– **Table of Contents**

– CONTENTS

Ultrasonic Diffraction

Subtitle

Dishant Tanmay (23110100), Faayza Vora (23110109), Goraksh Bendale (23110118), Haravath Saroja (23110127)

***Abstract*—This ultrasonic diffraction experiment is used to determine the velocity of ultrasonic waves, the bulk modulus(β) and estimate the compressibility of a liquid κ. The ultrasonic waves by the transducer pass through the crystal to the liquid and form a stationary wave pattern due to resonance. The wavelength of the ultrasonic wave is calculated using the diffraction pattern, and the velocity is determined using the crystal oscillator's frequency and the laser's wavelength. Then, calculate the bulk modulus and, hence, the compressibility of the given liquid. Thus, this experiment allows us to know about ultrasonic waves and the properties of liquid.**

***Index Terms*—adiabatic compressibility (κ), Bulk modulus (β, ultrasonic waves, order of diffraction.**

I. Objective

The Experiment aims to :

1) Find the velocity of the ultrasonic waves in liquid using the ultrasonic diffraction apparatus.
2) Find the bulk modulus of the liquid used
3) Estimate the compressibility of the liquid.

II. Materials and Methods

A. Materials

Laser, laser mount, glass tank with liquid, crystal with mount, glass tank holder, optical rail(1500mm), RF oscillator, Cell mount with linear translation stage and Pinhole detector, Output measurement unit

B. Methods

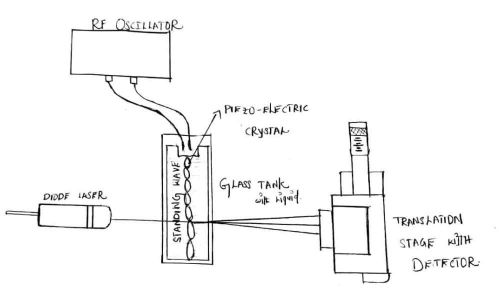

Fig. 1. Optical system for observation of diffraction by Ultrasonic waves

The Transducer generates ultrasonic waves, which are reflected from the walls of the glass container. The interference of incident and reflected rays leads to the formation of Standing waves in the glass container.

We can calculate the speed of the ultrasonic waves by using the formula:

$$V = \nu\Lambda \tag{1}$$

where V is the velocity of the ultrasonic waves, ν is the frequency of the crystal oscillator, which is set to 3MHz, and Λ is the wavelength of sound which is given by the formula:

$$\Lambda = \frac{n\lambda}{\sin\theta} \tag{2}$$

where n is the order of diffraction, λ is the wavelength of the source that is laser diode which is 650nm and θ is the angle of diffraction given by:

$$\tan^{-1}\frac{D}{L} \tag{3}$$

where D is the order of length, and L is the distance between the crustal oscillator and the detector.

In order to calculate the Bulk modulus (β) of the given liquid, we use the formula:

$$\beta = \rho V^2 \tag{4}$$

where ρ is the density of the liquid (which is assumed to be $1000\frac{kg}{m^3}$) and V is the velocity of the ultrasonic wave in the liquid.

The adiabatic compressibility can be found out by taking reciprocal of the bulk modulus

$$\kappa = \frac{1}{\beta} = \frac{1}{\rho V^2} \tag{5}$$

III. Procedure

1) Set up the ultrasonic diffraction apparatus.
2) Mount and fix the glass tank with the liquid in the glass tank holder. Place the crystal with the mount in the liquid and place it perpendicular to the optical rail.
3) Use the RF oscillator and crystal to produce ultrasonic waves in the liquid. These waves will generate a standing wave which can be used as a diffraction grating for the laser diode.
4) Pass a diode laser beam through the liquid at a right angle to the wave. The liquid will diffract light at angles determined by the wavelength of the ultrasonic waves.
5) Use the kinematic laser mount to position the laser so that the diffracted light strikes the pinhole detector.
6) Use the linear translation stage to scan the detector's position and note the micrometre reading.
7) You can use the output measurement unit to take the data and calculate the velocity of the ultrasonic waves and the bulk modulus of the liquid using the formulas $V = \nu\Lambda$ and $\beta = \rho V^2$, where V is the velocity of ultrasonic wave, ν is the frequency of crystal oscillator, ρ is the liquid

density, and Λ is the wavelength of the sound which is given by:

$$\Lambda = \frac{n\lambda}{\sin\theta} \tag{6}$$

where n is the order of diffraction, λ is the wavelength of the laser used and θ is the angle of diffraction.

8) Now we can calculate the compressibility (κ) of the liquid, which is the reciprocal of bulk modulus
The graph, when plotted, shows the distance (D) between the central bright light and the nth order spot.

IV. Observation and Graphs

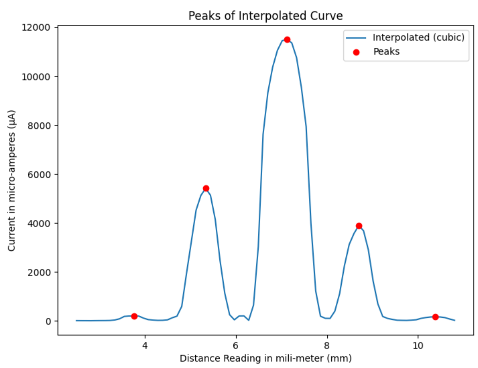

Fig. 2. Current detected plotted as a function of distance.

V. Calculations and Discussion

A. Calculations

As discussed above,

$$\Lambda = \frac{n\lambda}{\sin\theta}$$

where θ is the angle of diffraction given by:

$$\theta = \tan^{-1}(\frac{D}{L})$$

The following values were obtained:
$n = 1, 2$
$\nu = 3.02$ MHz
$\lambda = 650$ nm

For order length (D), we shall take the mean

Order	|Left - Central|	|Right - Central|	Mean Reading (D)
1st	1.53 mm	1.58 mm	1.55 mm
2nd	3.36 mm	3.26 mm	3.31 mm

Order	D	$\frac{D}{L}$	$\tan^{-1}(D/L)$	$\sin(\theta)$	λ	$\Lambda = \frac{n\lambda}{\sin(\theta)}$	V (m/s) = $\nu\Lambda$
1	0.00155 m	0.00127	0.00127	0.0012 rad	$650 * 10^{-9}$ m	0.00051 m	1540.2 m/s
2	0.00331 m	0.00271	0.00271	0.0027 rad	$650 * 10^{-9}$ m	0.00048 m	1447.0 m/s

Therefore,

$$\text{Mean Velocity} = (1540.2 + 1447.0)/2 = 1493.5 \text{ m/s}$$

B. Discussions

From the above calculations, we can clearly see that the Velocity of sound in the liquid is 1493.5 m/s Given this velocity we can also find out the bulk modulus and adiabatic compressibility:

$$\text{Velocity} = 1493.5 \text{ m/s}$$

$$\text{Bulk Modulus} = \beta = \rho V^2 = 1000\times(1493.5)^2 = 2.23\times10^{-9} \text{ Pa}$$

$$\text{Adiabatic Compressibility} = K = \frac{1}{\rho V^2} = \frac{1}{\beta} = 4.48\times10^{8} \text{ Pa}^{-1}$$

VI. Error analysis

VII. Finding Error in V mathematically

We know:

$$V = \nu\Lambda \tag{7}$$

Taking logs on both sides, we get:

$$log(V) = log(\nu) + log(\Lambda) \tag{8}$$

we also know that:

$$\Lambda = \frac{n\lambda}{\sin\theta} \tag{9}$$

Substituting $\theta = \tan^{-1}\frac{D}{L}$ in the above equation we get:

$$\Lambda = \frac{n\lambda}{\sin\tan^{-1}\frac{D}{L}} \tag{10}$$

$$\Lambda = \frac{n\lambda}{\sin(\sin^{-1}\frac{D}{\sqrt{D^2+L^2}})} \tag{11}$$

$$\therefore \Lambda = \frac{n\lambda}{\frac{D}{\sqrt{D^2+L^2}}} \tag{12}$$

$$\therefore \Lambda = \frac{n\lambda \times \sqrt{D^2+L^2}}{D} \tag{13}$$

Taking log on both sides and the magnitude of each term, we get:

$$log(\Lambda) = log(n) + log(\lambda) + \frac{1}{2}log(D^2+L^2) + log(D) \tag{14}$$

Substituting eqn (16) in eqn (10) we get:

$$log(V) = log(\nu)+log(n)+log(\lambda)+\frac{1}{2}log(D^2+L^2)+log(D) \tag{15}$$

Differentiating both sides, we get:

$$\frac{\Delta V}{V} = \frac{\Delta\nu}{\nu} + \frac{\Delta n}{n} + \frac{\Delta\lambda}{\lambda} + \frac{1}{2}\frac{2\Delta D + 2\Delta L}{D^2+L^2} + \frac{\Delta D}{D} \tag{16}$$

Since, n,λ are constant, thus $\frac{\Delta n}{n} = 0$ and $\frac{\Delta\lambda}{\lambda} = 0$
Now, for V = 1540.2 m/s, D= 1.55mm, L=1220 mm we get:

$$\frac{\Delta V}{1540.2} = \frac{0.01}{3} + \frac{1}{2}\frac{2(0.01)+2(1)}{1.55^2+1220^2} + \frac{0.01}{1.55} \tag{17}$$

$$\therefore \Delta V = 15.1 m/s \tag{18}$$

Now, for V = 1447.0 m/s, D= 3.31mm, L=1220 mm we get:

$$\frac{\Delta V}{1447.0} = \frac{0.01}{3} + \frac{1}{2}\frac{2(0.01)+2(1)}{3.31^2+1220^2} + \frac{0.01}{3.31} \tag{19}$$

$$\therefore \Delta V = 14.2 m/s \tag{20}$$

Thus, the average value of ΔV comes out to be:

$$\Delta V_{mean} = 14.6 m/s \tag{21}$$

Thus the final value of velocity is:

$$V = 1493.5 \pm 14.6 \text{ m/s} \tag{22}$$

VIII. Finding Error in β mathematically

We know that:

$$\beta = \rho V^2 \tag{23}$$

Taking log on both the sides we get:

$$log(\beta) = log(\rho) + 2log(V) \tag{24}$$

Differentiating bother the sides we get:

$$\frac{\Delta\beta}{\beta} = \frac{\Delta\rho}{\rho} + 2\frac{\Delta V}{V} \tag{25}$$

Since, n,ρ are constant, thus $\frac{\Delta\rho}{\rho} = 0$

$$\therefore \Delta\beta = 2.23 \times 10^{-9} \times (\frac{2 \times 14.6}{1493.5}) \tag{26}$$

$$\therefore \Delta\beta = 0.04 \times 10^{-9} Pa \tag{27}$$

Thus the final value of β is $(2.23 \pm 0.04) \times 10^{-9}$

IX. Finding Error in κ mathematically

We know that:

$$\kappa = \frac{1}{\beta} \tag{28}$$

Taking log on both sides and magnitude of each term, we get:

$$log(\kappa) = log(\beta) \tag{29}$$

Differentiating bother the sides, we get:

$$\frac{\Delta\kappa}{\kappa} = \frac{\Delta\beta}{\beta} \tag{30}$$

$$\therefore \Delta\kappa = 4.48 \times 10^{9} \times \frac{0.04 \times 10^{-9}}{2.23 \times 10^{-9}} \tag{31}$$

$$\therefore \Delta\kappa = 0.08 \times 10^{9} Pa \tag{32}$$

Thus the final value of κ is $(4.48 \pm 0.08) \times 10^9$

A. Conclusion

The ultrasonic diffraction experiment successfully calculates the velocity of ultrasonic waves in the given liquid is about 1493.5 ± 14.6
Also, the bulk modulus of the liquid (β) comes out to be $(2.23 \pm 0.04) \times 10^{-9}$ and the adiabatic compressibility (κ) is $(4.48 \pm 0.08) \times 10^9$. This experiment demonstrates a practical application of the diffraction phenomenon and provides a deeper understanding of standing waves and diffraction.

B. Future prospects

The future scope of this experiment may focus on:

1) Improving and clarifying the fringes by adjusting the setup's precision, and the distance between the laser and the cell mount.
2) Regulating brightness and temperature to reduce the fluctuations caused by changes in light intensity.
3) Minimising the external disturbances, as the given liquid generates standing waves on the basis of the frequency given by the RF oscillator.
4) Improving the management of liquids.
5) Using a transducer with a higher frequency to examine ultrasonic characteristics in liquids with shorter wavelengths.
6) Taking more readings to reduce the inaccuracy,

X. AUTHOR CONTRIBUTIONS

Name	Roll number	Contribution	Signature
Faayza Vora	23110109	Material and Methods, Procedure, Error analysis and Taking readings in lab	
Goraksh Bendale	23110118	Lab readings/equipment handling, Discussions, Calculations. observations graph	
Dishant Tanmay	23110100	Abstract, Objective , Procedure	
Haravath Saroja	23110127	Conclusion and future prospects	

TABLE I
AUTHOR'S CONTRIBUTION