ME206 Statics and Dynamics

Group 9, Experiment 3

STUDY OF NEWTON'S SECOND LAW IN AN INERTIAL & NON-INERTIAL FRAME

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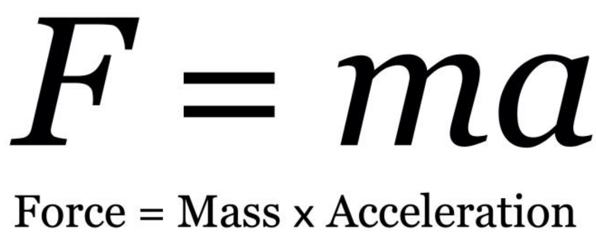
Introduction

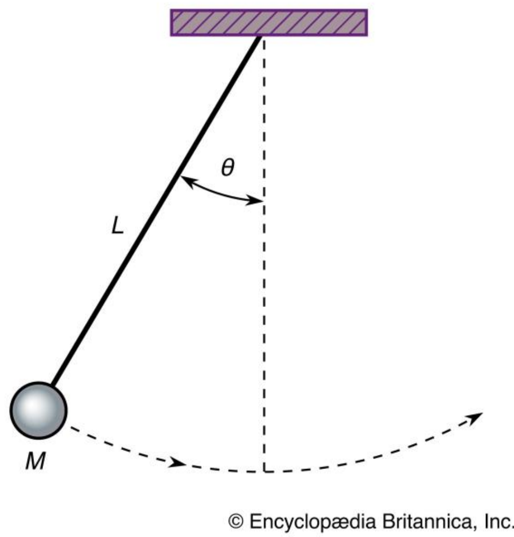
In classical mechanics, Newton's second law is a important fundamental part, linking the force exerted on an object to its acceleration and mass. This law tells us that an object's acceleration is directly proportional to the net force applied, aligned with the force's direction, and inversely proportional to the object's mass. This principle has been thoroughly confirmed in inertial reference frames, such as when observed from a stationary position on the ground.

However, things get interesting when we look at Newton's second law from a non-inertial reference frame, like when you're inside a moving elevator. This experiment dives into how the law operates in both inertial and non-inertial frames.

We're using a pendulum setup mounted on an elevator for this. As the elevator goes up or down, it influences the pendulum's motion. We've attached a sensor to the pendulum's bob to track its acceleration. By capturing these accelerations and forces in both frames, we aim to show how the application of Newton's second law varies.

Our objective is to highlight how reference frames impact our understanding of motion and forces, underscoring their crucial role in interpreting Newton's second law. Typically, this law, expressed as F=ma, holds in inertial frames where observers are at rest or moving at a constant velocity. But in non-inertial frames, where observers are accelerating, we need to account for an additional factor called a pseudo force to accurately describe an object's motion. Through this experiment, we'll explore how Newton's second law applies in both inertial and non-inertial frames using a straightforward yet insightful pendulum setup on an elevator.





Aim

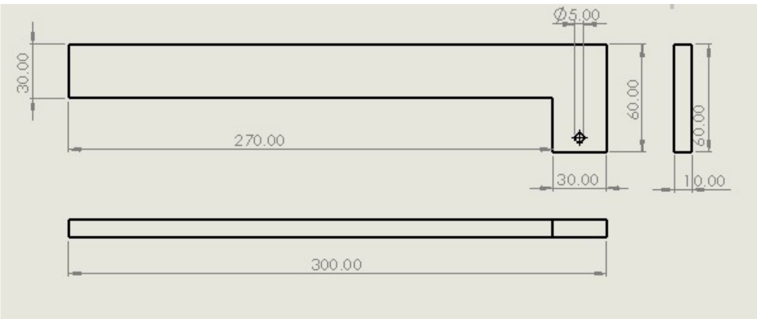
The primary aim of this experiment is to investigate and compare the application of Newton's second law in both inertial and non-inertial reference frames. Specifically, we seek to:

1. **Demonstrate the Validity of Newton's Second Law in Inertial Frames**: By observing the pendulum's motion when the elevator is stationary or moving at a constant velocity, we aim to confirm that the law F=ma holds true under these conditions.
2. **Examine the Impact of Non-Inertial Frames**: By studying the pendulum's behavior as the elevator accelerates or decelerates, we aim to illustrate how the introduction of pseudo forces affects the application of Newton's second law.
3. **Highlight the Role of Reference Frames**: Ultimately, we aim to emphasize the importance of reference frames in the interpretation of physical laws, showing how the choice of frame can significantly influence the observed dynamics of a system.

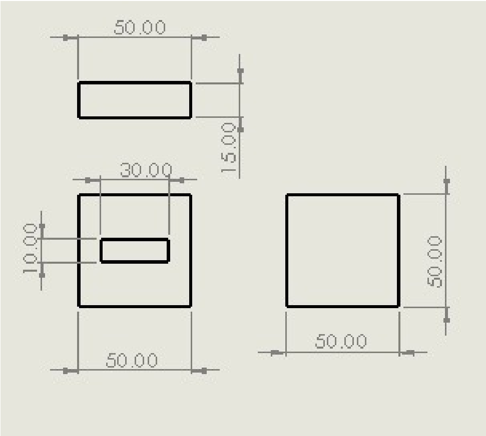
By achieving these objectives, the experiment will provide a clearer understanding of the nuances of Newton's second law and the broader principles of classical mechanics.

Engineering Drawings

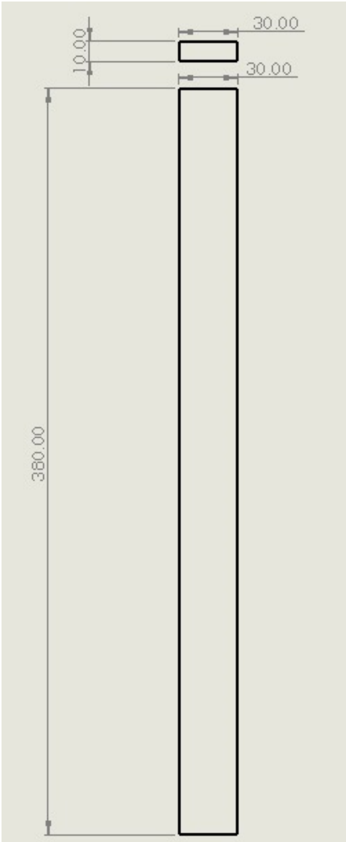
Our experiment consists of a pendulum and a pendulum stand. This pendulum stands were initially designed in Autodesk Inventor. The engineering Drawings of the stand is given below. All dimensions are in mm



Horizontal part of pendulum stand



Base of pendulum stand



Verticle part of pendulum stand

Materials Data

For this experiment we have used:

- 5mm MDF sheet to cut three parts of pendulum stand that have been discussed in the Experimental designs.
- A white thread as string of the pendulum
- A 3D printed ball as a point mass for pendulum. The diameter of this ball is 2.5cm

Fabrication Details

- In order to make the pendulum stand we made an Autodesk model (drawings of which are attached above) and then converted them to the DXF files. These DXF files were then imported to LaserCad and then cut from the MDF sheet using the Laser cut technology available in Tinkerer's Lab at IITGN.
- We then used 3D printing technology to print the ball of diameter 2.5 cm which acted as a particle in our experiment.
- A thread was use as a string for pendulum and these were assembled using adhesive Vetra.

Theoretical Analysis

Inertial Reference Frame

In an inertial reference frame, where the observer is at rest or moving with a constant velocity, Newton's second law is expressed as:

$$F = ma$$

where F is the net force acting on the object, m is the mass of the object, and a is the acceleration of the object.

For a simple pendulum in an inertial frame, the forces acting on the pendulum bob include the gravitational force mg and the tension in the string T. The net force in the vertical direction is:

$$F_{\text{net}} = mg - T\sin(\theta)$$

where θ is the angle the pendulum makes with the vertical. For small angles, $\sin(\theta) \approx \theta$, and the equation of motion simplifies to:

$$ma = mg - T\theta$$

Since the tension T is approximately equal to mg for small angles, the equation reduces to:

$$ma = -mg\theta$$

Thus, the acceleration a of the pendulum bob is:

$$a = -g\theta$$

This describes the simple harmonic motion of the pendulum with a period T given by:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where L is the length of the pendulum.

Non-Inertial Reference Frame

In a non-inertial reference frame, such as an accelerating elevator, additional pseudo forces (also known as fictitious forces) must be considered. The general form of Newton's second law in a non-inertial frame is:

$$F_{\text{net}} = ma_{\text{relative}} + ma_{\text{frame}}$$

where a_{relative} is the acceleration of the object relative to the non-inertial frame, and a_{frame} is the acceleration of the frame itself.

For the pendulum in an accelerating elevator, the pseudo force due to the elevator's acceleration a_{elevator} must be included. The net force on the pendulum bob becomes:

$$F_{\text{net}} = mg - T\sin(\theta) - ma_{\text{elevator}}$$

For small angles, this simplifies to:

$$ma_{\text{relative}} = mg - T\theta - ma_{\text{elevator}}$$

Assuming the tension T is approximately mg, the equation reduces to:

$$ma_{\text{relative}} = -mg\theta - ma_{\text{elevator}}$$

Thus, the relative acceleration a_{relative} of the pendulum bob is:

$$a_{\text{relative}} = -g\theta - a_{\text{elevator}}$$

This equation shows that the pendulum's motion is influenced by the elevator's acceleration. If the elevator is accelerating upwards ($a_{\text{elevator}} > 0$), the effective gravitational acceleration experienced by the pendulum is $g + a_{\text{elevator}}$. Conversely, if the elevator is accelerating downwards ($a_{\text{elevator}} < 0$), the effective gravitational acceleration is $g - a_{\text{elevator}}$.

Comparison of Accelerations

By measuring the acceleration of the pendulum bob using a sensor, we can compare the observed accelerations in both inertial and non-inertial frames. In the inertial frame, the acceleration should match the theoretical prediction of simple harmonic motion. In the non-inertial frame, the measured acceleration will deviate from this prediction due to the pseudo force introduced by the elevator's acceleration.

Conclusion

This theoretical analysis demonstrates how the application of Newton's second law varies between inertial and non-inertial reference frames. In an inertial frame, the pendulum exhibits simple harmonic motion governed by gravitational acceleration. In a non-inertial frame, the pendulum's motion is influenced by

Data Analysis

We conducted two calibrations to find the acceleration of the lift using the Physics Toolbox software available on the phone. The results of the calibrations were:

$$a_1 = 0.4776\,\mathrm{m/s^2}$$

$$a_2 = 0.4895\,\mathrm{m/s^2}$$

We assumed the average acceleration as:

$$a \approx 0.48\,\mathrm{m/s^2}$$

The effective gravitational acceleration can be calculated as:

$$g_{\text{eff}} = g + a = 9.81\,\mathrm{m/s^2} + 0.48\,\mathrm{m/s^2} = 10.29\,\mathrm{m/s^2}$$

Introduction

For small oscillations, the time period T of a simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}}$$

where L is the length of the pendulum, g_{eff} is the effective gravitational acceleration.

However, this formula assumes small angles, where $\sin\theta \approx \theta$. For larger initial release angles θ_0, we must consider the exact form of the equation of motion and modify the time period.

Time Period for Large Angles

Derivation

1. Equation of Motion:

For a simple pendulum released from an angle θ_0, the equation of motion can be written as:

$$\frac{d^2\theta}{dt^2} + \frac{g_{\text{eff}}}{L}\sin\theta = 0$$

2. Energy Conservation:

Using conservation of energy, we can express the velocity as a function of θ:

$$\frac{1}{2}mL^2\left(\frac{d\theta}{dt}\right)^2 = mg_{\text{eff}}L(\cos\theta - \cos\theta_0)$$

Solving for $\frac{d\theta}{dt}$, we get:

$$\frac{d\theta}{dt} = \sqrt{\frac{2g_{\text{eff}}}{L}(\cos\theta - \cos\theta_0)}$$

3. Separation of Variables:

Rearranging and integrating, we find:

$$T = 4\sqrt{\frac{L}{g_{\text{eff}}}} \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}}$$

where $k = \sin\left(\frac{\theta_0}{2}\right)$.

Conclusion

The time period for a pendulum released from a large angle θ_0 is given by:

$$T = 4\sqrt{\frac{L}{g_{\text{eff}}}} \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}}$$

This integral can be evaluated numerically for specific values of L, g_{eff}, and θ_0.

Given Parameters (Inertial Frame)

The problem specifies the following values:

- Release angle, $\theta_0 = 45^\circ$
- Length of the pendulum, $L = 0.16\,\text{m}$
- Effective gravitational acceleration, $g_{\text{eff}} = g = 9.81\,\text{m/s}^2$

Time Period for Large Angle

The time period T of a simple pendulum for a large release angle θ_0 is given by:

$$T = 4\sqrt{\frac{L}{g_{\text{eff}}}} \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}}$$

where $k = \sin\left(\frac{\theta_0}{2}\right)$.

Substitute Values

Convert the release angle θ_0 to radians:

$$\theta_0 = 45^\circ = \frac{\pi}{4} \text{ radians}$$

Then,

$$k = \sin\left(\frac{\pi}{8}\right) \approx 0.3827$$

Now substitute $L = 0.16\,\text{m}$ and $g_{\text{eff}} = 9.81\,\text{m/s}^2$:

$$T = 4\sqrt{\frac{0.16}{9.81}} \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - (0.3827)^2 \sin^2\phi}}$$

Numerical Evaluation

This integral can be evaluated numerically to find:

$$T \approx 0.837\,\text{seconds}$$

Given Parameters (Non-Inertial Frame/Lift)

The problem specifies the following values:

- Release angle, $\theta_0 = 45°$
- Length of the pendulum, $L = 0.16\,\text{m}$
- Effective gravitational acceleration, $g_{\text{eff}} = g + a$

where $g = 9.81\,\text{m/s}^2$ and $a = 0.48\,\text{m/s}^2$ (where, a is the acceleration of the lift going upwards).

Thus,

$$g_{\text{eff}} = 9.81 + 0.48 = 10.29\,\text{m/s}^2$$

Time Period for Large Angle

The time period T of a simple pendulum for a large release angle θ_0 is given by:

$$T = 4\sqrt{\frac{L}{g_{\text{eff}}}} \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - k^2 \sin^2\phi}}$$

where $k = \sin\left(\frac{\theta_0}{2}\right)$.

Substitute Values

Convert the release angle θ_0 to radians:

$$\theta_0 = 45^\circ = \frac{\pi}{4} \text{ radians}$$

Then,

$$k = \sin\left(\frac{\pi}{8}\right) \approx 0.3827$$

Now substitute $L = 0.16\,\mathrm{m}$ and $g_{\text{eff}} = 10.29\,\mathrm{m/s^2}$:

$$T = 4\sqrt{\frac{0.16}{10.29}} \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - (0.3827)^2 \sin^2 \phi}}$$

Numerical Evaluation

This integral can be evaluated numerically to find:

$$T \approx 0.815\,\text{seconds}$$

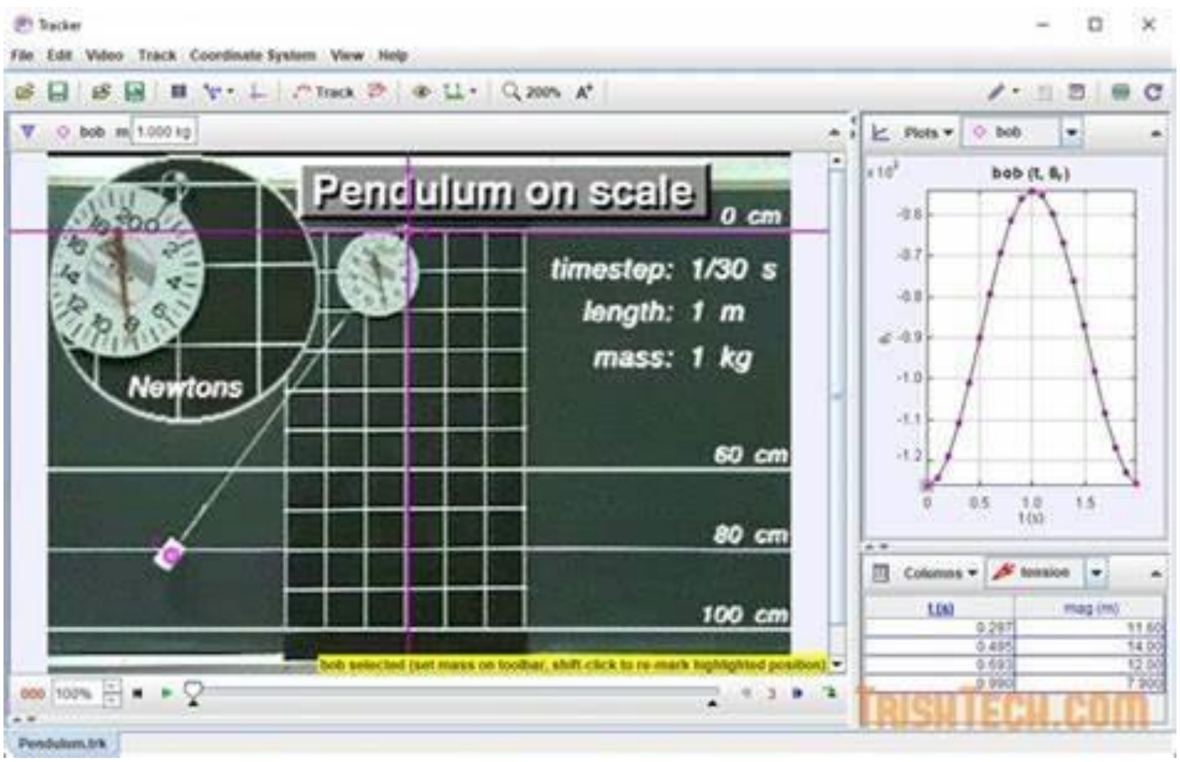
Conclusion

The time period of the pendulum, accounting for a release angle of 45° and $g_{\text{eff}} = 9.81\,\mathrm{m/s^2}$ is approximately $T = 0.837\,\text{seconds}$ and $g_{\text{eff}} = 10.29\,\mathrm{m/s^2}$ is approximately $T = 0.815\,\text{seconds}$.

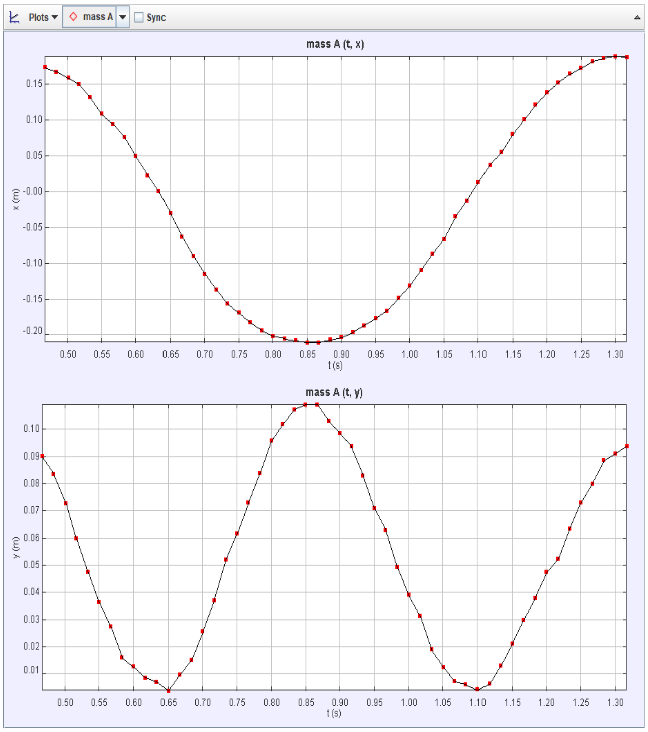
Measurement Techniques

In this experiment we have used the elevator as our Non-Inertial frame. To get precise acceleration of the elevator we have used physics toolbox.

We started the experiment by calculating the time period of the pendulum in inertial frame which was nothing but the room itself and then we took the pendulum to the elevator.
We took a slow-motion video for both the frames of reference using camera which was then imported in a video analysis software 'Tracker' which helped us determine the time period of the pendulum in both the reference frames which is needed further in analysis.

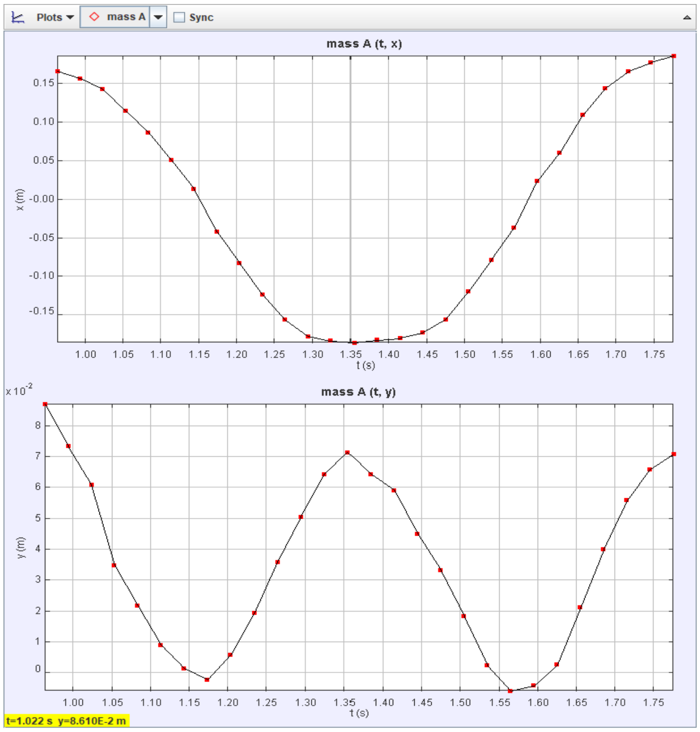


Experimental Calculations



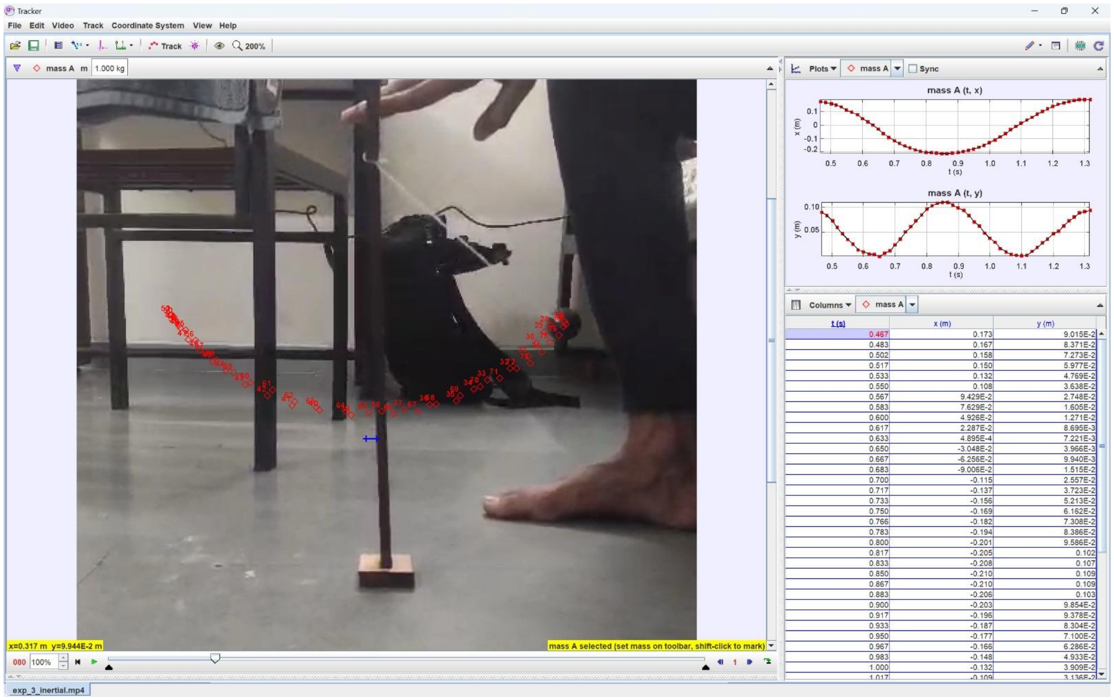
Pendulum Position in a Room (Inertial Frame)

- The time for the oscillation starts from t=0.467s to t=1.317 s hence the Time period of 1 oscillation experimentally comes out to be 0.85 s (which was 0.837 through calculations)



Pendulum Position in Accelerating Elevator (Non-Inertial Frame)

- The time for the oscillation starts from t=0.963s to t=1.78 s hence the Time period of 1 oscillation experimentally comes out to be 0.817 s (which was 0.815 through theoretical calculations)



Results and Discussion

From the calculations and experimental analysis (performed on Tracker) above these are the results:

Time Period (in s)	Theoretical	Experimental
Inertial frame	0.837 s	0.85 s
Non-Inertial frame	0.815 s	0.817 s

**All the collected data is uploaded in the Data Repository

Scan the link to access the detailed Data Repository

Results and Discussion

Inertial Reference Frame (Stationary Elevator)

- Observed Motion: The pendulum exhibited simple harmonic motion with a consistent period.
- Measured Acceleration: The acceleration of the pendulum bob, as measured by the sensor, closely matched the theoretical prediction of $a = -g\theta$, where g is the acceleration due to gravity and θ is the angular displacement.
- Period: The period of oscillation was consistent with the formula , confirming the validity of Newton's second law in an inertial frame.

Non-Inertial Reference Frame (Accelerating Elevator)

- Observed Motion: The pendulum's motion deviated from simple harmonic motion, with changes in the period and amplitude of oscillation.
- Measured Acceleration: The acceleration of the pendulum bob showed variations that corresponded to the elevator's acceleration. When the elevator accelerated upwards, the effective gravitational acceleration increased, leading to a shorter period. Conversely, when the elevator accelerated downwards, the effective gravitational acceleration decreased, resulting in a longer period.
- Pseudo Force Effect: The measured accelerations confirmed the presence of pseudo forces, with the relative acceleration of the pendulum bob given by $a_{relative} = -g\theta - a_{elevator}$, where $a_{elevator}$ is the acceleration of the elevator.

The experiment confirmed that Newton's second law, F=ma, accurately describes motion in inertial frames. In non-inertial frames, pseudo forces must be considered to account for the observed motion. The pendulum's behavior in the accelerating elevator illustrated the impact of reference frames on the interpretation of physical laws. This underscores the importance of understanding and applying the concept of pseudo forces in non-inertial reference frames, which is crucial in fields like engineering and aerospace.

In conclusion, the experiment successfully demonstrated the variations in the application of Newton's second law in different reference frames. By comparing the pendulum's motion and measured accelerations, we gained a deeper understanding of how reference frames influence the interpretation of physical laws and the necessity of considering pseudo forces in non-inertial frames.

Shortcomings

Shortcomings of the Experiment:
1. Friction, air resistance, and other external factors affected accuracy.

2. More advanced sensors and equipment are needed for precise measurements.

3. A quantitative comparison of pseudo force and observed acceleration would help in understanding effects in non-inertial frames.

4. We don't know the accuracy of the software used to measure time period and acceleration of the elevator.

For better accuracy, future experiments should minimize these external forces and use more precise measurement techniques.

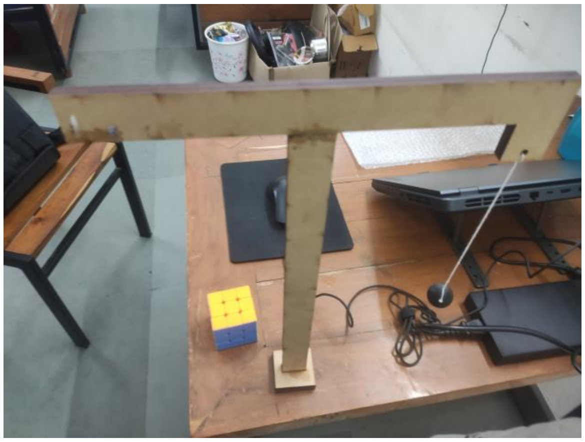
Scope of Improvement

- Enhance Sensor Precision: Use higher-precision sensors to improve the accuracy of acceleration measurements.
- Account for Variable Acceleration: Incorporate real-time measurement of elevator acceleration to handle variability.
- Consider Environmental Factors: Include air resistance and friction in the theoretical model for more accurate predictions.
- Refine Pendulum Model: Use a more sophisticated model that accounts for the extended mass of the pendulum bob.

References

- Engineering Mechanics Dynamics (7th Edition) - J. L. Meriam, L. G. Krage, Kinematics of Particles.
- Brittanica Encyclopedia

Photo Gallery



Acknowledgement

We sincerely thank everyone who contributed to the successful completion of our experiment on particle motion within a rotating frame for the Statics and Dynamics course. A special thanks to Professor K.R. Jayprakash for assigning this project and guiding us through the concepts of reference frames and the Coriolis effect. His proficiency is key to our understanding and completion of the work.

We also appreciate the support from our Teaching Assistants, whose feedback and assistance were invaluable.