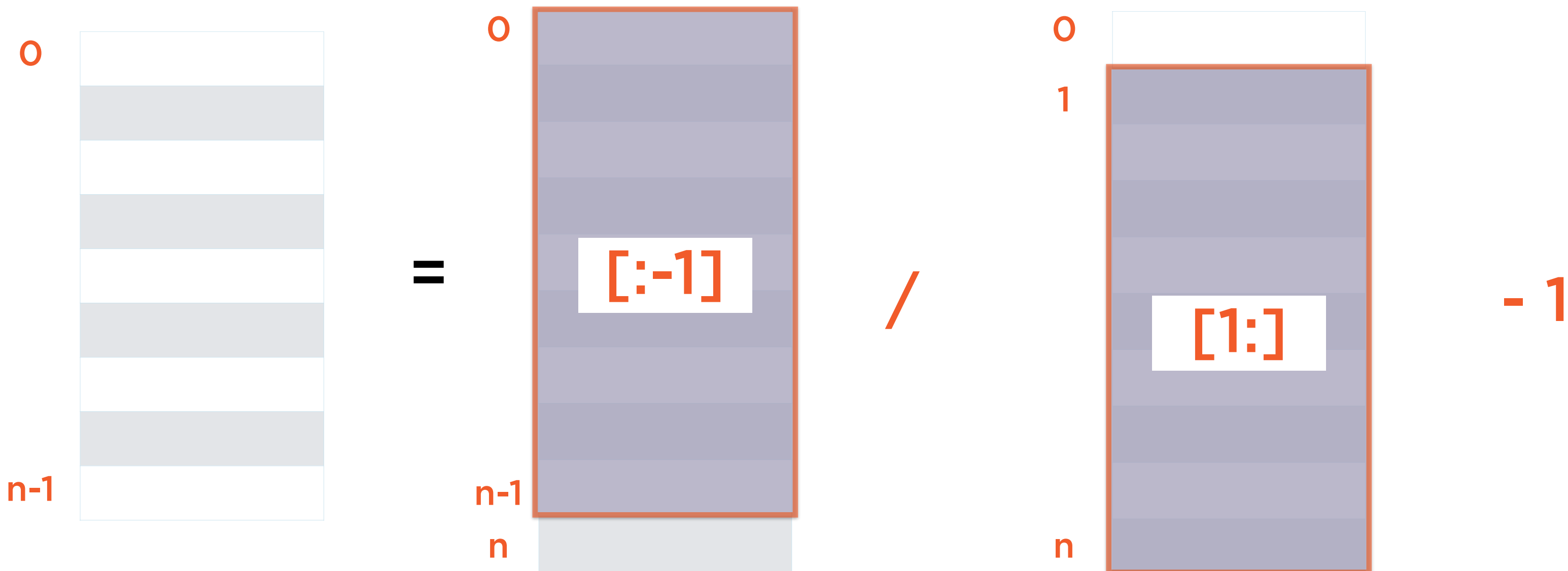
**Statsmodels for regression**

**Matplotlib for plots**

# Negative Indices In Python

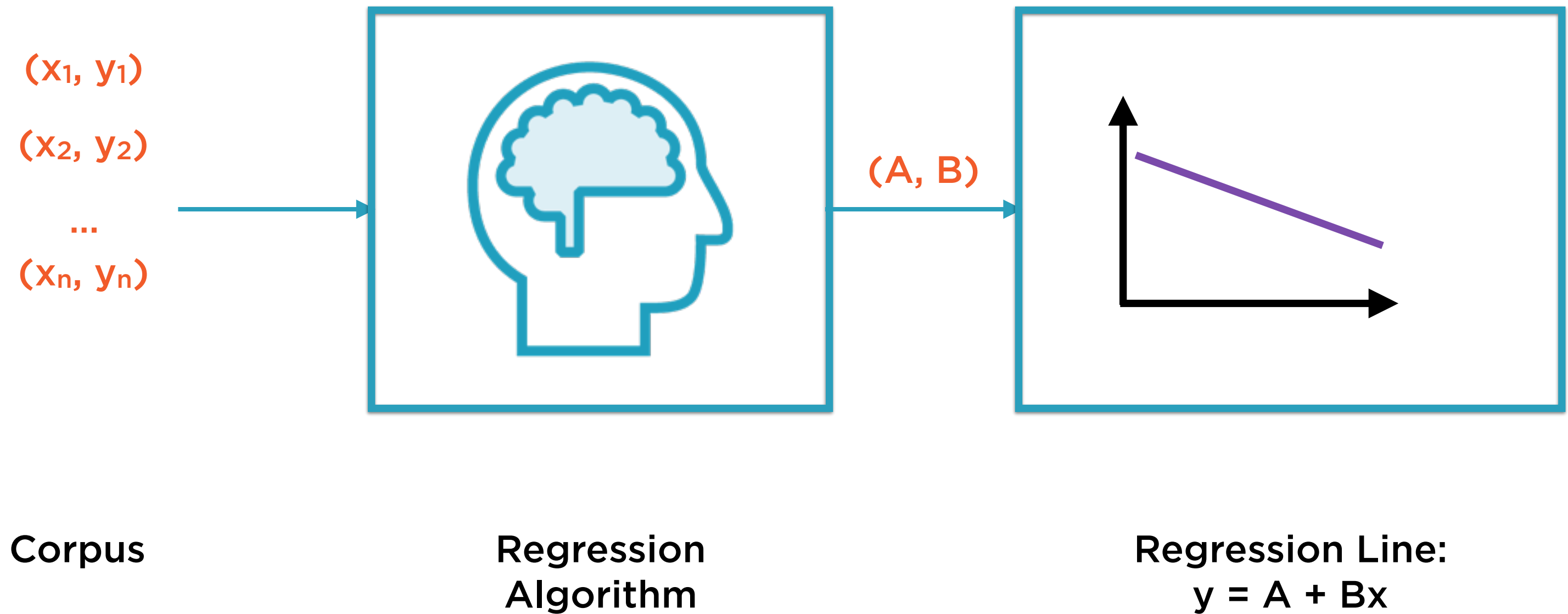


# Prices to Returns



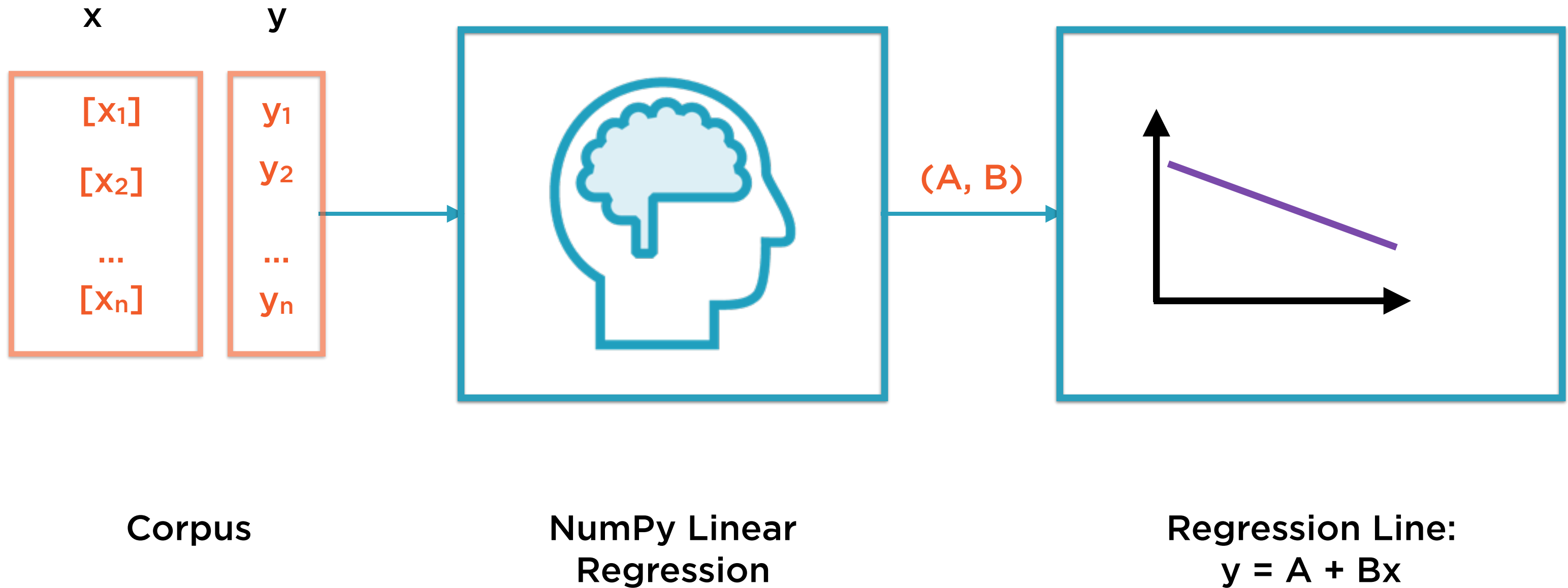
**Returns = Prices[: -1] / Prices[1:] - 1**

# ML-based Regression Model

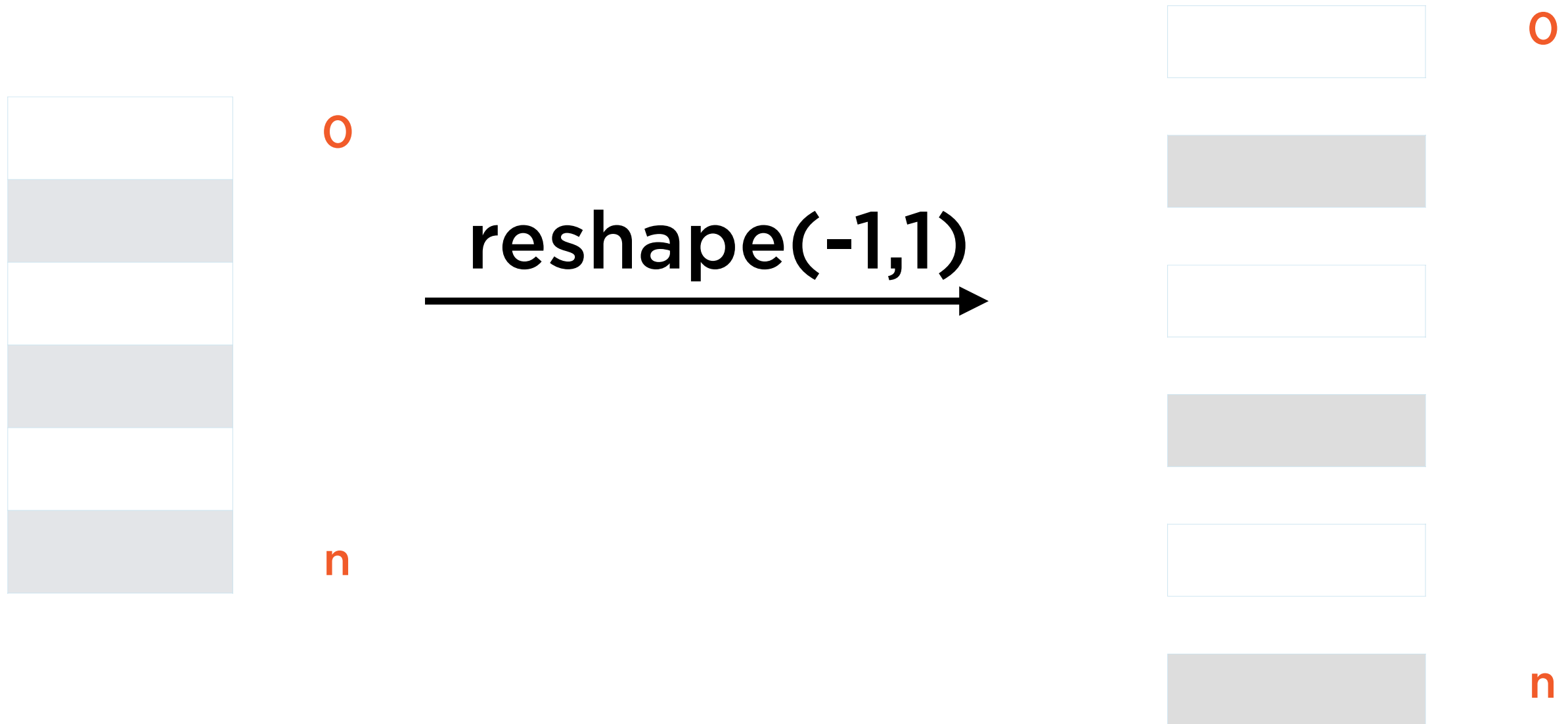




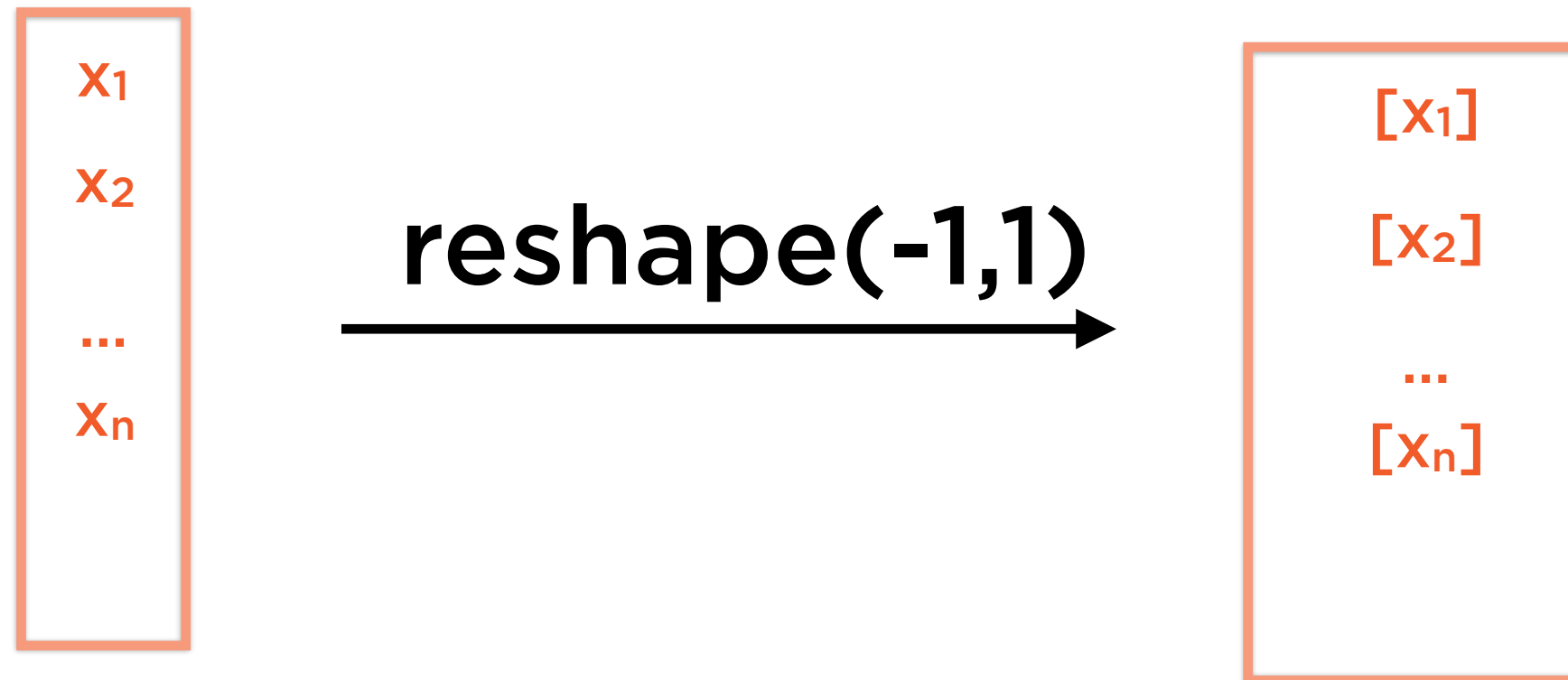
# ML-based Regression Model



# Reshaping in NumPy



# Reshaping in NumPy



# Implementing Regression in TensorFlow

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Non-TensorFlow implementation

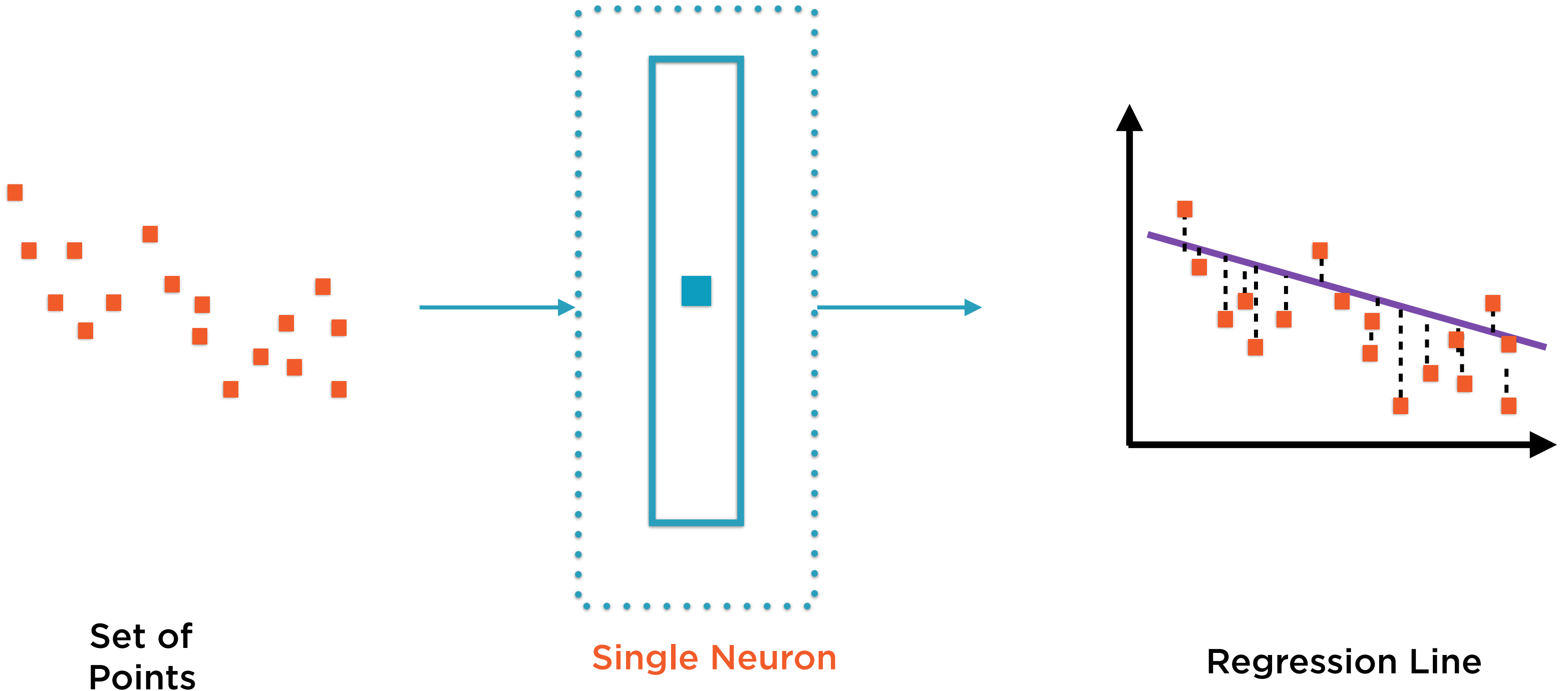
Regular python code

## Computation Graph

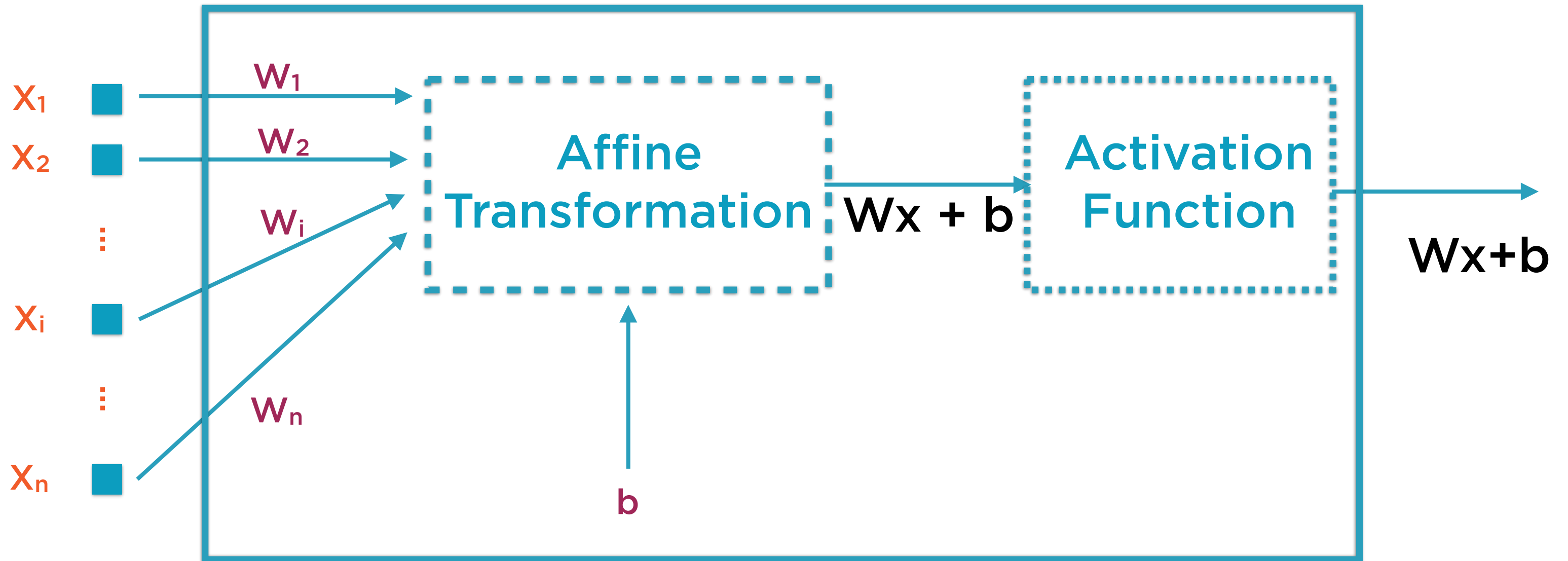
Neural network of 1 neuron

Affine transformation suffices

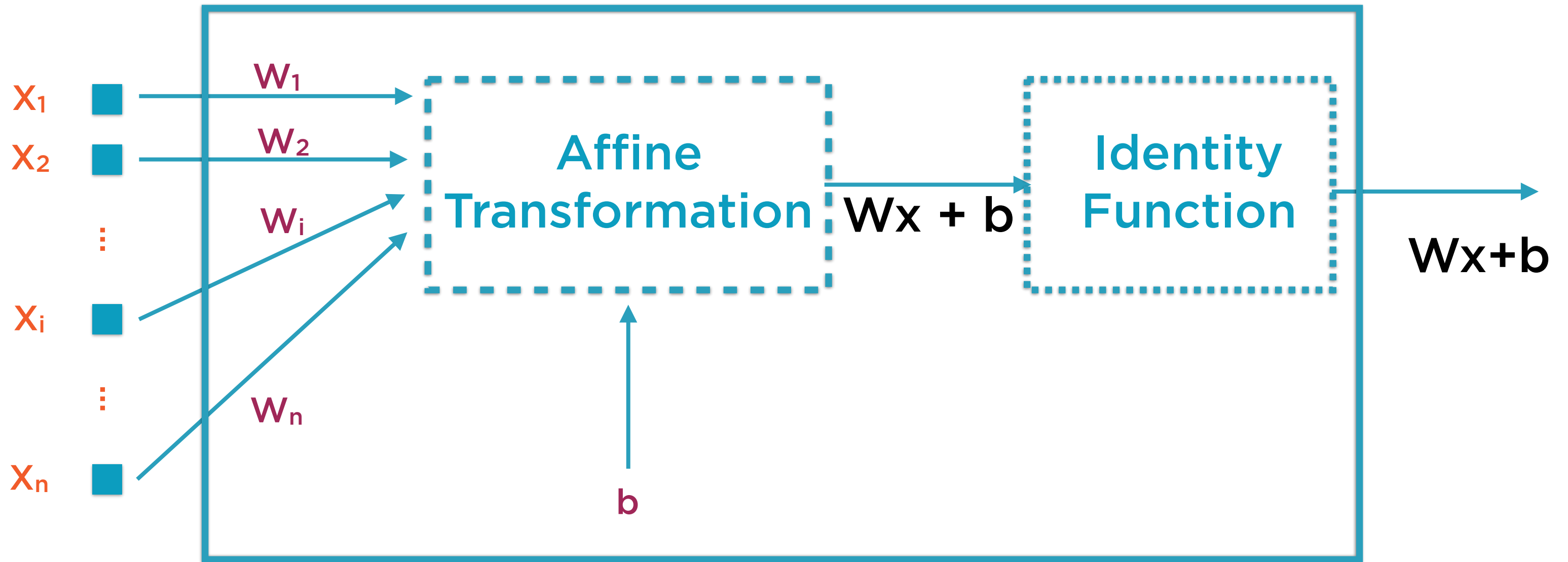
# Regression: The Simplest Neural Network



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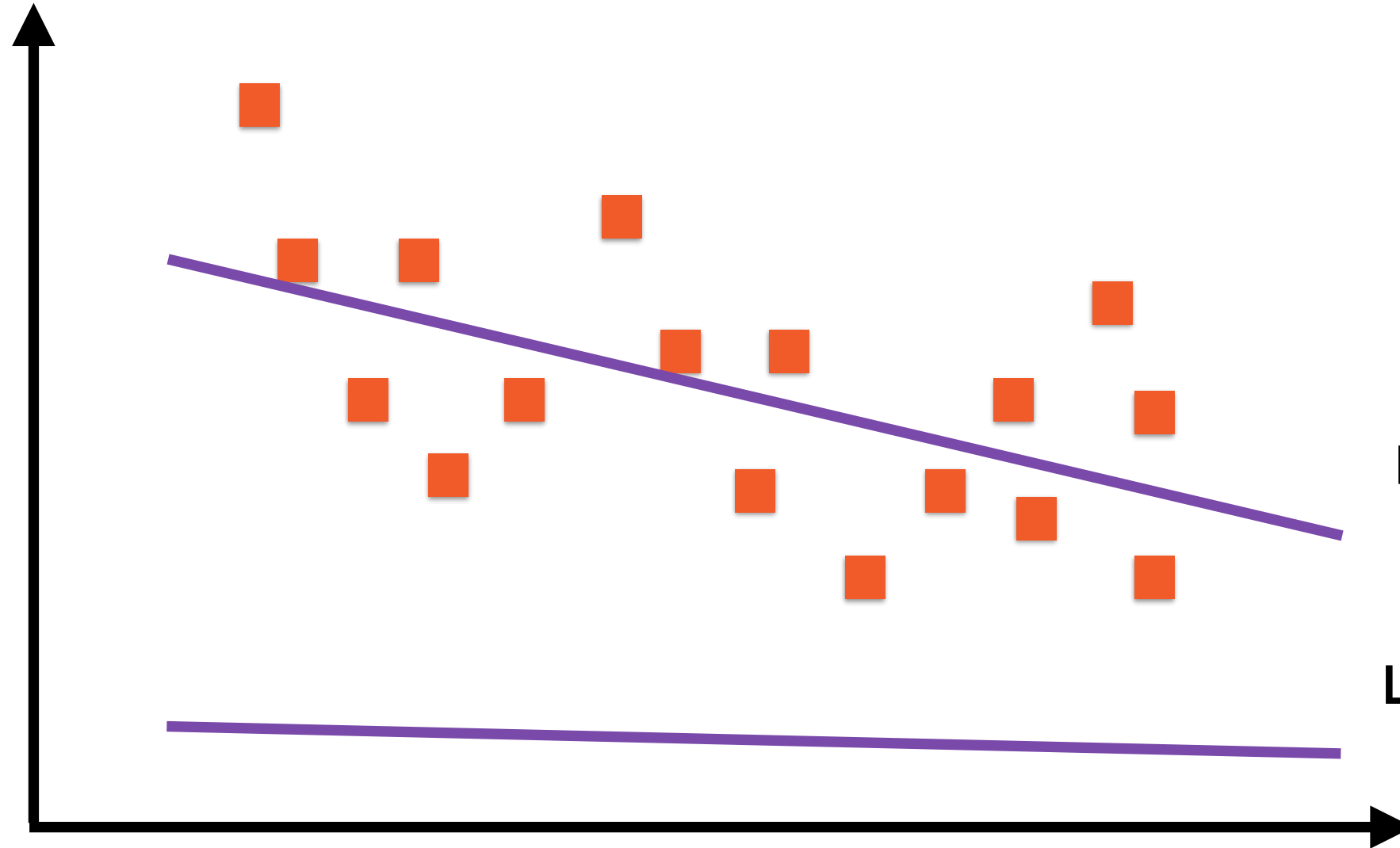
Neural network of 1 neuron  
Affine transformation suffices



# The “Best” Regression Line



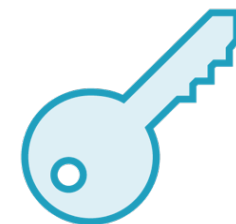
Y



Line 1:  $y = A_1 + B_1x$

Line 2:  $y = A_2 + B_2x$

X

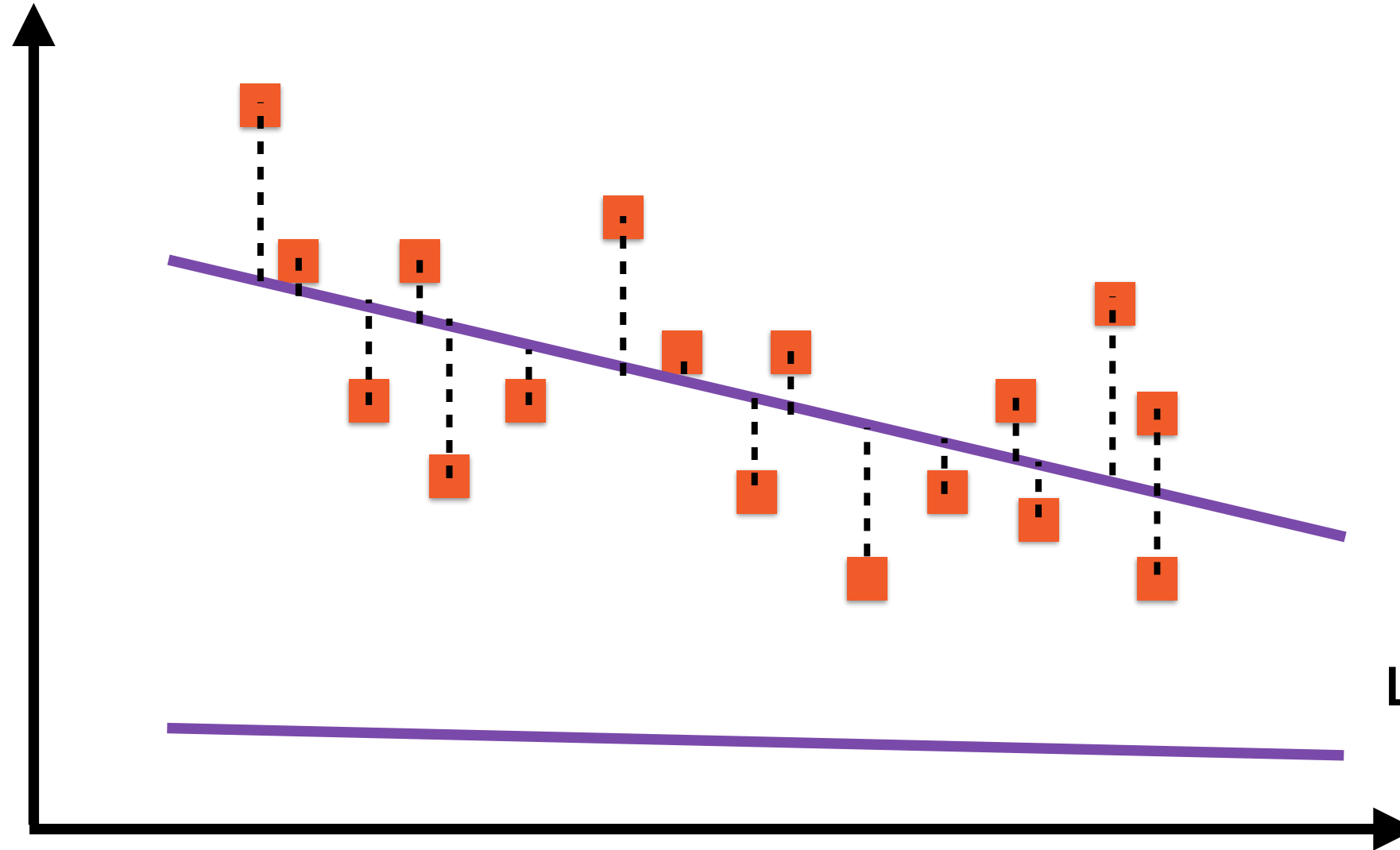


Let's compare two lines, Line 1 and Line 2

# Minimising Least Square Error



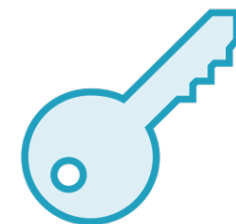
Y



Line 1:  $y = A_1 + B_1x$

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X

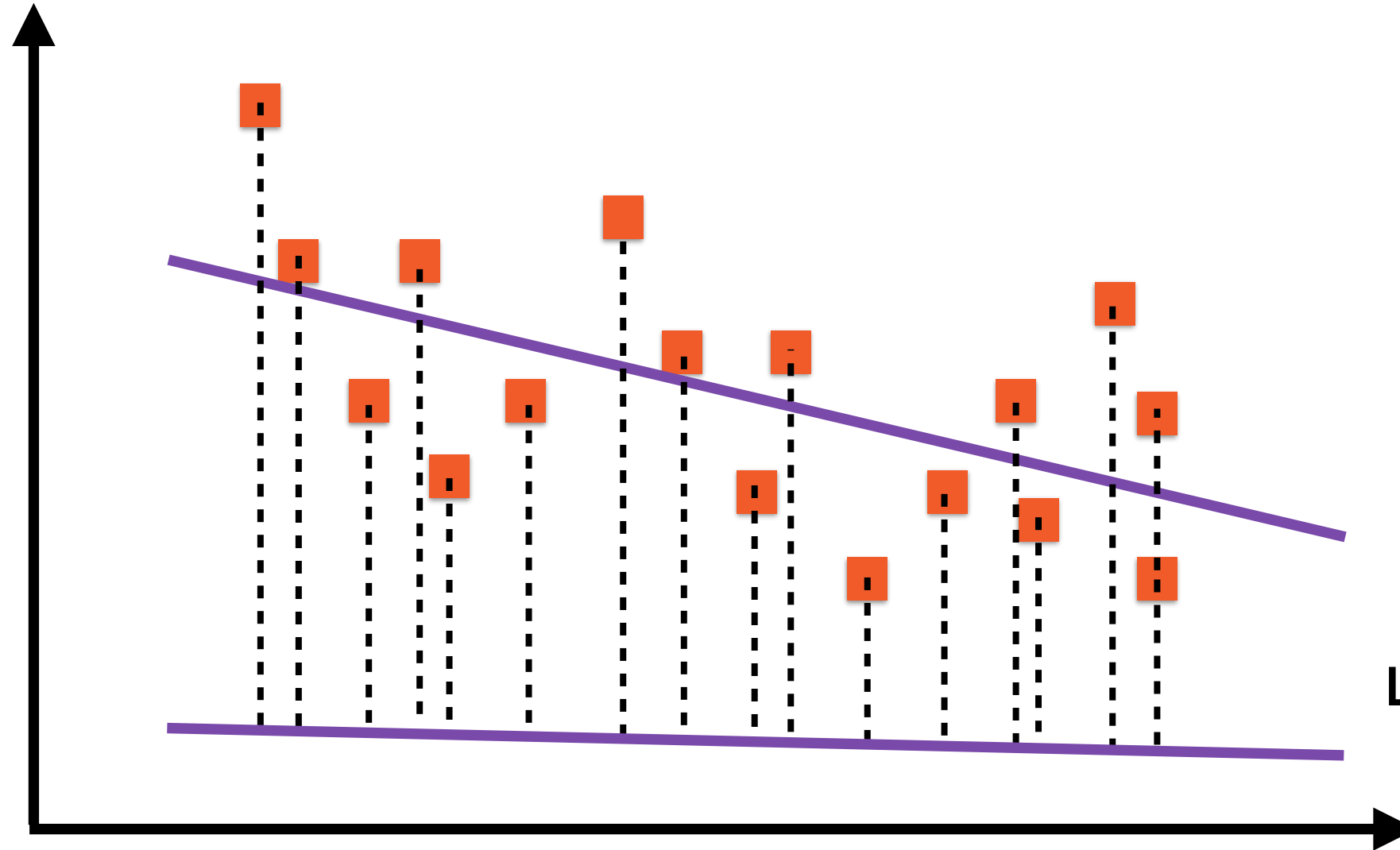


Drop vertical lines from each point to  
the lines A and B

# Minimising Least Square Error



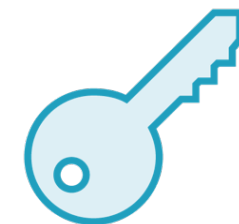
Y



Line 1:  $y = A_1 + B_1x$

Line 2:  $y = A_2 + B_2x$

X



Drop vertical lines from each point to  
the lines A and B

# Simple Regression

**Regression Equation:**

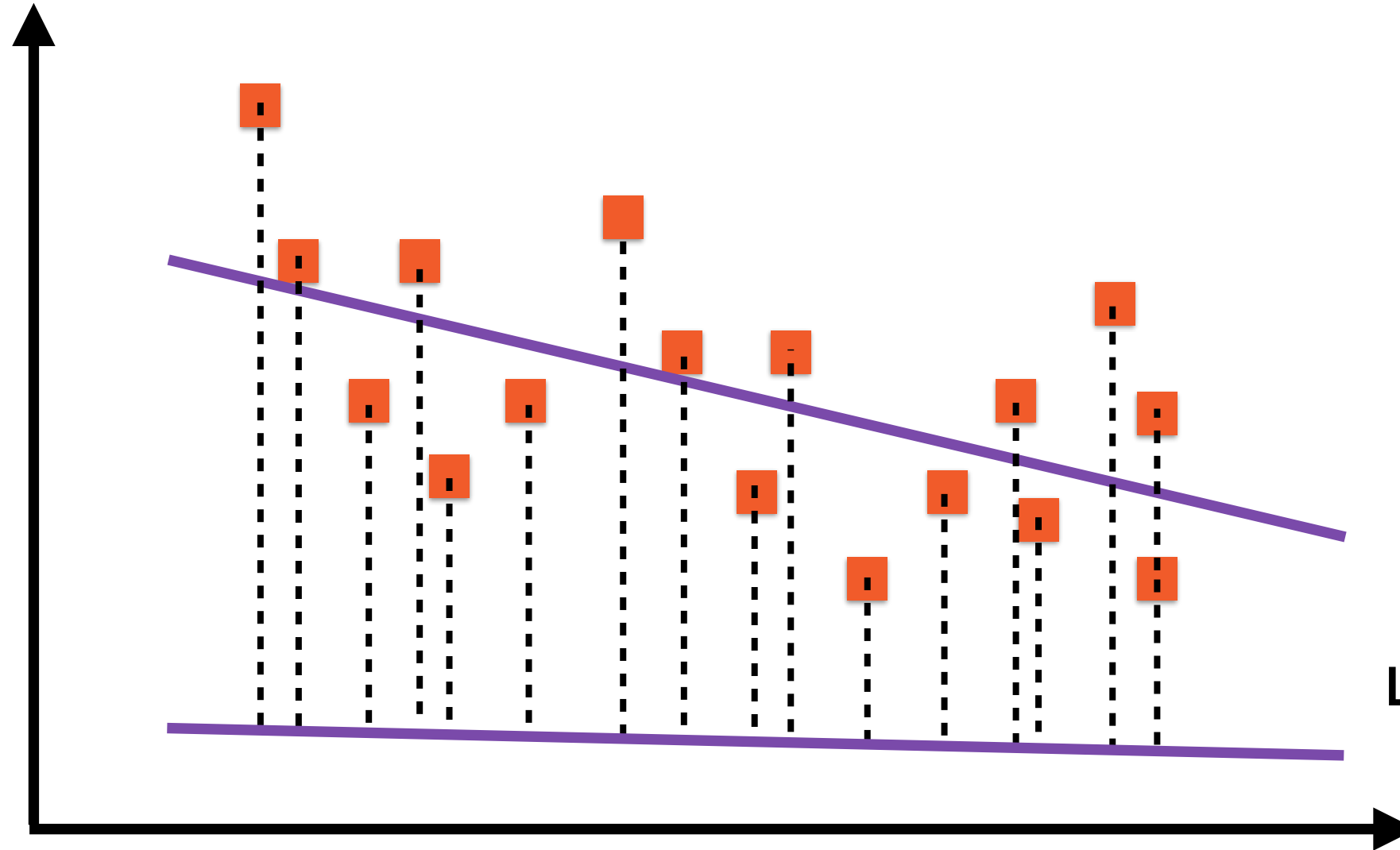
$$y = A + Bx$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \dots \\ y_n \end{bmatrix} = A \begin{bmatrix} 1 \\ 1 \\ 1 \\ \dots \\ 1 \end{bmatrix} + B \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \dots \\ x_n \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \dots \\ e_n \end{bmatrix}$$

# Minimising Least Square Error



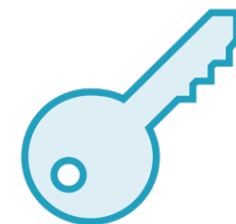
Y



Line 1:  $y = A_1 + B_1x$

Line 2:  $y = A_2 + B_2x$

X

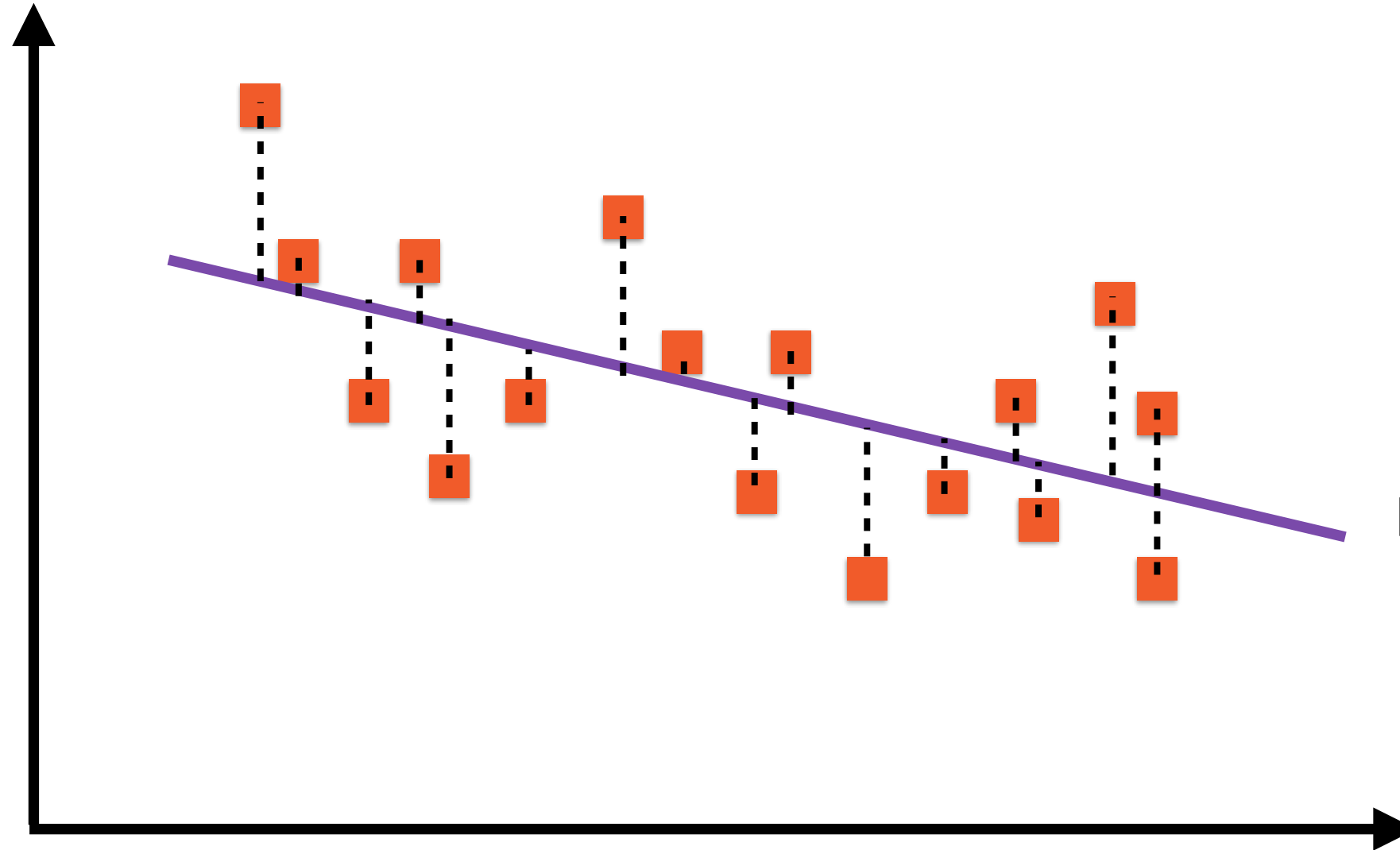


The “best fit” line is the one where the sum of the squares of the lengths of these dotted lines is minimum

# Minimising Least Square Error

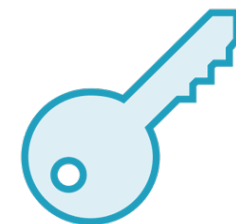


Y



Regression Line:  
 $y = A + Bx$

X



The “best fit” line is called the  
regression line

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# Why Choosing Is Complicated



What do we really  
want to achieve?



What is slowing us  
down?



What do we really  
control?

**Choosing involves answering complicated questions**



# Why Optimization Helps



What do we really  
want to achieve?



What is slowing us  
down?



What do we really  
control?

**Optimization forces us to mathematically pin down  
answers to these questions**

# Framing the Optimization Problem



## Objective Function

What we would like to achieve



## Constraints

What slows us down



## Decision Variables

What we really control

Collectively, these answers constitute the optimization problem

# Linear Regression as an Optimization Problem



## Objective Function

Minimize variance of  
the residuals (MSE)



## Constraints

Express relationship as  
a straight line

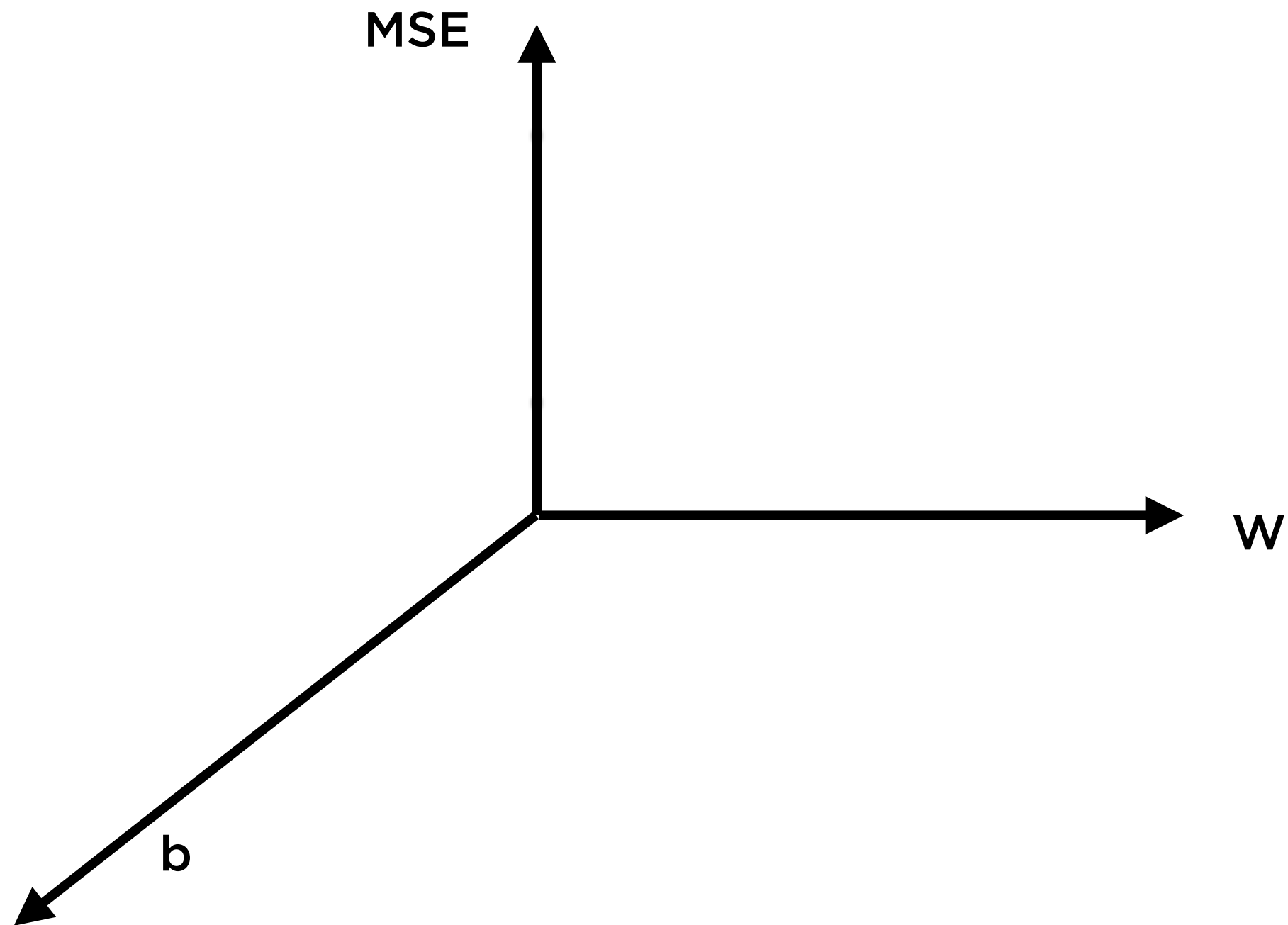
$$y = Wx + b$$



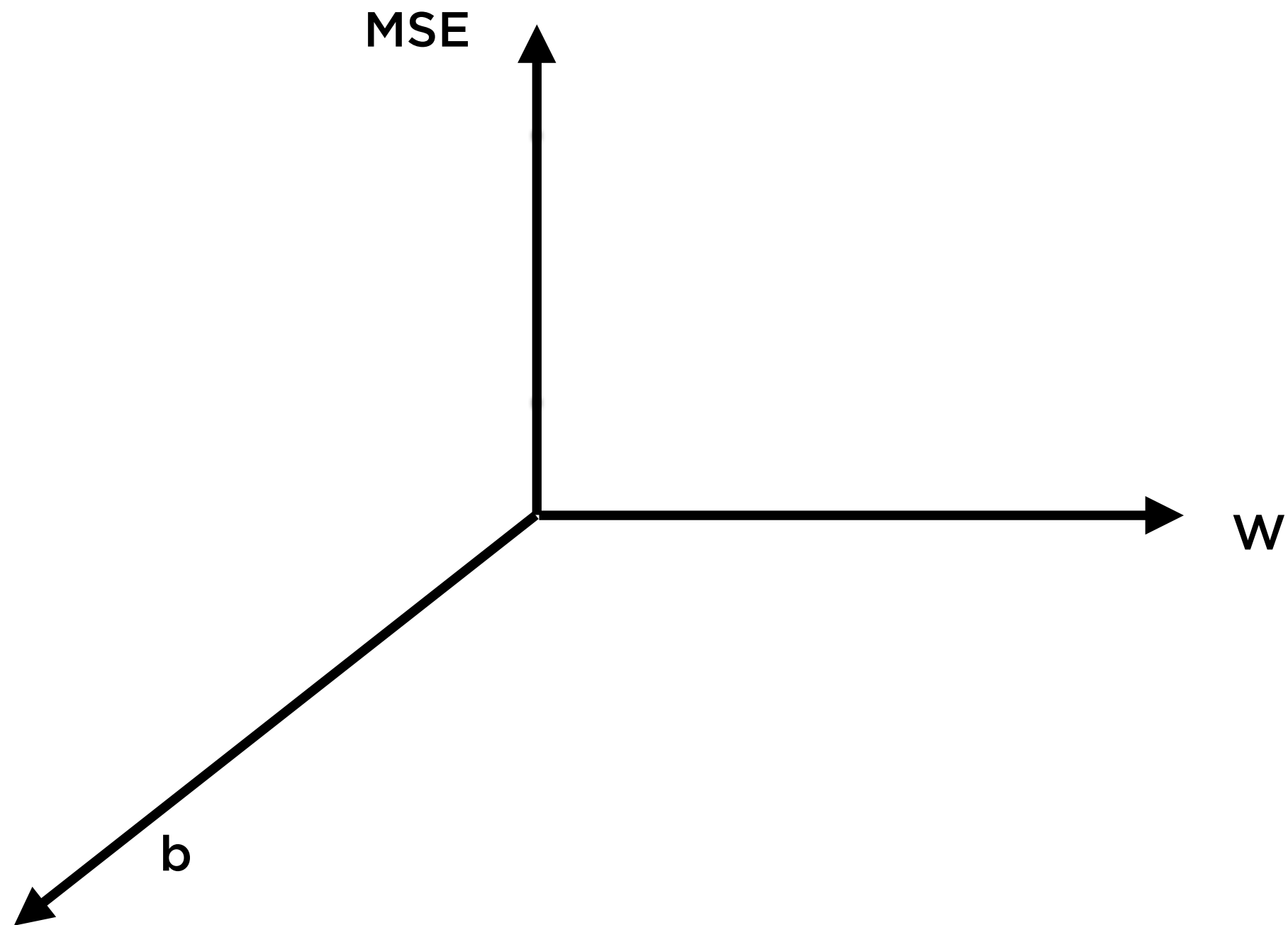
## Decision Variables

Values of  $W$  and  $b$

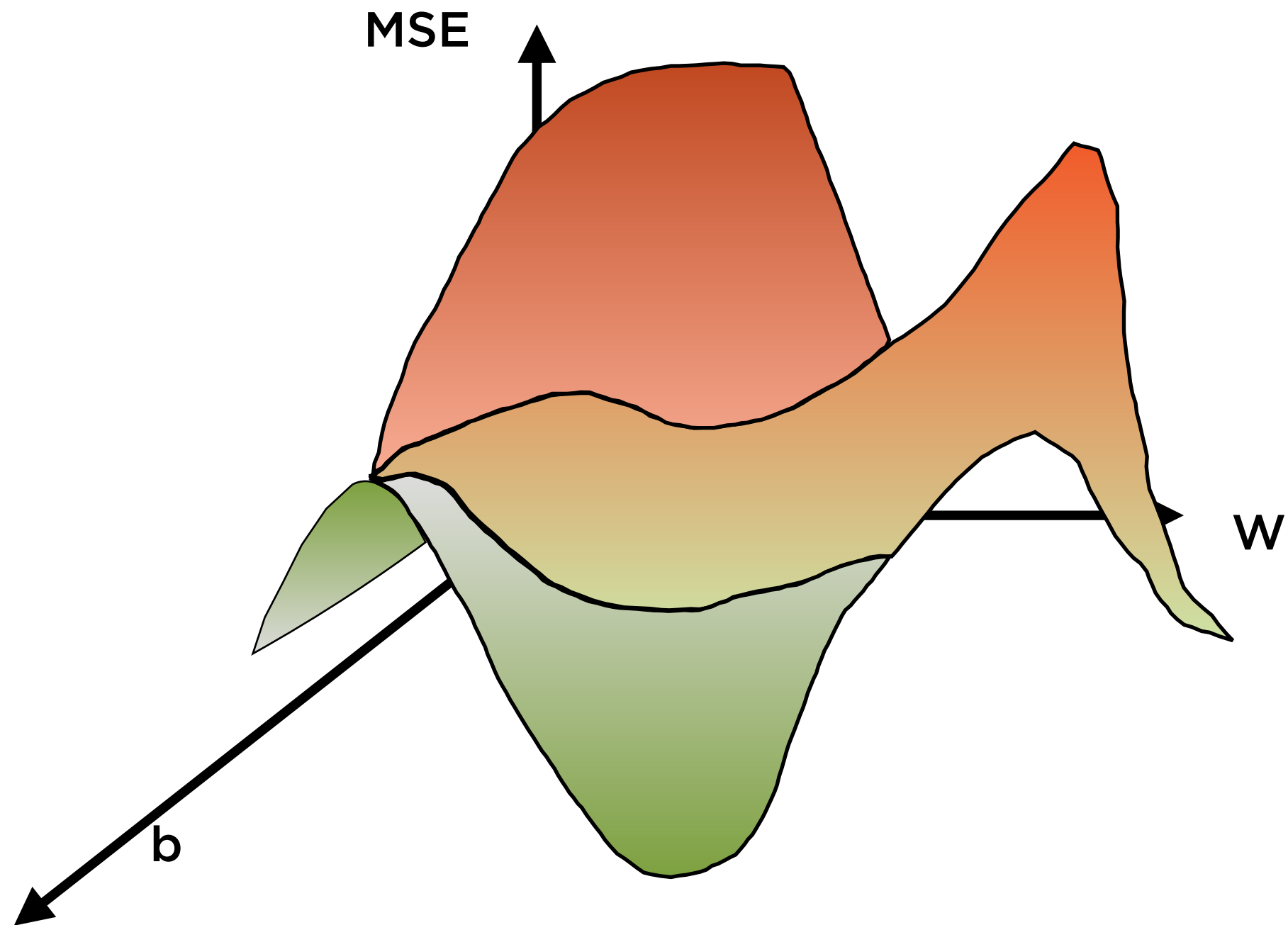
# Minimizing MSE



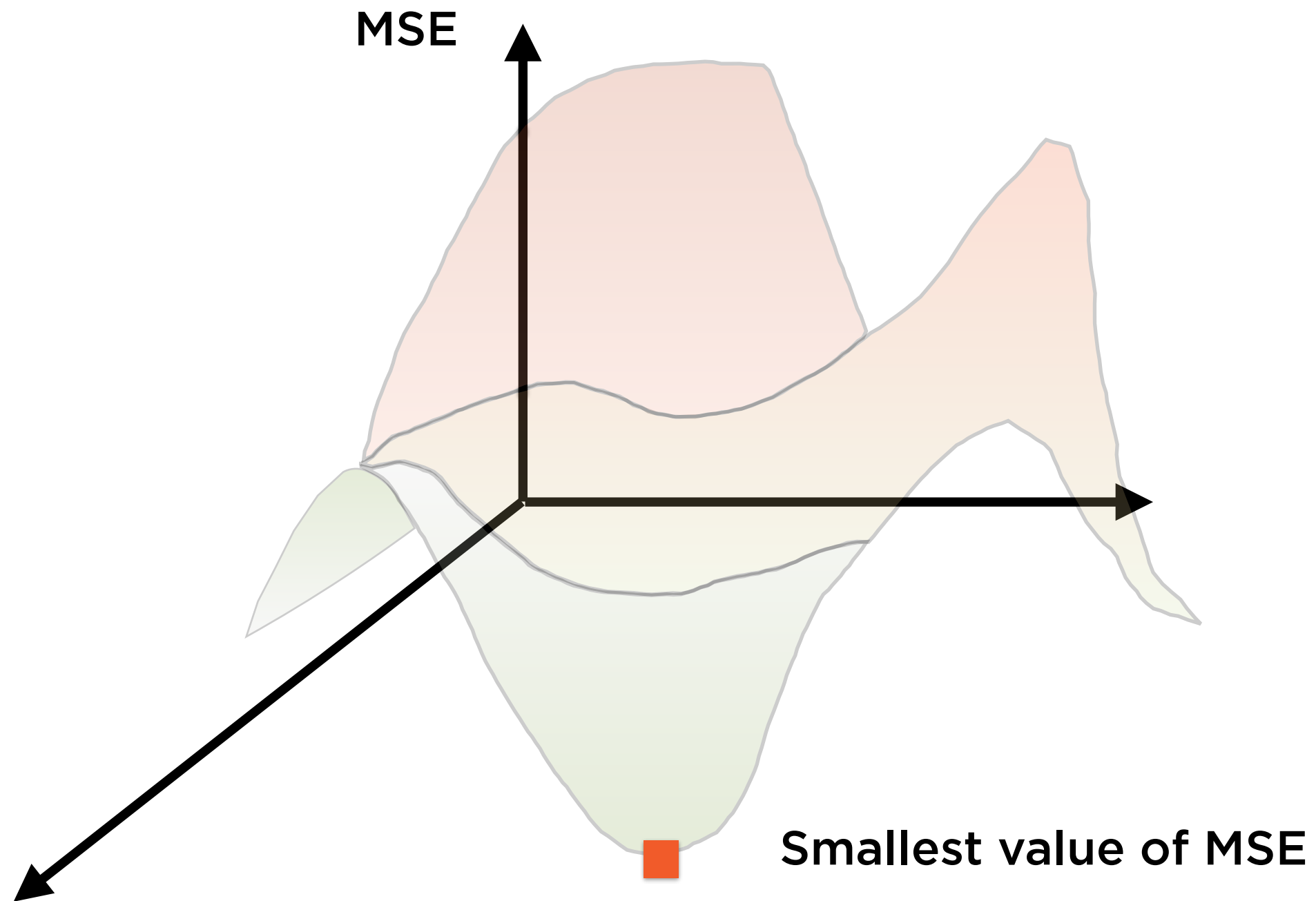
# Minimizing MSE



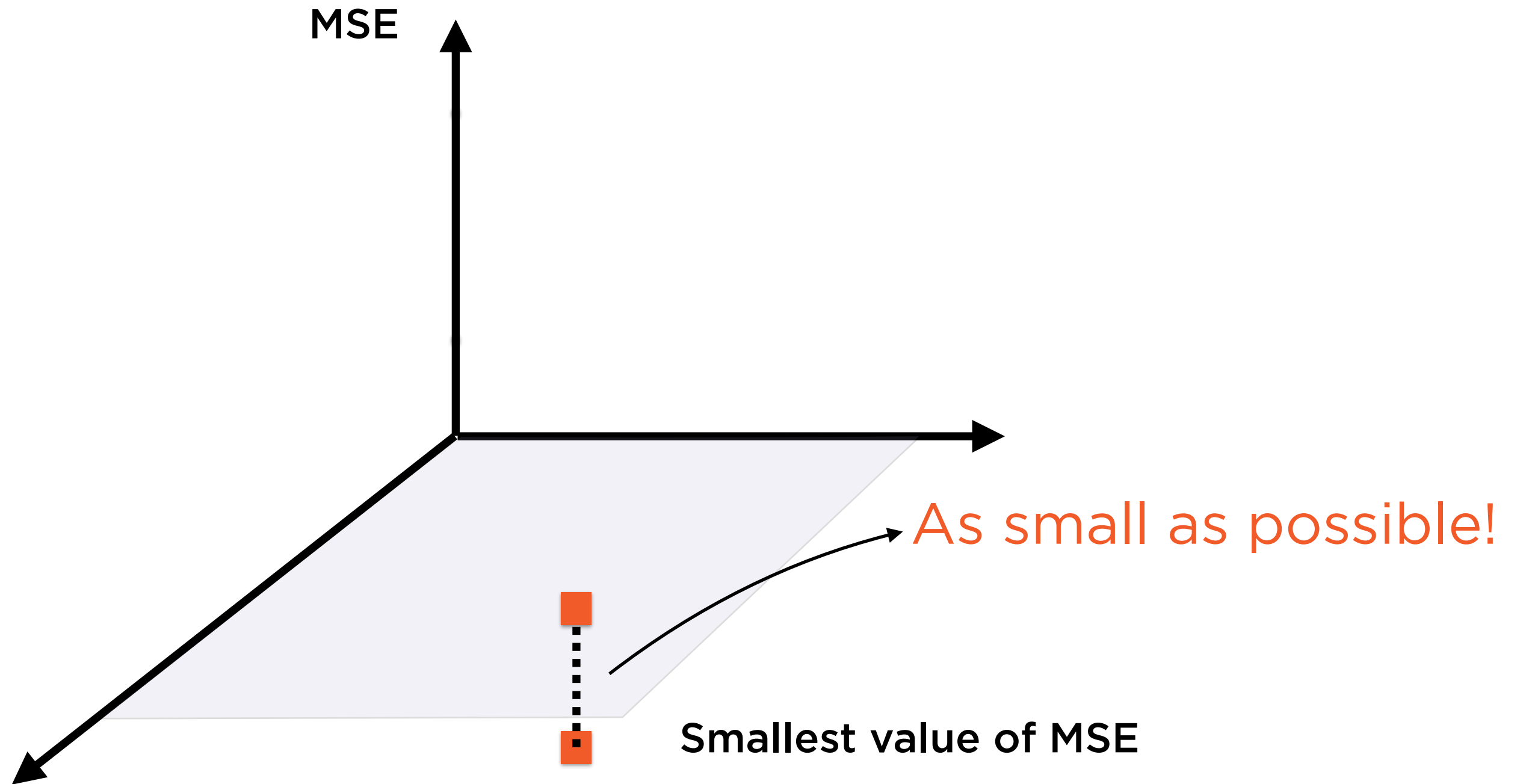
# Minimizing MSE



# Minimizing MSE

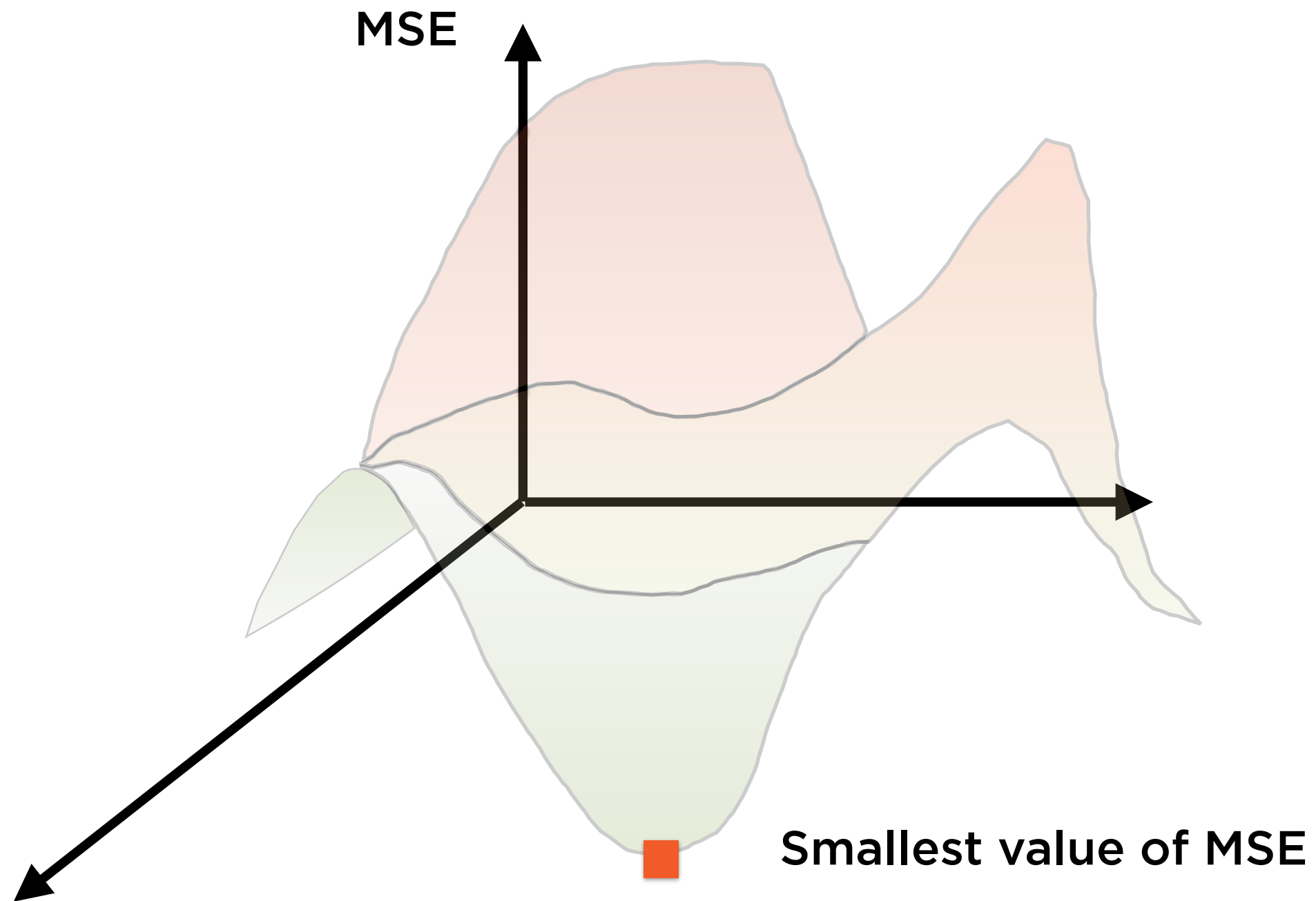


# Minimizing MSE

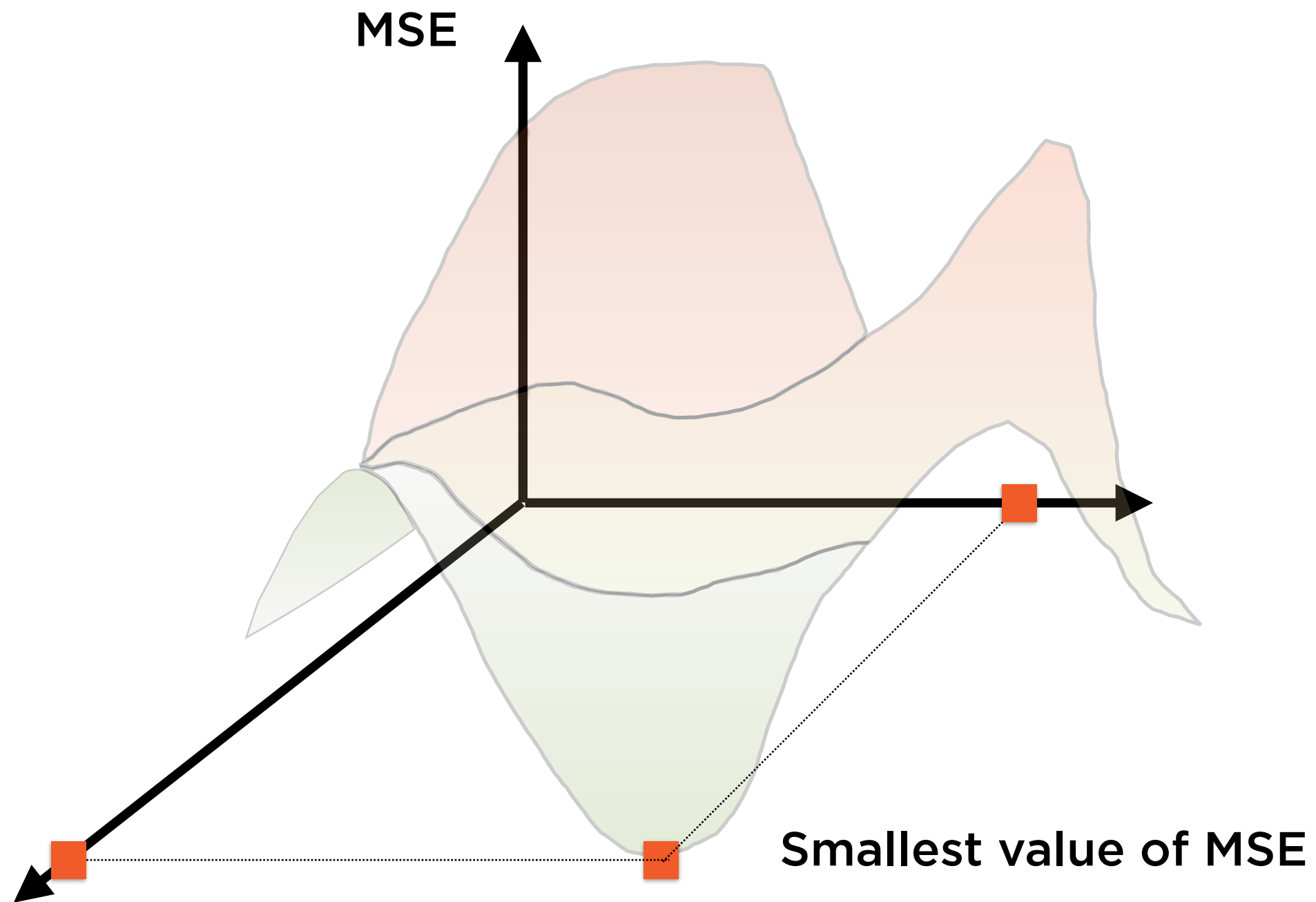




# Minimizing MSE

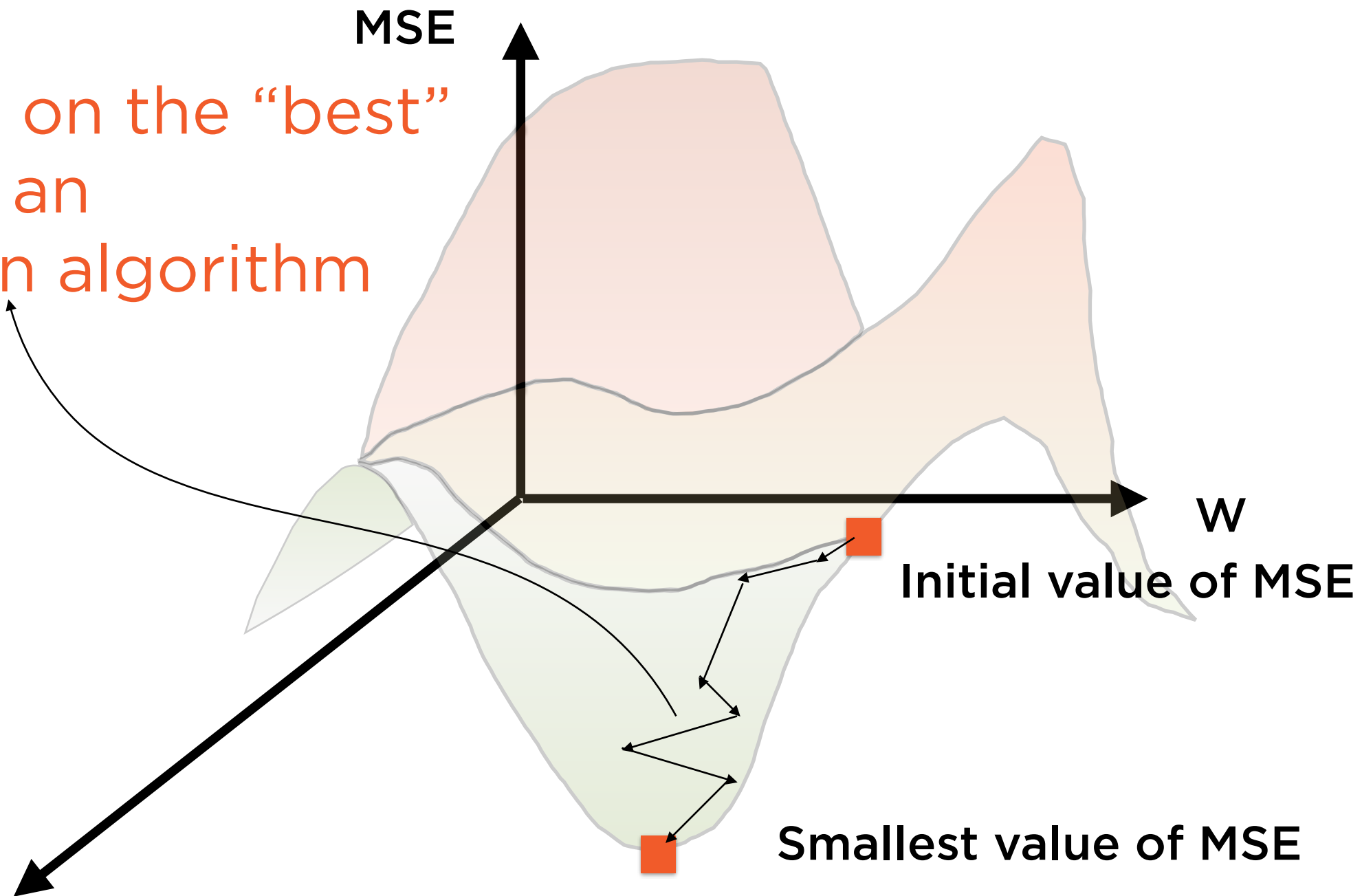


# Minimizing MSE

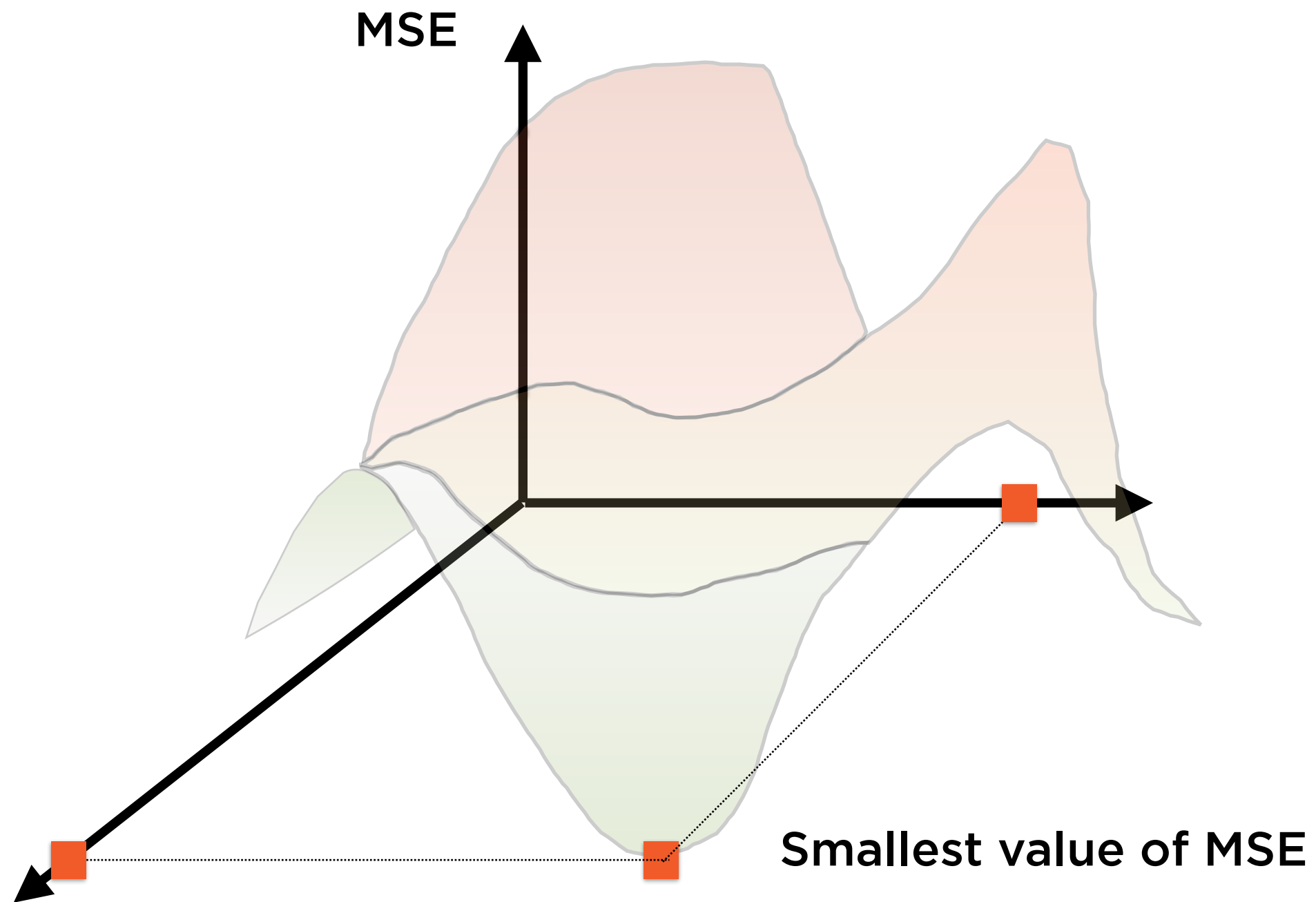


# “Gradient Descent”

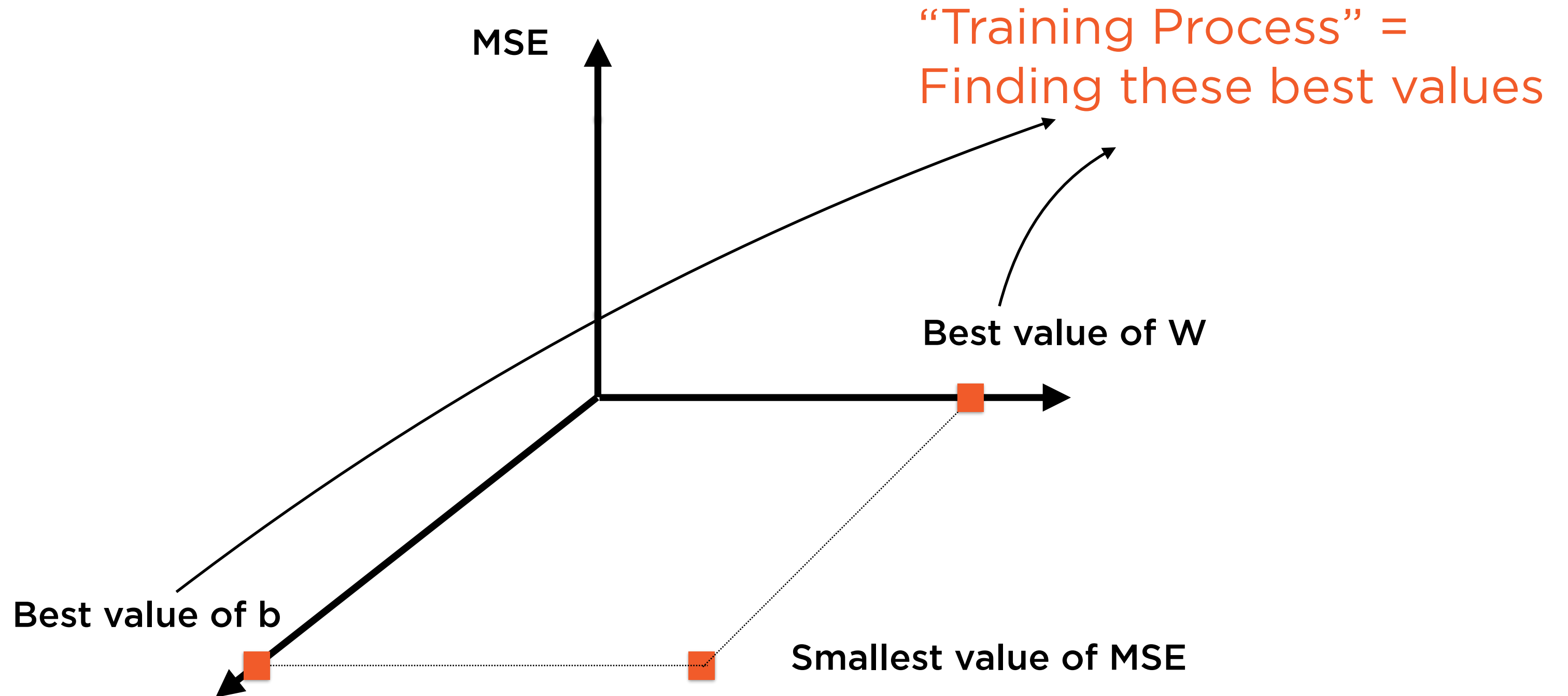
Converging on the “best”  
value using an  
optimization algorithm



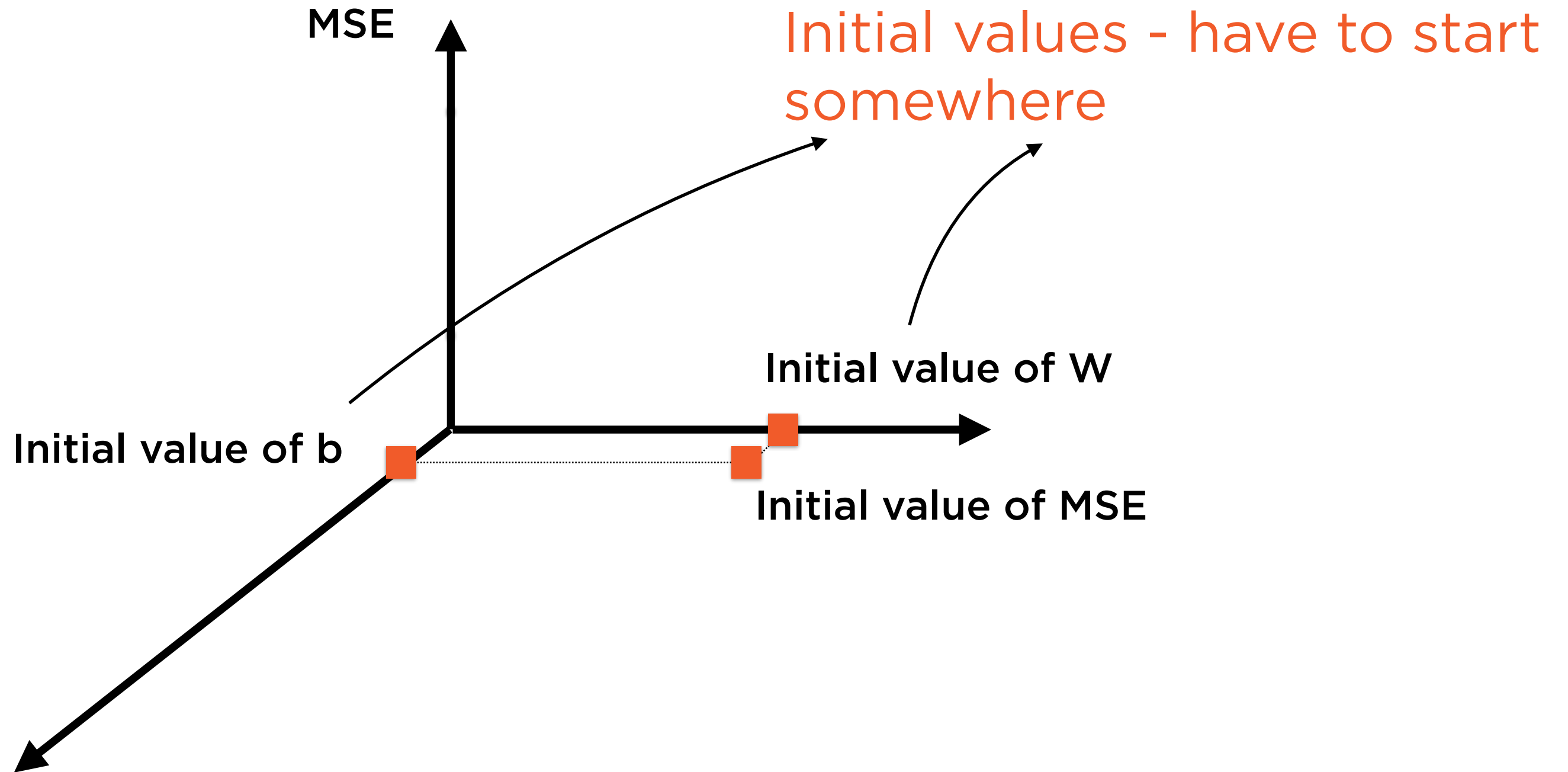
# Minimizing MSE



# “Training” the Algorithm

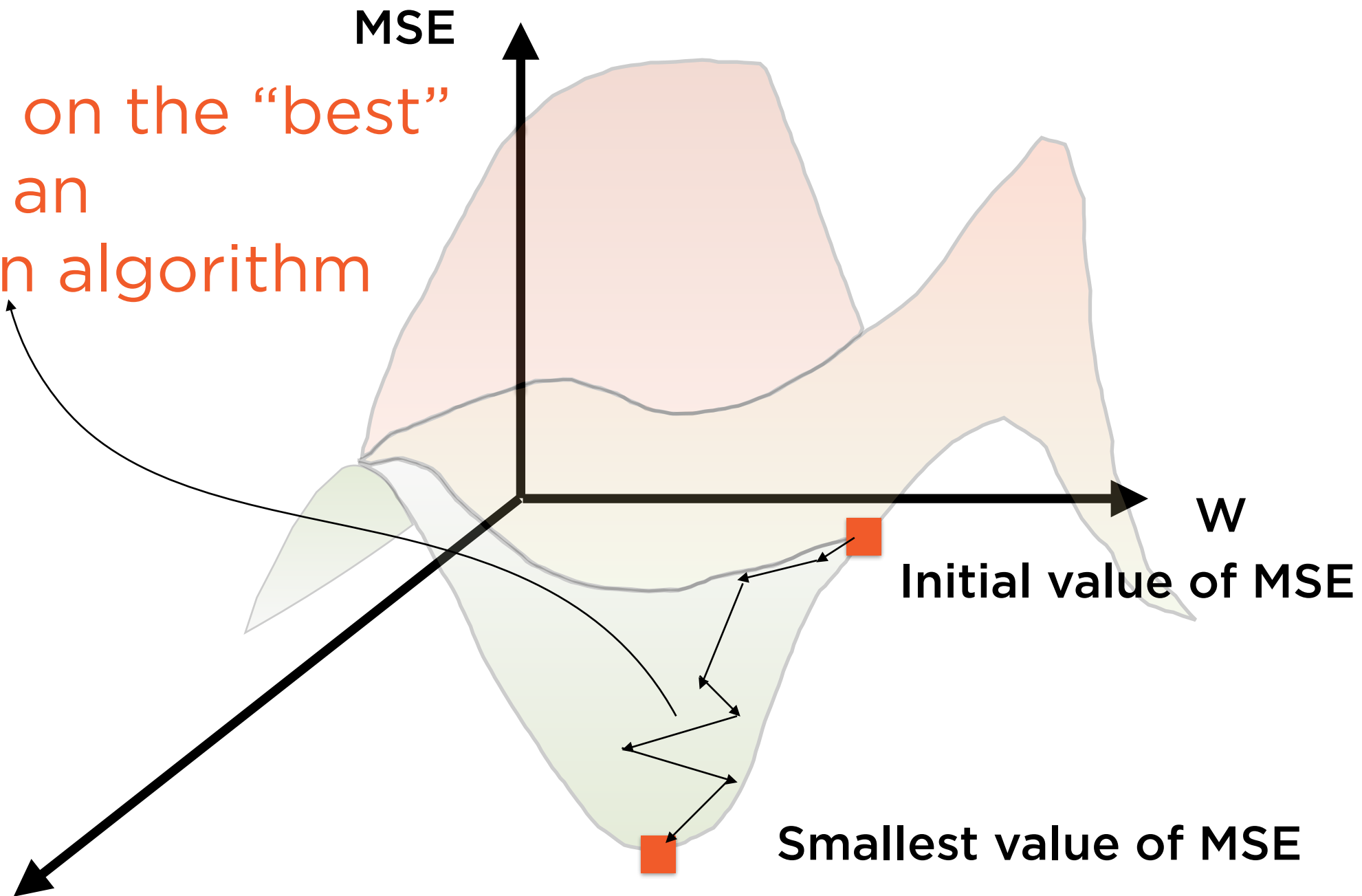


# Start Somewhere



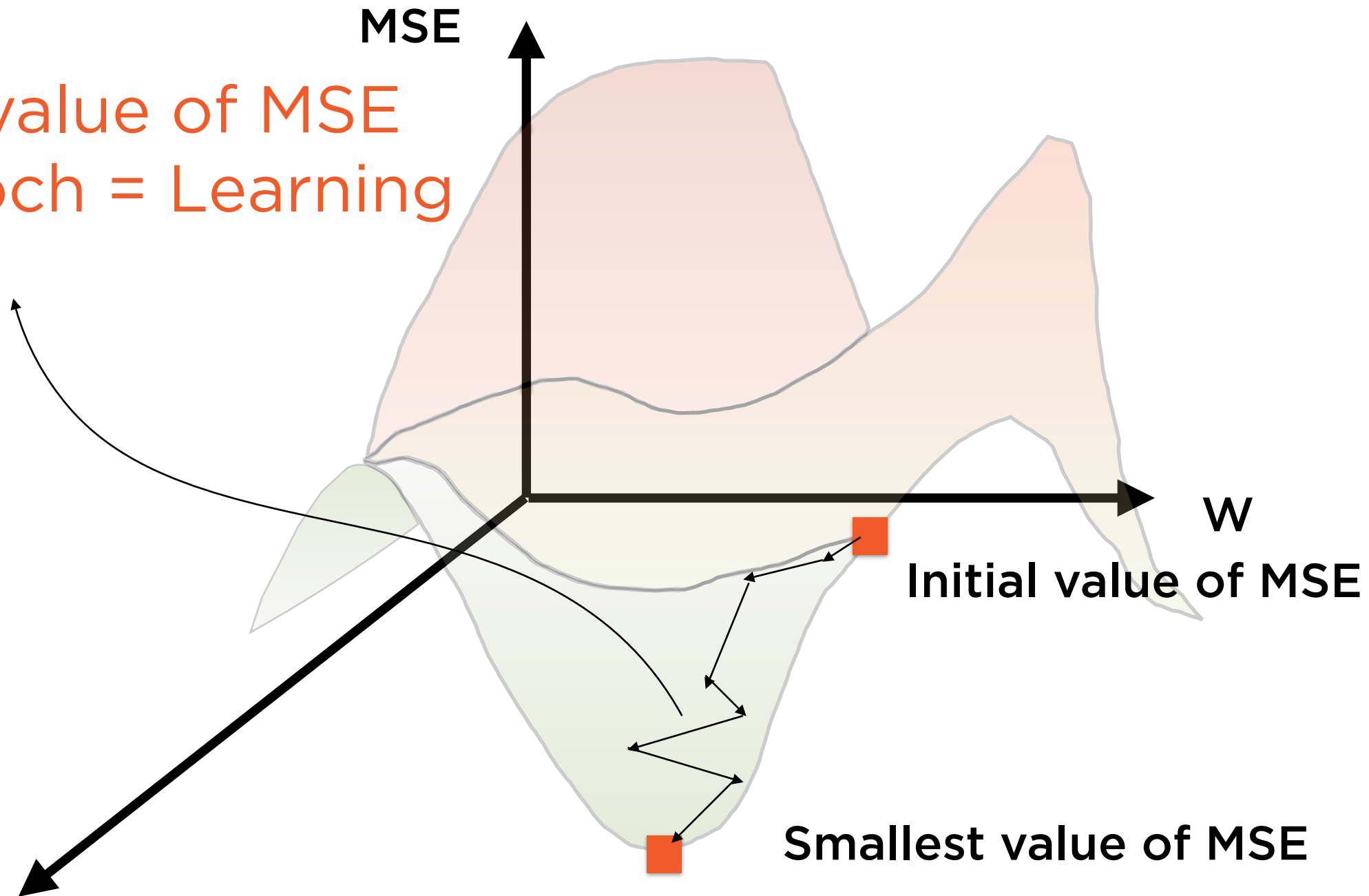
# “Gradient Descent”

Converging on the “best”  
value using an  
optimization algorithm



# “Learning Rate”

Change in value of MSE  
in each epoch = Learning  
Rate





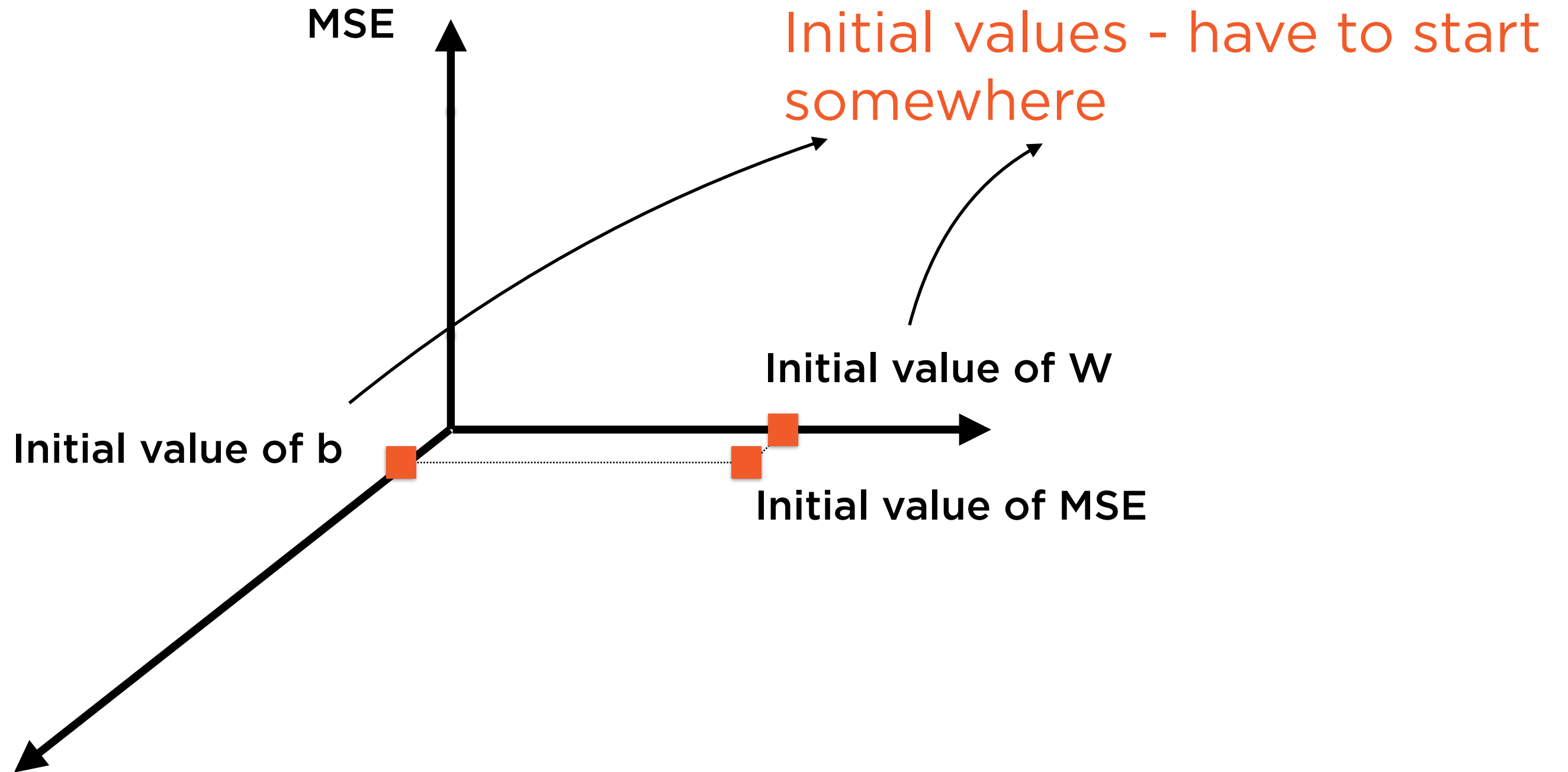
# Gradient Descent Optimizers

**tf.train.GradientDescentOptimizer**

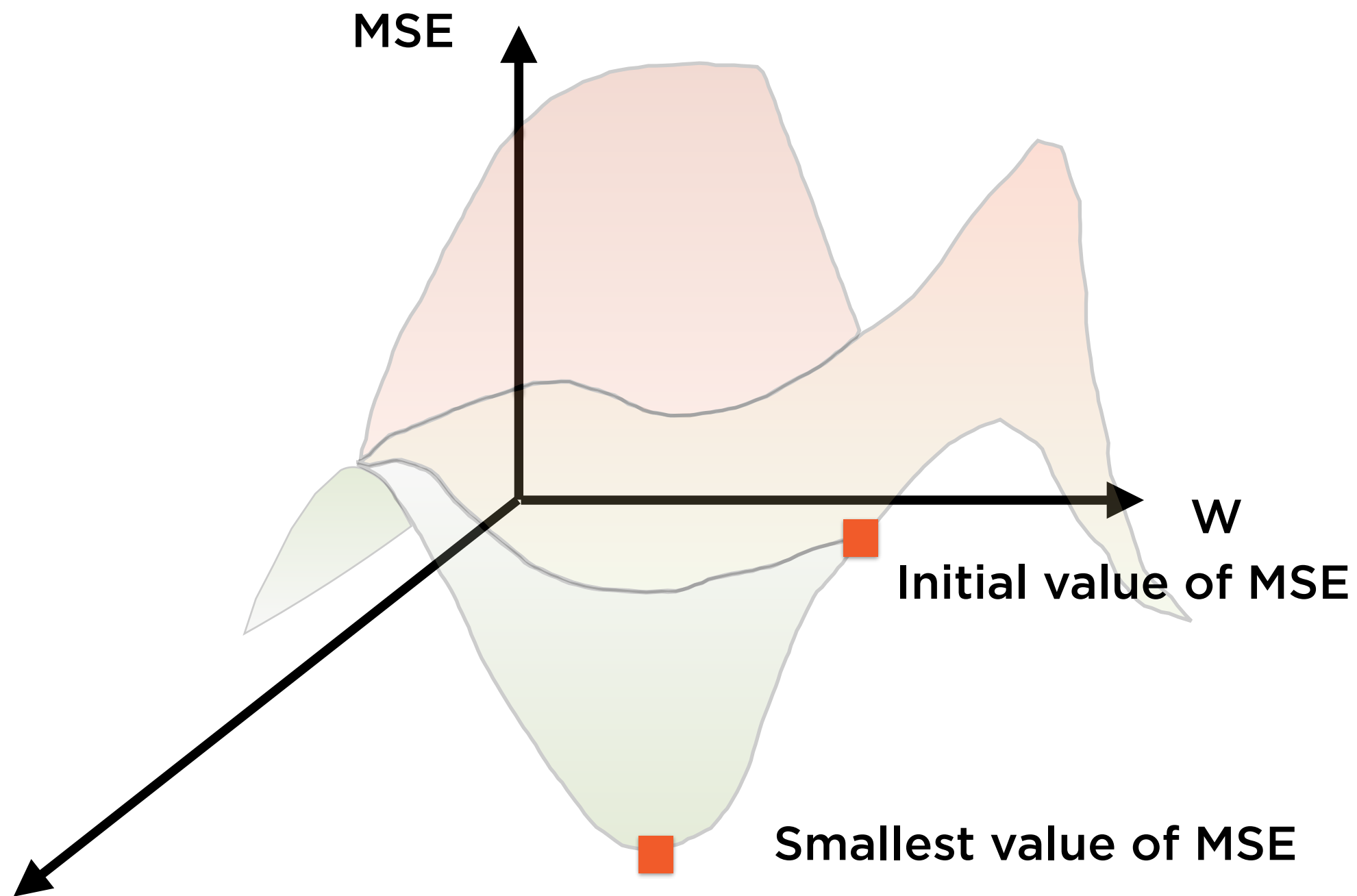
**tf.train.AdamOptimizer**

**tf.train.FtrlOptimizer**

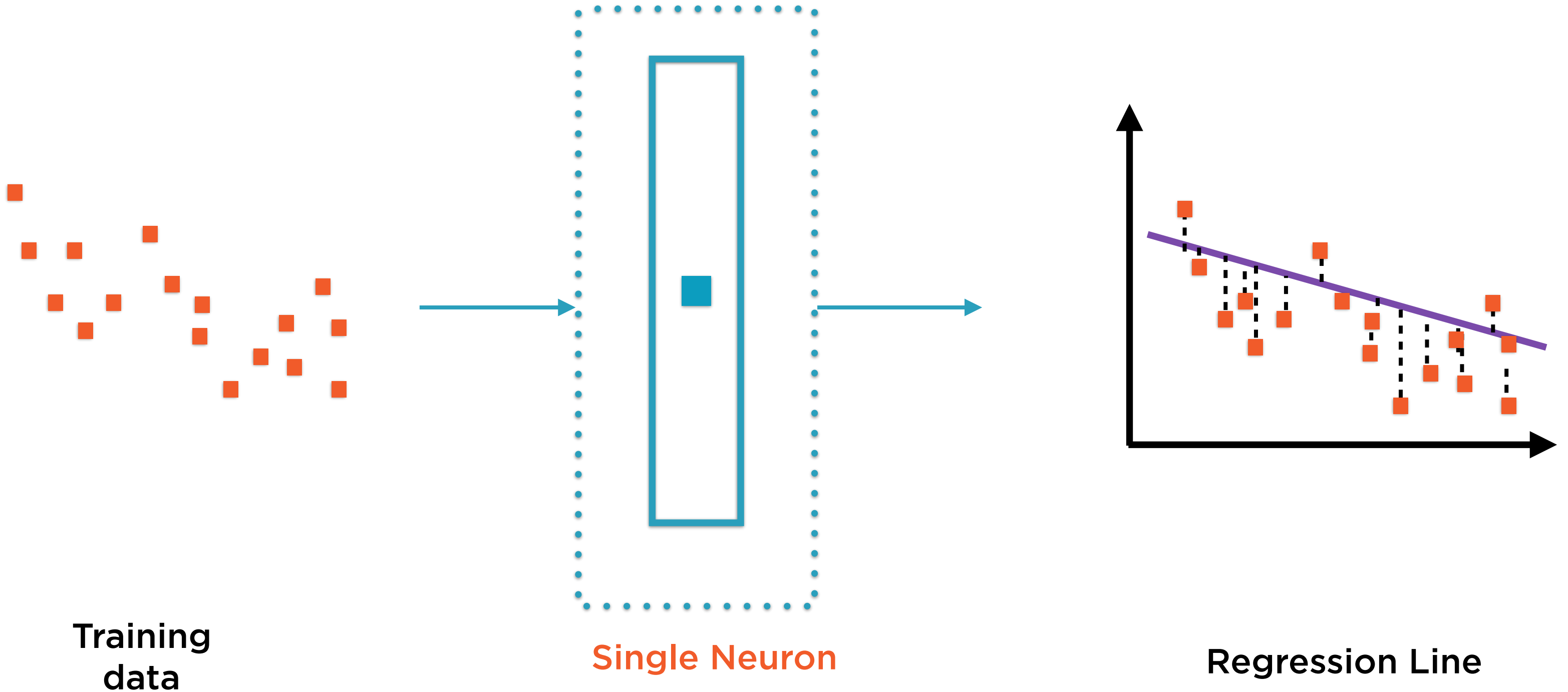
# Start Somewhere



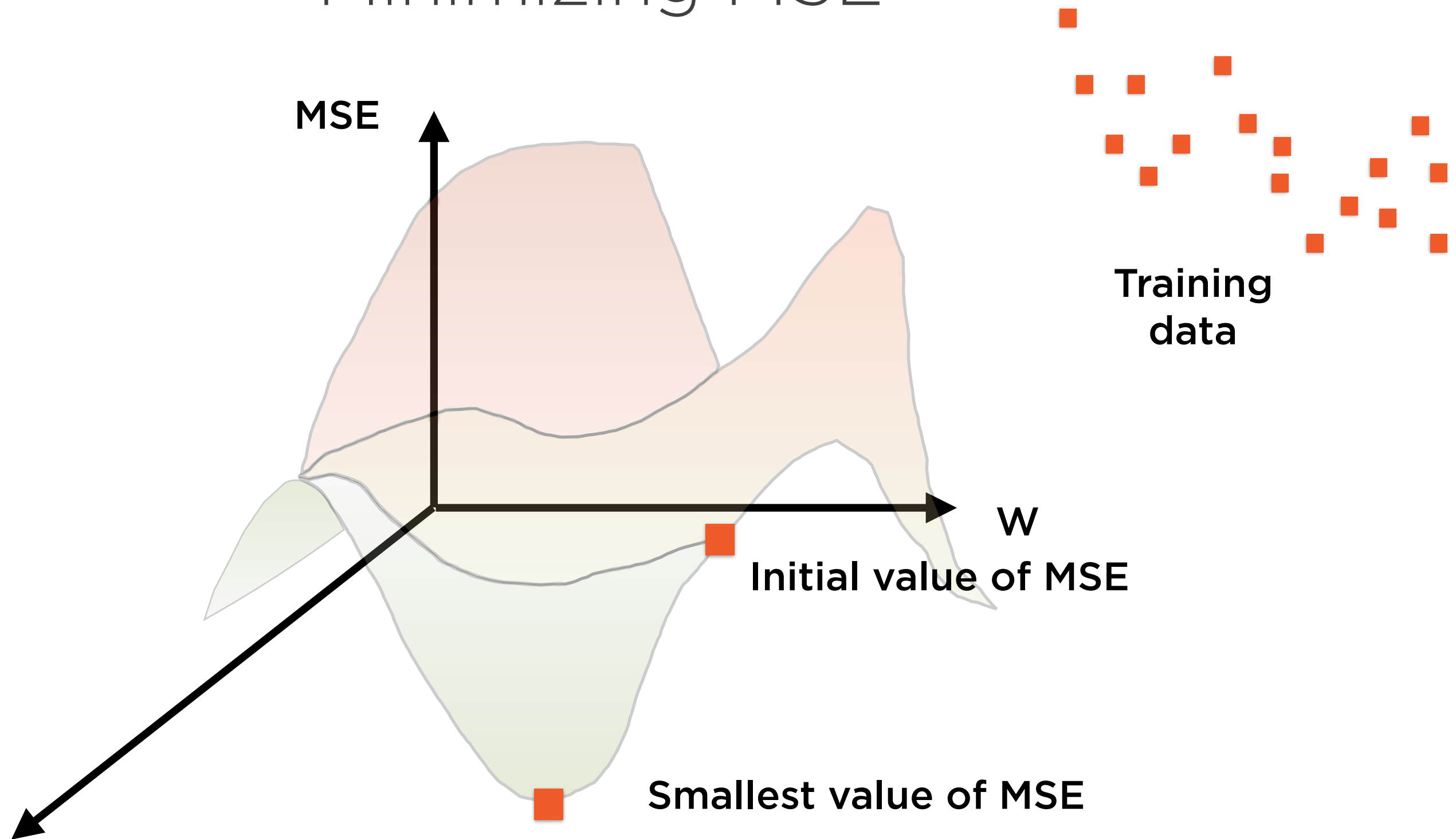
# Minimizing MSE



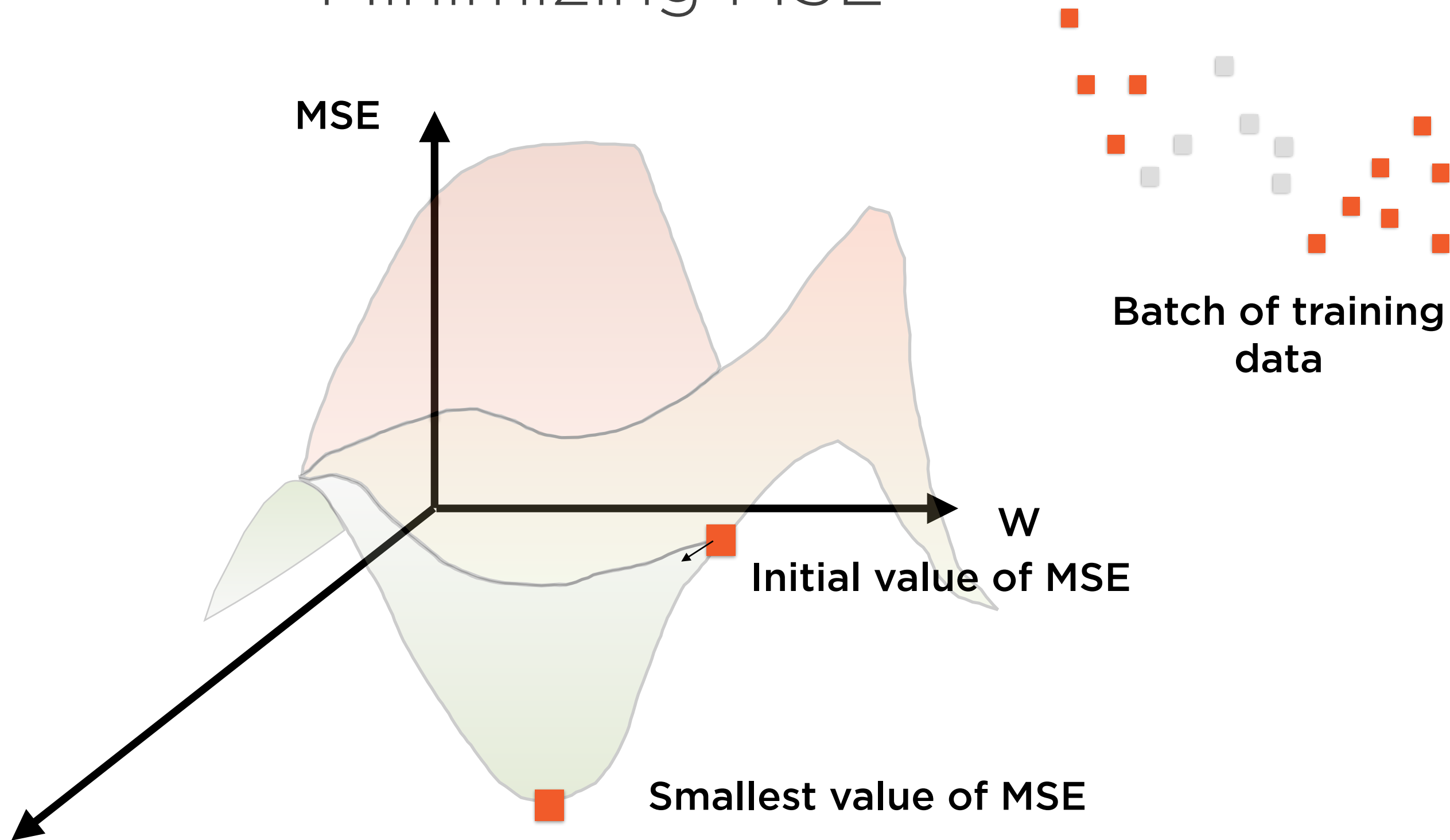
# Regression: The Simplest Neural Network



# Minimizing MSE



# Minimizing MSE



“Epoch”

MSE

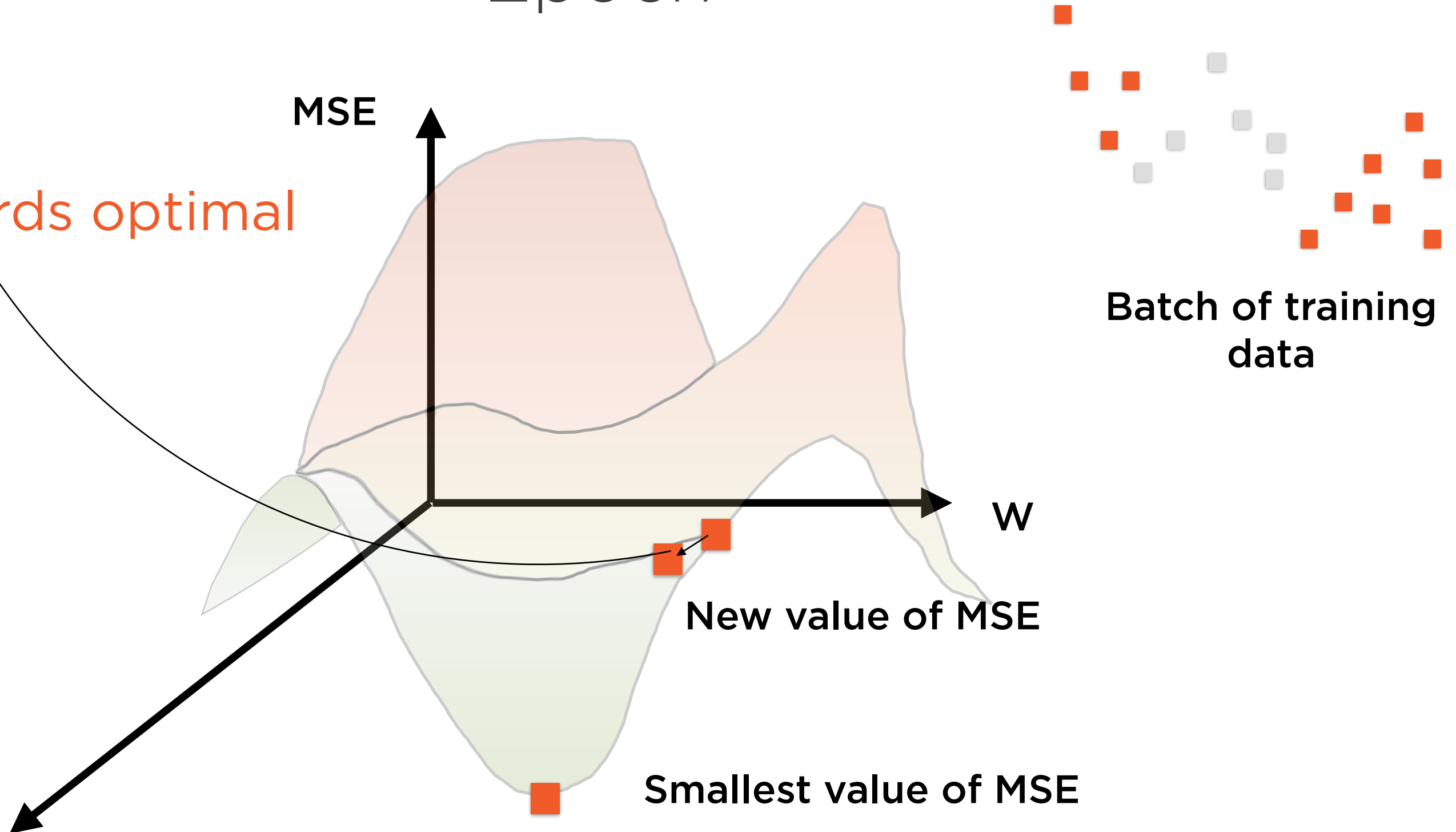
1 step towards optimal  
= 1 epoch

Batch of training  
data

W

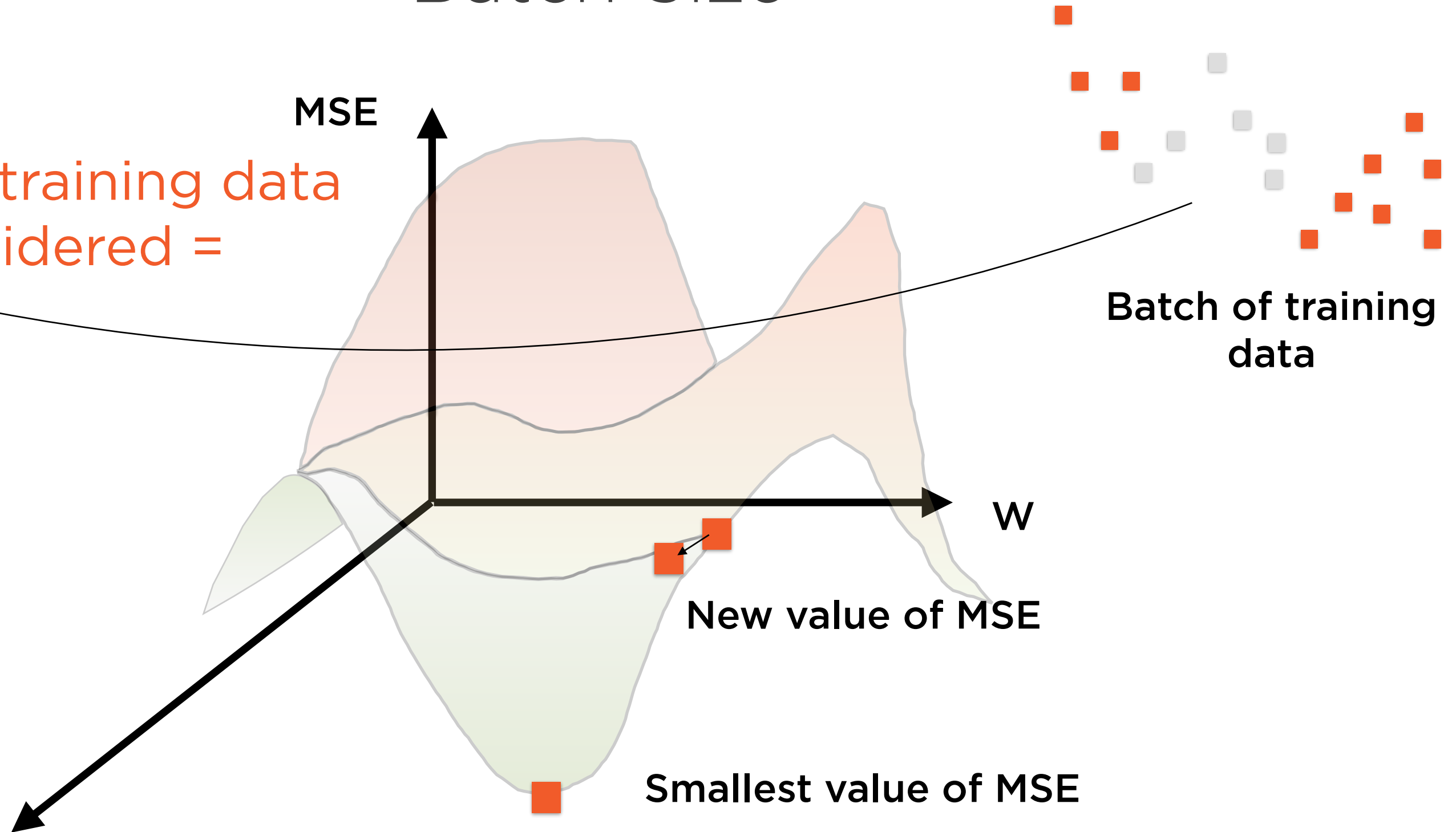
New value of MSE

Smallest value of MSE



“Batch Size”

Number of training data  
points considered =  
batch size





# “Batch Size”



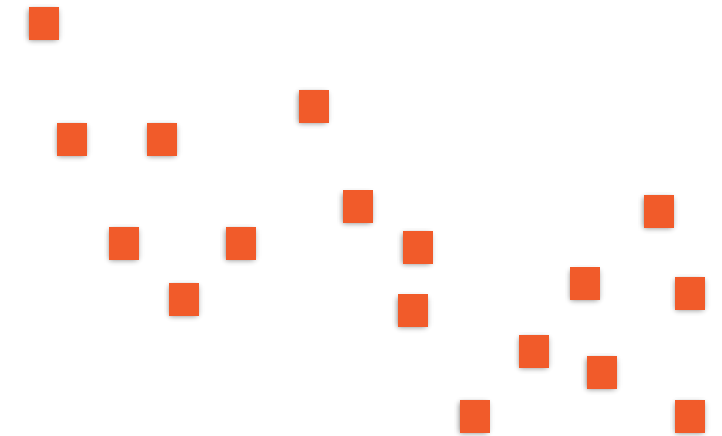
**Stochastic Gradient  
Descent**

1 point at a time



**Mini-batch Gradient  
Descent**

Some subset in each  
batch



**Batch Gradient  
Descent**

All training data in  
each batch

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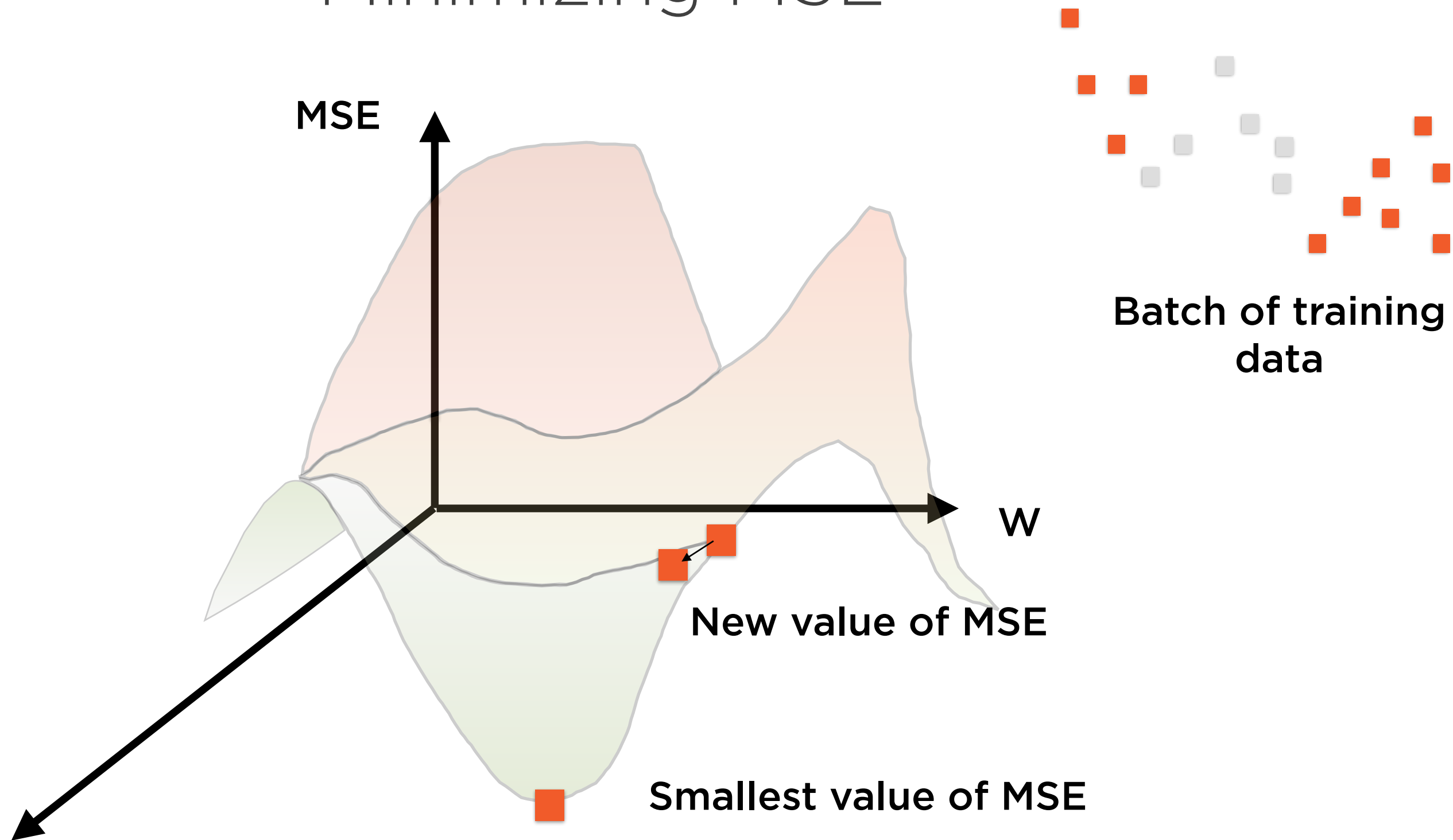
## Computation Graph

Neural network of 1 neuron  
Affine transformation suffices

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Improving goodness-of-fit

# Minimizing MSE



# Decisions in Training

**Initial values**

**Type of optimizer**

**Number of epochs**

**Batch size**

# Simple Regression



**Cause**

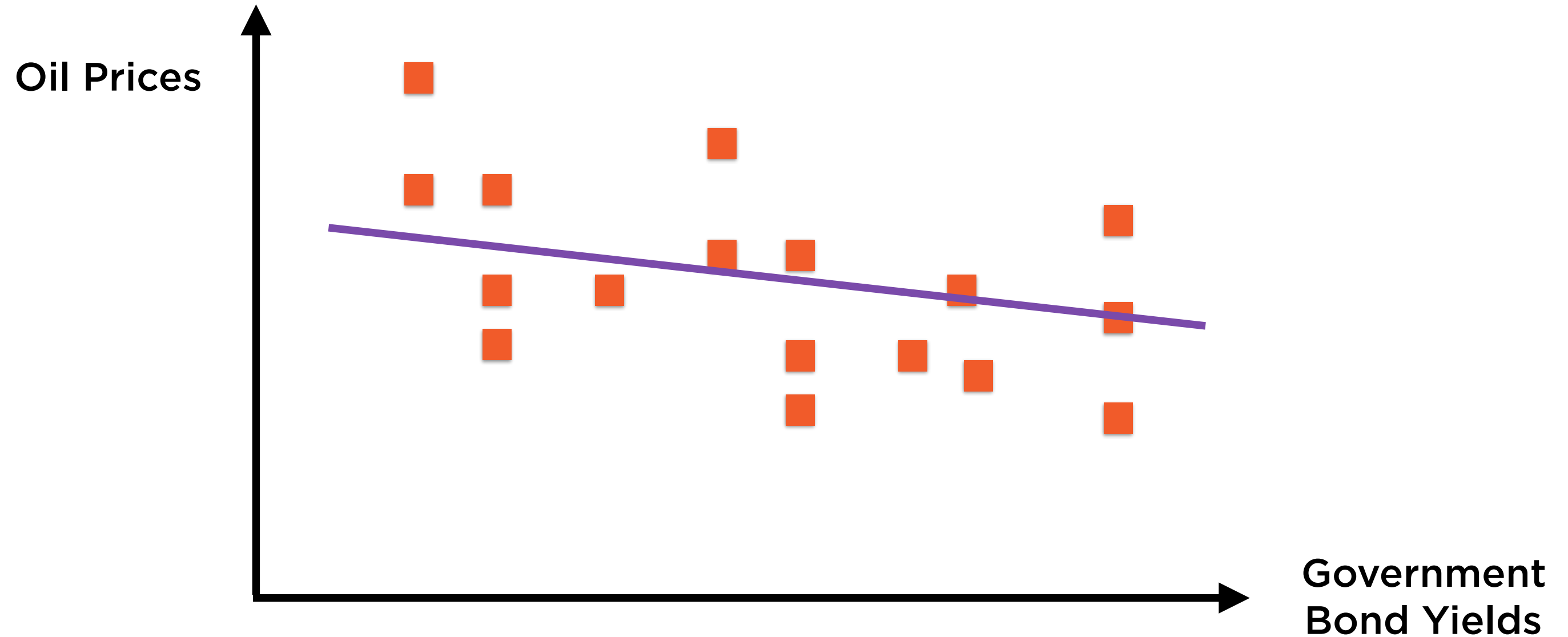
**Independent variable**



**Effect**

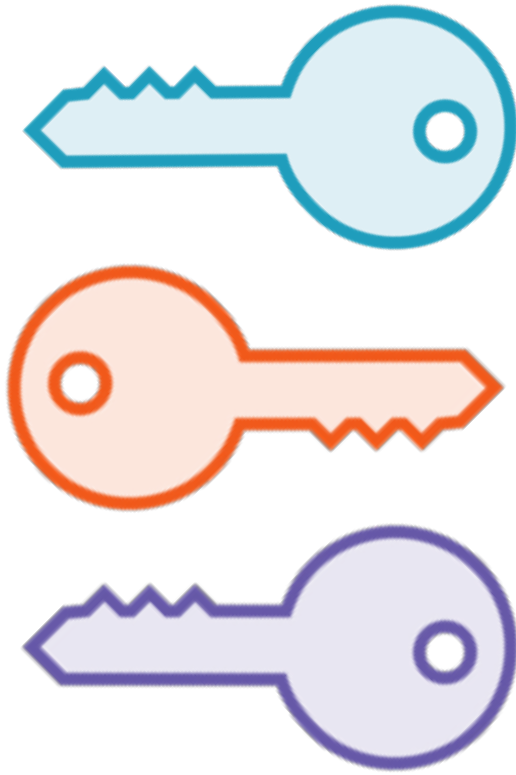
**Dependent variable**

# Simple Regression



One cause, one effect

# Multiple Regression



**Causes**

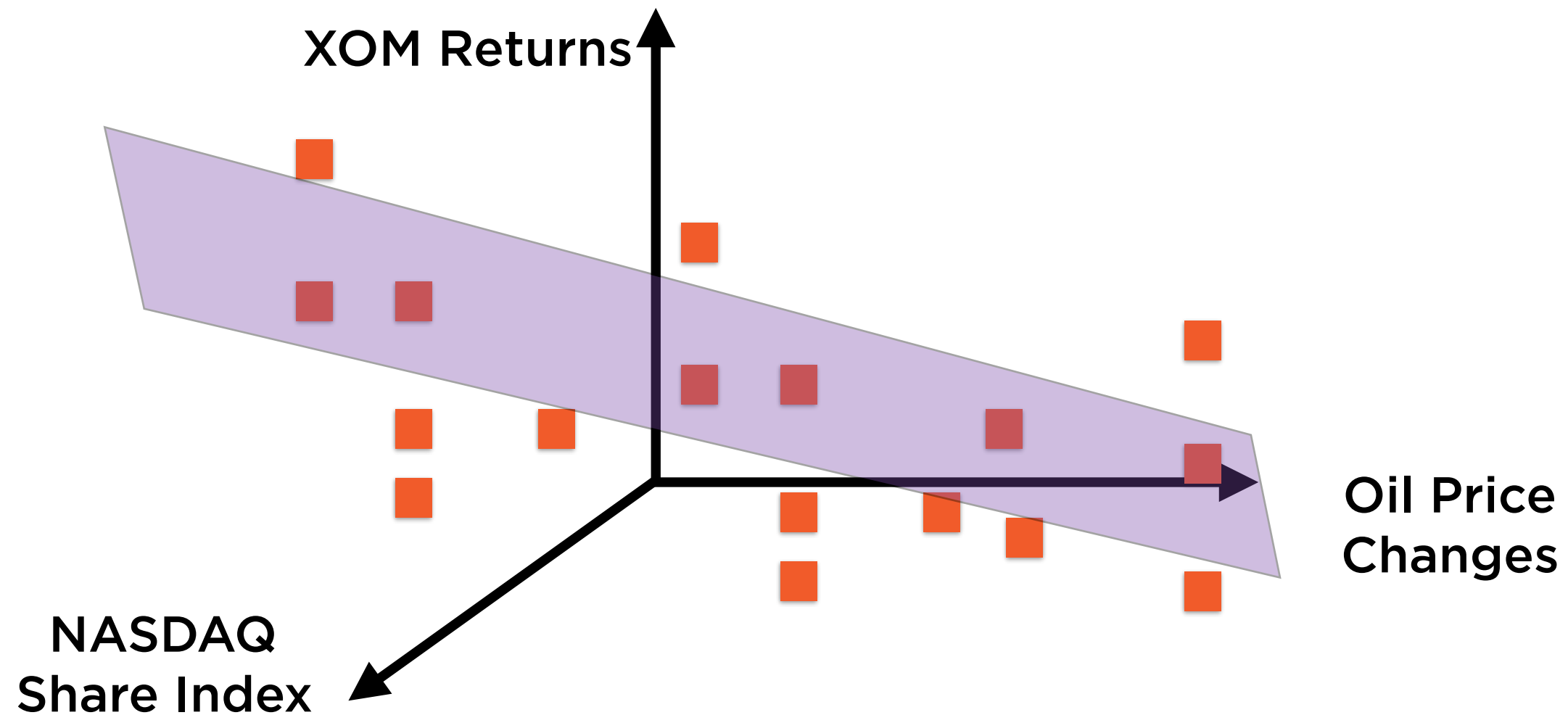
**Independent variables**



**Effect**

**Dependent variable**

# Multiple Regression



Many causes, one effect



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## Converged Model

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Compare to baseline

# Summary

**Implement linear regression without using TensorFlow**

**Define computation graph of just one neuron**

**Set up the cost function**

**Use various gradient descent optimizers**

**Understand gradient descent process**

**Train the model to get a converged regression model**