ME 207: Fluid Dynamics

Pressure field across curved streamline

GROUP 11

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Abstract—This report examines the pressure field across curved streamlines, where fluid motion is influenced by centripetal acceleration and velocity variations. Unlike straight streamlines, curvature introduces pressure gradients due to centrifugal effects. Using Bernoulli's equation and Navier-Stokes equations, this study highlights the impact of curvature on fluid behavior, with applications in aerodynamics and hydrodynamics.

Objectives of the experiment

Our main objective is to analyse the pressure field across a curved streamline by analysing how pressure varies due to centrifugal forces in fluid motion. It requires our strong theoretical foundations of fluid dynamics, specifically the Navier-Stokes equation, to derive the mathematical relationship between pressure gradients and velocity. Additionally, it examines the key influencing factors such as curvature, velocity, density, and viscosity, offering a broader understanding of how these elements shape fluid behaviour beyond the idealised inviscid assumptions.

Introduction

Fluid moves differently in curved paths as compared to straight ones because it needs a force to keep turning. In straight flow, Bernoulli's principle says the total energy along a streamline stays the same. But when fluid moves in a curve, it must keep accelerating toward the center, which creates a pressure difference. This makes pressure higher on the outer side of the bend than on the inner side. Understanding this is important for designing pipes, turbines, and aircraft surfaces.

This study uses a U-shaped pipe with a uniform rectangular cross-section, where water flows steadily. To measure pressure changes, five ports are placed—two before the curve, one at the curve's peak, and two after it. These measurements help track pressure differences across the bend. The uniform pipe shape ensures that any pressure variations are caused only by the curve, while extra measurements confirm the flow speed stays the same.

The theory behind this is based on the Navier-Stokes equation and a modified Bernoulli equation for curved streamlines. Comparing real pressure readings with calculations helps confirm how curvature affects pressure. This experiment also teaches important skills, like setting up pressure taps, calibrating equipment, checking flow uniformity, and analyzing errors. Through this study, students learn key fluid dynamics concepts and gain hands-on experience in testing real-world fluid systems.

Preliminary theoretical discussion/calculations

In fluid dynamics, a streamline represents the path a fluid particle follows in steady flow, where velocity is always tangential to the streamline. When streamlines curve, fluid elements undergo centripetal acceleration, requiring a force balance to maintain their motion.

Curved streamlines result in a radial pressure gradient, which arises due to the need for a centripetal force to keep the fluid moving along the curved path. As a result, the pressure is higher on the outer side of the curvature compared to the inner side. This pressure variation is fundamental in aerodynamic flows, curved pipe flows, and rotating fluid systems.

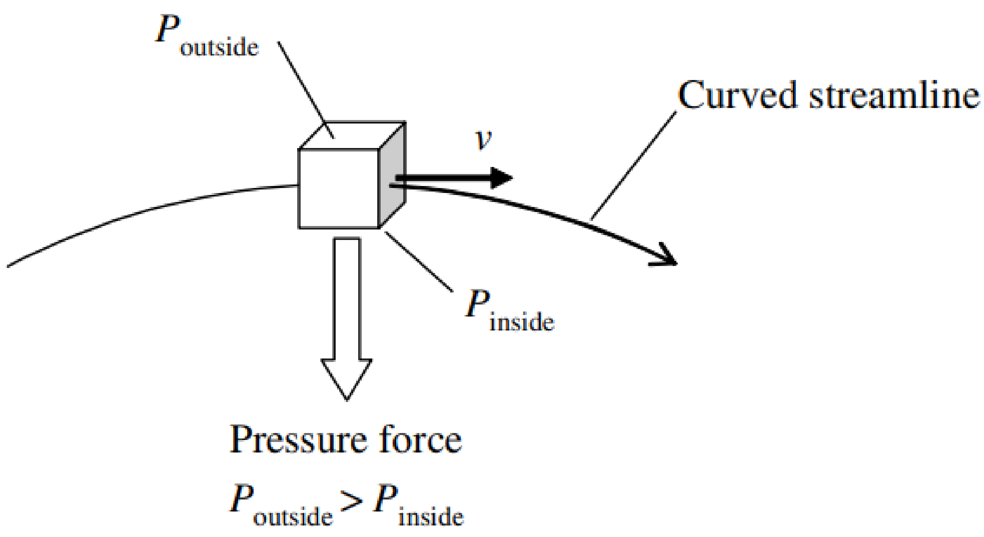

Fig.Fluid particle travelling along a curved streamline.

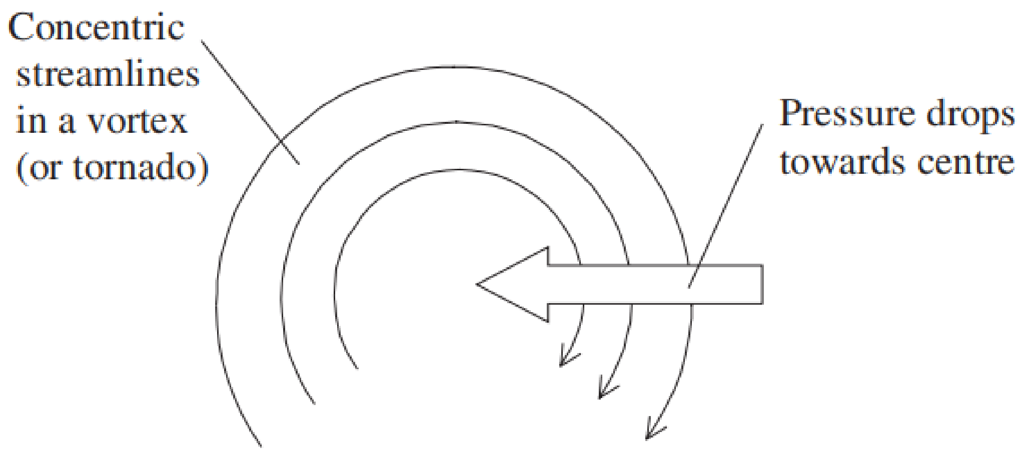

Fig. Pressure gradient across streamlines in a vortex(An example of curved streamline but we didnt use a vortex for the experiment)

The Navier-Stokes equation for a compressible, viscous fluid in cylindrical coordinates (r=radius of curvature,θ,z) includes terms for acceleration, pressure forces, viscous forces, and external forces:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r}\right) = -\frac{\partial P}{\partial r} + \mu\left(\nabla^2 v_r\right) + F_r$$

Where:

ϱ= fluid density
V$_r$ = radial velocity component
V$_\theta$ = tangential velocity component
P = pressure
μ= viscosity coefficient
F_r = external body force in the radial direction
$(\nabla^2)vr$ = viscous diffusion term
r= radius of curvature

To derive the equation for inviscid, steady, purely rotational flow, we make the following **assumptions**:

1. **Steady Flow → ** No time dependence

   $\partial v_r/\partial t = 0$

2. **Inviscid Flow → ** Ignore viscosity effects

   $\mu=0$

3. **Purely Rotational Motion → ** Negligible radial velocity

   $v_r=0$

4. **No External Forces → ** Ignore body forces in the radial direction

   $F_r = 0$

Applying these assumptions simplifies the Navier-Stokes equation to:

$$\boxed{\frac{dp}{dr} = \rho \frac{v_\theta^2}{r}}$$

This equation shows that pressure increases outward in curved streamline flow. The pressure gradient balances the inward-directed centripetal acceleration required for curved motion.

Based on the measurements done at the location, the volume flow rate was found to be approximately 300 cm^3/s, which is about 3×10^{-6} m^3/s. The area of the rectangular cross-section is 15 x 4.5 cm.

Velocity = Volume flow rate/ Area of cross-section
= $300 \times 10^{-6} / (15 \times 4.5 \times 10^{-4})$
= 0.0444 m/s

Using the simplified Navier-Stokes Equation we derived earlier, we get pressure variation across the curved section with varying radius as follows, also assuming the density of the water to be 1000 kg/m^3:

$$\frac{dp}{dr} = \frac{1000 \cdot (0.0444)^2}{0.15} = 13.1412$$

$$dp = \frac{\rho V_0^2}{r} \cdot dr$$

Integrate on both sides,

$$\int_{p_1}^{p_2} dp = \rho V_0^2 \int_{r_1}^{r_2} \frac{dr}{r}$$

If we consider the variation between points 1 and 2, then the limits of dp come out to be p1 and p2 and radius r1 and r2, respectively, and the final equation comes out to be as follows:

Consider r_1 to be 15cm and r_2 to be 30cm:

$$p_2 - p_1 = \rho V_0^2 ln(r)|_{r_1}^{r_2}$$

$$p_2 - p_1 = 1000 \cdot (0.0444)^2 \cdot ln(\frac{0.3}{0.15})$$
$$p_2 - p_1 = 1.36644$$

units of p_2 - p_1 is Pa.

Now, to visualise how much height difference is observed between the nearest and the furthest point of the curve, we follow these mathematical steps:

$$1.36644 = \rho \cdot g \cdot h_{difference}$$
$$h_{difference} = \frac{1.36644}{9.81 \cdot 1000}$$
$$h_{difference} = 0.1392\ mm$$

As we can see, the height difference is very small.

EXPERIMENTAL SETUP:

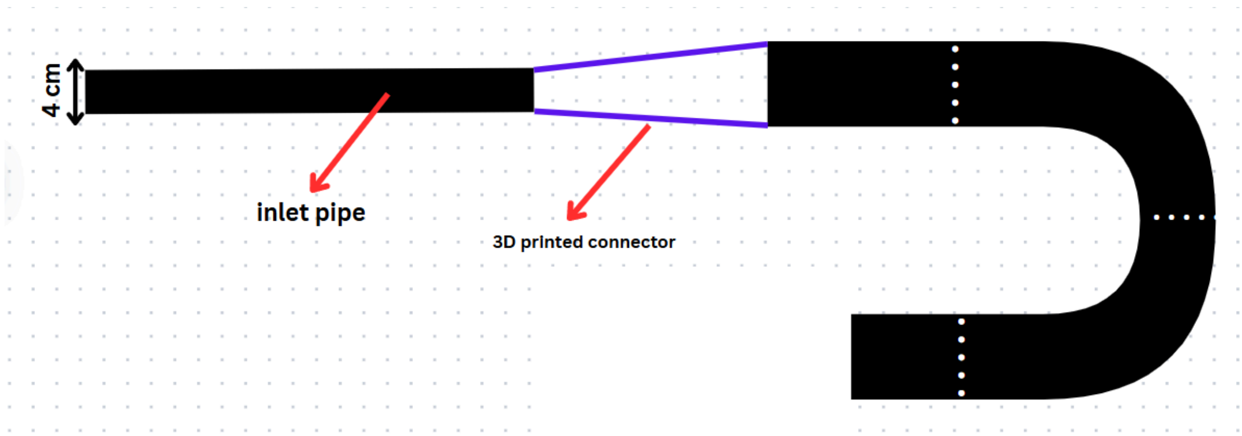

Fig. Entire setup including the U-shaped Pipe

The experimental setup consists of a PVC pipe with a diameter of 40 mm, serving as the water inlet. This pipe is connected to another PVC pipe via a 3D-printed connector, ensuring a secure and efficient transition between components. The connector is then linked to a U-shaped tube, which has a rectangular cross-section of 15 cm × 4.5 cm.

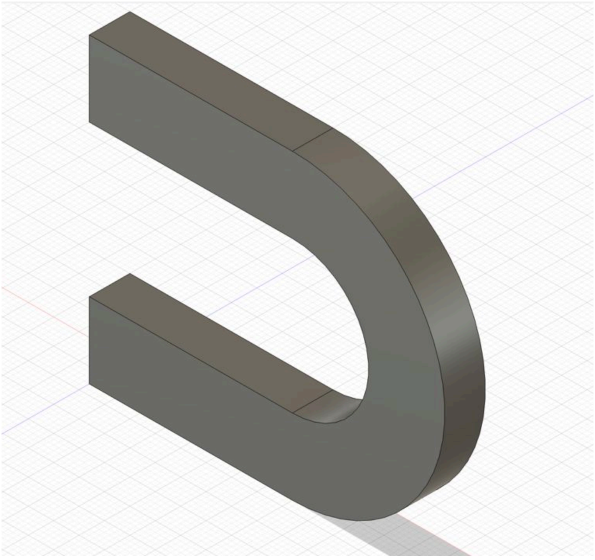

Fig. U shaped pipe

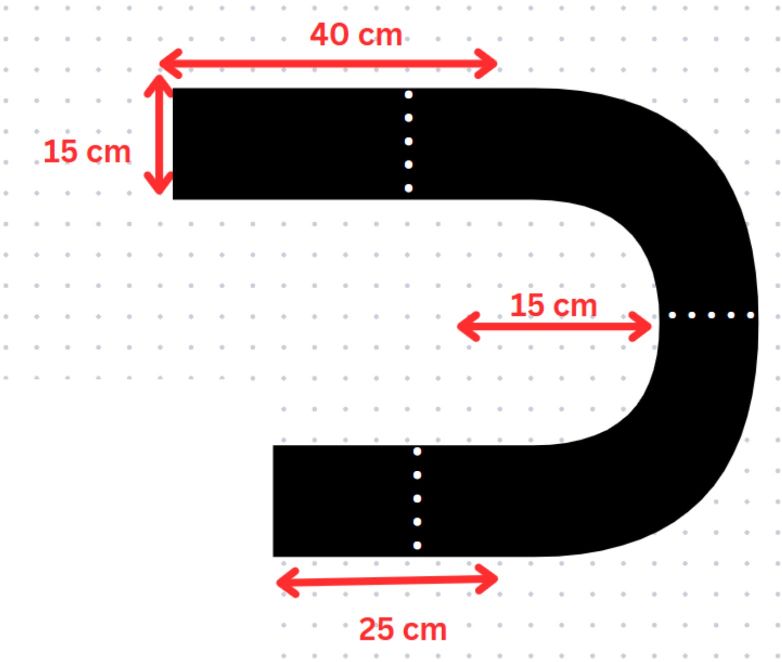

Fig. Top view of the pipe

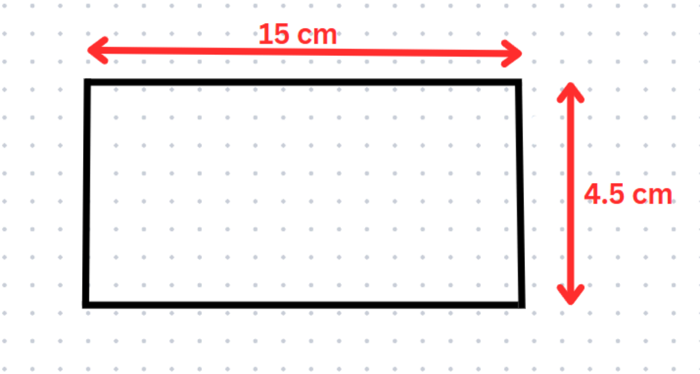

Fig. Side view of the pipe

Fig. U-shaped pipe

Fig. setup

Fig. Pressure tubes

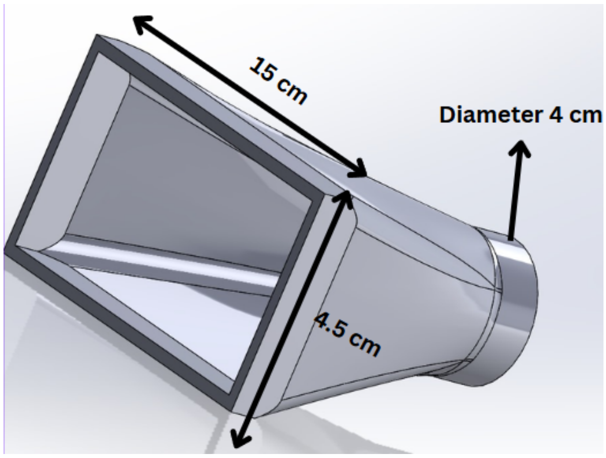

Fig. 3d printed connector

The U-shaped tube features a curved section forming a semi-circle with a radius of 15 cm, making it ideal for studying pressure variations due to streamline curvature. To measure pressure variations, three specific locations are selected:

1. Before the curved section
2. After the curved section
3. Within the curved section

At each of these measurement points, five pressure taps are installed, totaling 15 pressure taps across the setup. These taps enable detailed analysis of pressure distribution in response to fluid movement through the curved streamline.

PROCEDURE:

Water flow is allowed through a PVC section with a cross-section of 40mm radius, transitioning into a larger rectangular cross-section of 15 x 4.5 cm through a 3D printed attachment connecting the PVC pipe to our apparatus. Once the fluid fills the entire cross-section and begins to overflow through the pressure tappings, it is important to attach acrylic tubes to avoid air from becoming trapped in the acrylic tubes. Since the flow rate was not constant, the height of the fluid in the acrylic tubes fluctuated, prompting us to capture snapshots of these varying instances. To ensure a comprehensive understanding of the flow, we utilised 10 test pressure tappings strategically located both before and after the U-shaped curve to verify that the flow is consistent in velocity throughout.

RESULTS

Fig. Pressure variation across the curved region

These results demonstrate a gradual decline in pressure as the fluid moves from the outer side of the curve (where the baseline reading is taken) toward the inner side. Such a trend is consistent with the theoretical prediction that a radial pressure gradient develops to provide the necessary centripetal force.

The decision to adopt normalised height readings stems from the minimal variation in fluid column heights observed across the pressure taps, which aligns with our initial prediction. Since fluid pressure variations are subtle, direct height measurements in the acrylic tubes showed only minor differences, making it challenging to interpret absolute values meaningfully. By normalising the readings, the dataset is scaled relative to a reference value (e.g., the highest recorded height), allowing for easier comparison and ensuring that minor pressure differences are captured with greater clarity.

The normalised height readings obtained across the curve section are as follows:

Tap Position	Normalised Height
1 (furthest tap from the centre of the radius)	1
2	0.98
3	0.97
4	0.95
5	0.94

Fig. Pressure variation after the water flows through the curved surface

The normalised height readings obtained across the curve section are as follows:

Tap Position	Normalised Height

1 (furthest tap from the centre of the radius)	1
2	1
3	1.01
4	1
5	1.01

The nearly identical height levels of the tubes indicate that there is minimal pressure difference after the fluid exits the curved region.

DISCUSSIONS

The experimental observations align well with the theoretical expectations. The pressure decrease observed across the curved streamline validates the hypothesis that curvature induces a radial pressure gradient. Specifically, the pressure is highest on the outer side of the curve and progressively decreases toward the centre of the curvature - a direct consequence of the required centripetal force.

The reference reading (furthest pressure tap) is assumed to correspond to the region with maximum pressure (outer side), where the pressure gradient is initiated.

The subsequent lower readings (0.98 to 0.94) illustrate the consistent decline in pressure, confirming that the pressure gradient is not only measurable but also follows a systematic progression. This progression reflects the balance between inertial effects and the pressure force, as dictated by the equation.

$$\boxed{\frac{dp}{dr} = \rho \frac{v_\theta^2}{r}}$$

Uncertainty analysis of the results obtained

Although the gradient was visible, we were unable to establish a strong linear relationship.

Causes of error:

The experiment involves developing a setup to measure the pressure difference across a curved streamline. To do this, we designed a U-tube with a rectangular cross section made from acrylic tubes to measure the pressure heads at the curve—that is, at the apex of the U-tube. While doing so, the following are the errors that may have been induced:

1. A major issue plaguing the model, and a common occurrence with experiments involving water, is leakage. Despite our repeated efforts to seal any apertures with various adhesives like M-seal, Teflon tapes, gasket gel, Vetra, any tiny gaps left would give rise to leakages, thereby significantly affecting the water levels in the tubes.
2. The acrylic tubes, while rigid, are still prone to bending. A couple of tubes were not straight and slightly bent, hence their readings were not accurate.
3. As we directly supplied water into the U-tube through the valve using the water source instead of through a tank, there were a lot of air bubbles within the tubes, not allowing any gradient to form. Only after eliminating the air bubbles by blowing them out were we able to see some readings.
4. Head losses due to friction occur along the narrow tubes and the U-tube, which affect the pressure-head readings.
5. Due to unsteady and turbulent flow, the pressure head oscillates about a mean; this error was prevented by stabilising the flow and pressure heads.
6. Some measurements were taken close to the boundary, which can lead to viscous effects due to the boundary layer.
7. Dust and other contamination in the water can affect its density and viscosity.
8. Taking readings manually in a narrow tube can also induce parallax error.

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