ME206 Statics and Dynamics

Group 9, Experiment 2

STUDY OF MOTION IN ROTATING FRAME w.r.t STATIONARY FRAME

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INTRODUCTION

This experiment aims to study motion of a particle in different frames of reference. In non-inertial frames of reference like that of a rotating-frames the Coriolis and centrifugal forces come into play and understanding these forces are fundamental to various mechanical systems. The goal of this experiment is to build a physical model to simulate the system, and measure the particle's velocity and acceleration to compare experimental results with theoretical predictions. By doing so, we aim to observe the dynamic behavior in rotating frames.

In the experimental setup we have two layers one of which is connected to a DC motor which rotates the upper layer with constant angular velocity(Ω) and thus this layer acts as rotating frame of reference. The lower layer with grid pattern is stationary and serves as a reference frame similar to an inertial frame of reference. The particle is constrained to move along a straight line on the rotating frame by introducing a slot in it. While the particle moves in a straight line along the slot in the rotating frame, the grid allows us to observe how this straight-line motion in the rotating frame appears when viewed from the stationary frame, where fictious-forces like the Coriolis and centrifugal forces come into play.

In conclusion, this experiment explains how different reference frames change our perception of that motion. The use of a rotating and stationary frame in a controlled environment provides a clear demonstration of the Coriolis and centrifugal forces, thus helping us to understand basics of rotational dynamics.

EXPERIMENTAL DESIGNS

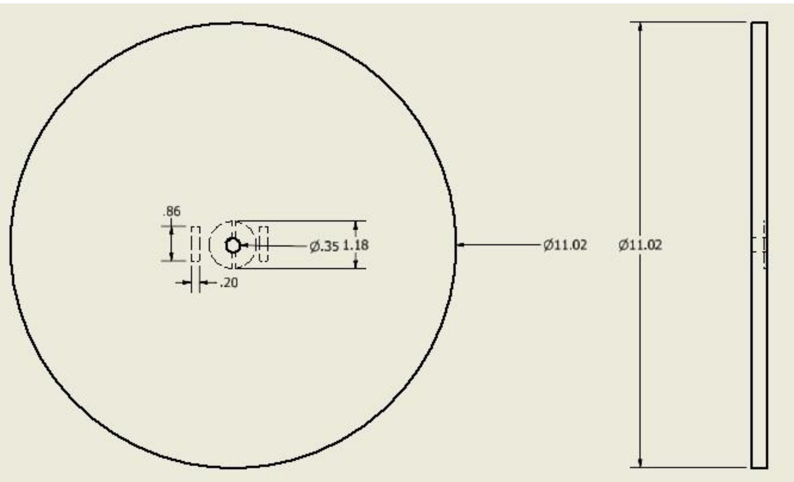
The design consists of a rotating acrylic disk mounted on a 12V 60rpm motor. The angular velocity of the motor is 2π i.e. 6.28 rad/s.

The acrylic disk rotates with the motor, also there is a MDF sheet with grid pattern below the acrylic sheet which acts as a stationary frame of reference. Both the disks are of diameter 28 cm. The small ball of diameter 2.5 cm is constrained with the help of two acrylic slits to move in a straight line. All the disks and slits were designed on Autodesk Inventor.

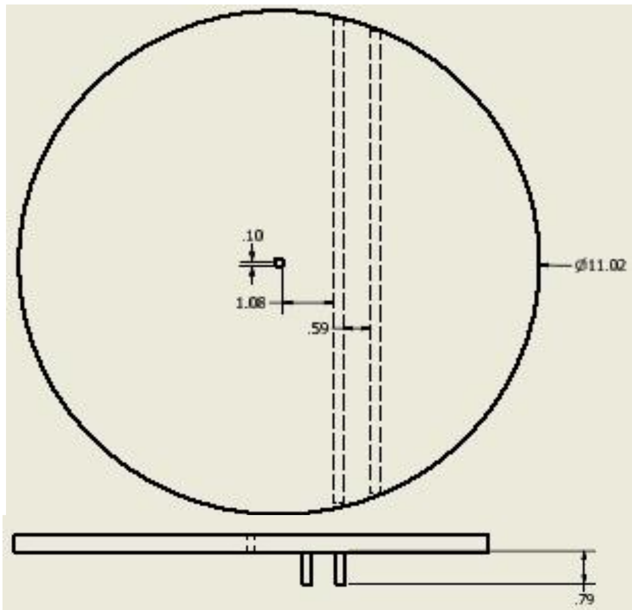
The objective is to measure particles velocity and acceleration. In order to do this, we recorded a video using phone and then analyzed using a software named 'Kinovea'.

ENGINEERING DRAWINGS

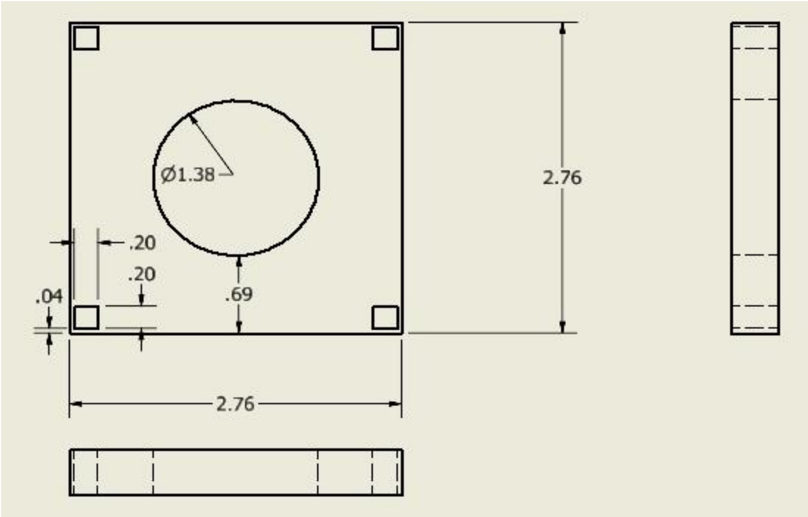
The following are the engineering drawings of different parts of model. All the dimensions are in inches.

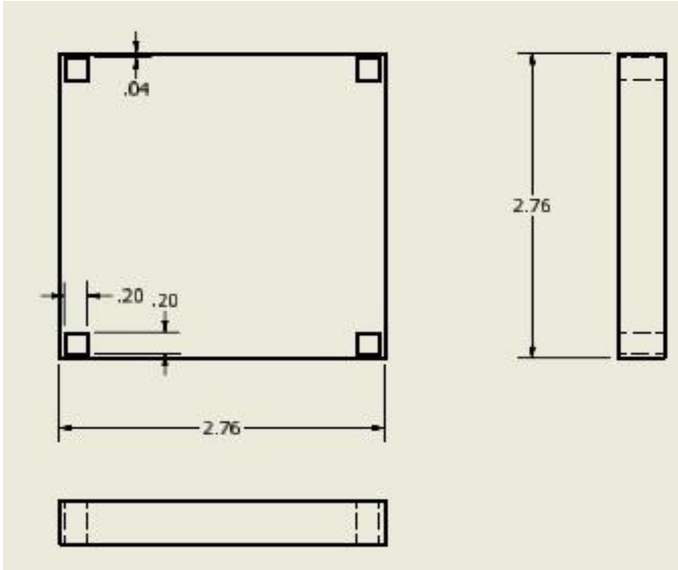


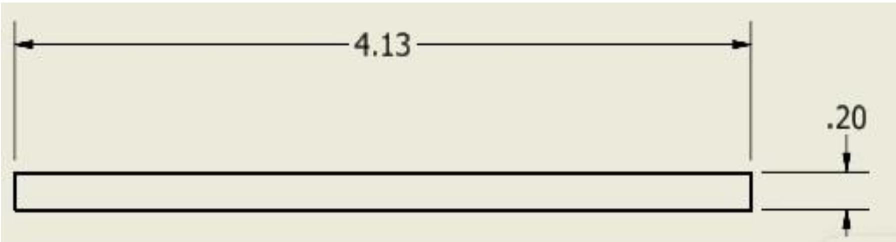
Disk made of MDF (Part of Stationary frame)



Disk made of acrylic part of rotating frame







Motor supports (base, top and pillars) part of stationary frame

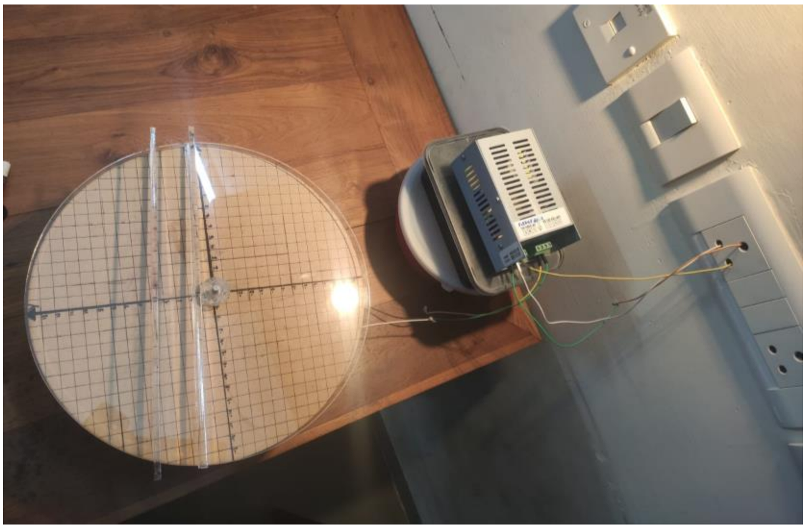
MATERIAL DATA

For this experiment we have used:

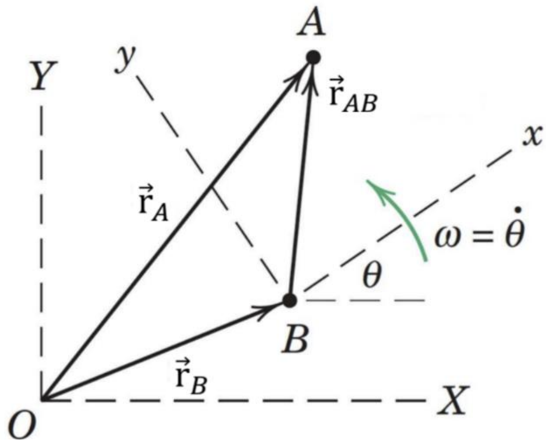
- 5mm MDF sheet to cut out a circle of diameter 28mm which acts as stationary frame of reference.
- 5mm Acrylic sheet to cut out a circle of diameter 28mm which acts as rotating frame.
- 5mm Acrylic sheet to cut out 7mm × 7mm square for motor support
- 12V 60rpm DC motor procured from the lab. (AB4/107).
- Switched Mode Power Supply (SMPS) to covert 230V to 12V.
- A glue gun and Vetra are used as adhesives.

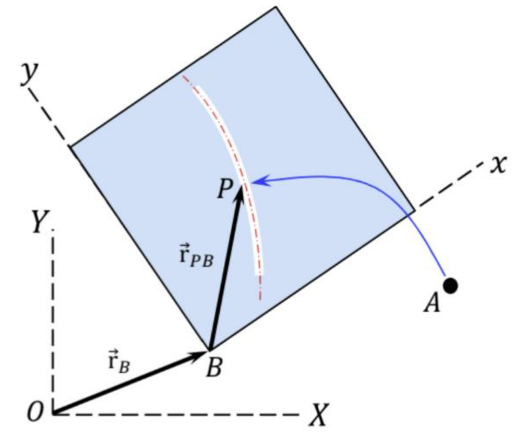
FABRICATION DETAILS

- In order to make the rotating frame we made an Autodesk model (drawings of which are attached above) with the slits and then converted them to the DXF files. These DXF files were then imported to LaserCad and then cut from the Acrylic sheet using the Laser cut technoligy available in Tinkerer's Lab at IITGN.
- Similarly the stationary frame was made using MDF sheet. Both the disk are 28 cm in diameter. Grids were made using black ball point pen.
- Also the supports for the motor were made using the same technology.
- We then used 3D printing technology to print the ball of diameter 2.5 cm which acted as a particle in our experiment.
- These disks were then carefully mounted on the motor procured from the lab.



THEORITICAL ANALYSIS





- OXY is the fixed reference frame and Bxy is a translating and rotating reference frame.

- The basis vectors $\hat{i}, \hat{j}, \hat{k}$ correspond to the $Bxyz$ reference frame and $\hat{I}, \hat{J}, \hat{K}$ correspond to the $OXYZ$ reference frame.

- We know that the rate of change of the unit vectors are:

$$\frac{d\hat{i}}{dt} = \vec{\omega} \times \hat{i}$$

$$\frac{d\hat{j}}{dt} = \vec{\omega} \times \hat{j}$$

$$\frac{d\hat{k}}{dt} = \vec{\omega} \times \hat{k}$$

Where

$$\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \ \hat{k}$$

- The position vector of a particle A is given by:

$$\vec{r}_A = \vec{r}_B + \vec{r}_{AB}$$

$$\therefore \vec{r}_A = X_B \hat{I} + Y_B \hat{J} + x_{AB} \hat{i} + y_{AB} \hat{j}.$$

- Thus, the rate of change of the position vector with respect to a fixed reference frame is.

$$
\begin{aligned}
\dot{\vec{r}}_A &= \dot{\vec{r}}_B + \dot{\vec{r}}_{AB} \\
&= \dot{\vec{r}}_B + \left(\dot{x}_{AB}\hat{\imath} + \dot{y}_{AB}\hat{\jmath}\right) + \left(x_{PB}\frac{d\hat{\imath}}{dt} + y_{PB}\frac{d\hat{\jmath}}{dt}\right) \\
&= \dot{\vec{r}}_B + \left(\dot{x}_{AB}\hat{\imath} + \dot{y}_{AB}\hat{\jmath}\right) + \left(x_{PB}\vec{\omega}\times\hat{\imath} + y_{PB}\vec{\omega}\times\hat{\jmath}\right) \\
&= \dot{\vec{r}}_B + \left(\dot{x}_{AB}\hat{\imath} + \dot{y}_{AB}\hat{\jmath}\right) + \vec{\omega}\times\left(x_{PB}\hat{\imath} + y_{PB}\hat{\jmath}\right) \\
&= \dot{\vec{r}}_B + \left(\dot{x}_{AB}\hat{\imath} + \dot{y}_{AB}\hat{\jmath}\right) + \vec{\omega}\times\vec{r}_{PB} \\
&= \dot{\vec{r}}_B + (\vec{v}_{AB})_{Bxy} + \vec{\omega}\times\vec{r}_{PB}
\end{aligned}
$$

$$
\therefore \dot{\vec{r}}_A - \dot{\vec{r}}_B = \left(\frac{d\vec{r}_{AB}}{dt}\right)_{Bxyz} + \vec{\omega}\times\vec{r}_{PB}
$$

$$
\therefore \left(\frac{d\vec{r}_{AB}}{dt}\right) = \left(\frac{d\vec{r}_{AB}}{dt}\right)_{Bxyz} + \vec{\omega}\times\vec{r}_{PB}
$$

- The rate of change of the velocity vector with respect to the fixed reference is

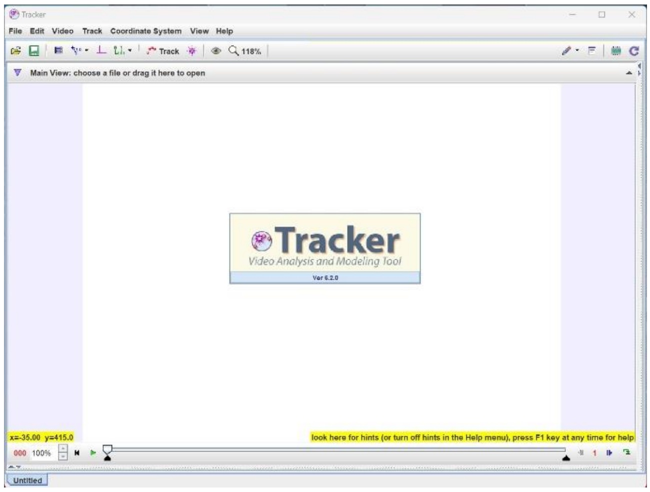
$$
\begin{aligned}
\ddot{\vec{r}}_A &= \ddot{\vec{r}}_B + \ddot{\vec{r}}_{AB} \\
&= \ddot{\vec{r}}_B + (\ddot{x}_{AB}\hat{\imath} + \ddot{y}_{AB}\hat{\jmath}) + \frac{d\vec{\omega}}{dt}\times\vec{r}_{AB} + \vec{\omega}\times\dot{\vec{r}}_{AB} \\
&= \ddot{\vec{r}}_B + (\ddot{x}_{AB}\hat{\imath} + \ddot{y}_{AB}\hat{\jmath}) + (\dot{x}_{AB}\vec{\omega}\times\hat{\imath} + \dot{y}_{AB}\vec{\omega}\times\hat{\jmath}) + \dot{\vec{\omega}}\times\vec{r}_{AB} \\
&\quad + \vec{\omega}\times\{(\dot{x}_{AB}\hat{\imath} + \dot{y}_{AB}\hat{\jmath}) + (\vec{\omega}\times\vec{r}_{AB})\} \\
&= \ddot{\vec{r}}_B + (\ddot{x}_{AB}\hat{\imath} + \ddot{y}_{AB}\hat{\jmath}) + (\vec{\omega}\times\dot{x}_{AB}\hat{\imath} + \vec{\omega}\times\dot{y}_{AB}\hat{\jmath}) + \dot{\vec{\omega}}\times\vec{r}_{AB} \\
&\quad + \vec{\omega}\times\{(\dot{x}_{AB}\hat{\imath} + \dot{y}_{AB}\hat{\jmath}) + (\vec{\omega}\times\vec{r}_{AB})\} \\
&= \ddot{\vec{r}}_B + (\ddot{x}_{AB}\hat{\imath} + \ddot{y}_{AB}\hat{\jmath}) + 2\vec{\omega}\times(\dot{x}_{AB}\hat{\imath} + \dot{y}_{AB}\hat{\jmath}) + \dot{\vec{\omega}}\times\vec{r}_{AB} \\
&\quad + \vec{\omega}\times\{(\vec{\omega}\times\vec{r}_{AB}+)\}
\end{aligned}
$$

$$
\therefore \ddot{\vec{r}}_A - \ddot{\vec{r}}_B = \left(\frac{d^2\vec{r}_{AB}}{dt^2}\right) + \dot{\vec{\omega}}\times\vec{r}_{AB} + 2\vec{\omega}\times\left(\frac{d\vec{r}_{AB}}{dt}\right) + \vec{\omega}\times\{(\vec{\omega}\times\vec{r}_{AB})\}
$$

$$
\therefore \frac{d}{dt}\left(\frac{d\vec{r}_{AB}}{dt}\right) = \left(\frac{d^2\vec{r}_{AB}}{dt^2}\right) + \dot{\vec{\omega}}\times\vec{r}_{AB} + 2\vec{\omega}\times\left(\frac{d\vec{r}_{AB}}{dt}\right) + \vec{\omega}\times\{(\vec{\omega}\times\vec{r}_{AB})\}
$$

MEASUREMENT TECHNIQUES

In order to accurately measure the velocity and acceleration of the particle we designed a grid on MDF with black ball point pen and a spacing of 1 cm. This provides a stable reference system for precise position tracking and analysis.



We also recorded a slow-motion video using a camera which was then imported in a video analysis software 'Tracker' which helped us determine the velocity and acceleration of the particle at various time instants. However, we don't really know how accurate this is. Such inaccuracies could have led to deviations of the experimental value from the theoretical ones.

We could have also used Inertial Measurement Units (IMUs) which should be attached to the particle. However, it would have added weight and could have altered the dynamics. Moreover, it is capable of measuring resultant acceleration and not its components.

RESULTS

The electric motor operates at 60rpm

$\therefore \omega = 2\pi \times \frac{60}{60}$

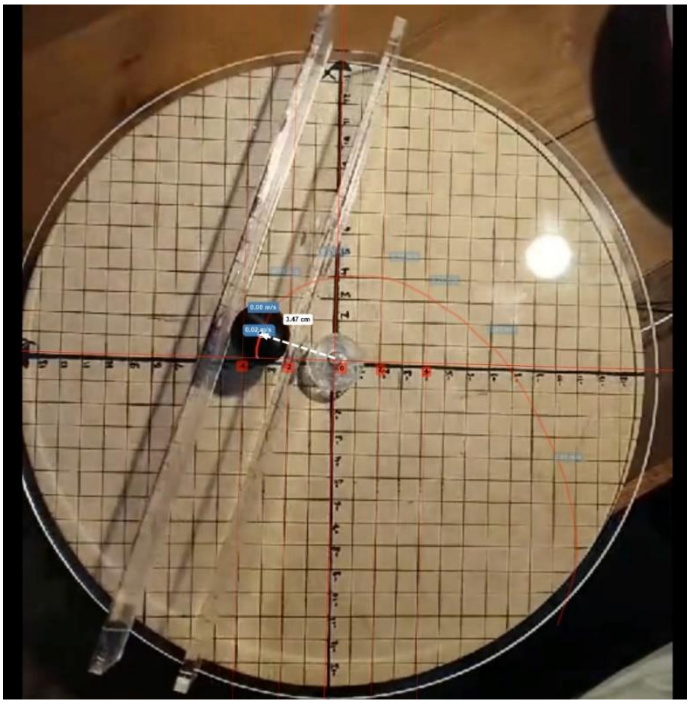
$\therefore \omega = 2\pi rad/sec$

$\therefore \omega = 6.28 rad/s$

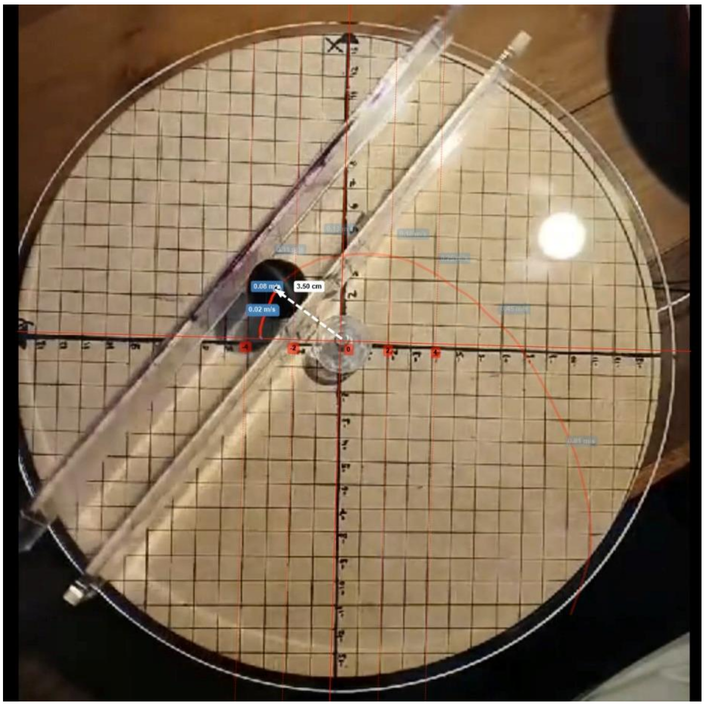
$\therefore \vec{\omega} = -6.28\hat{k}\ rad/s$

The snapshots taken at different intervals are as follows

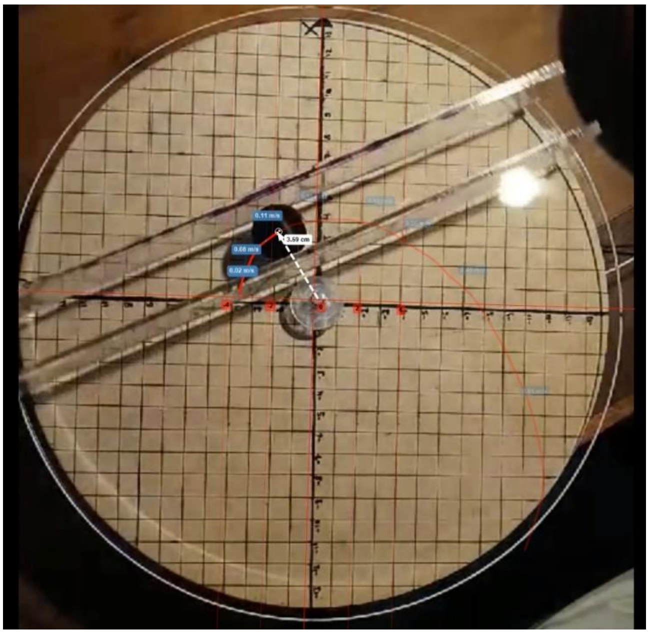
At t = 0.030 s



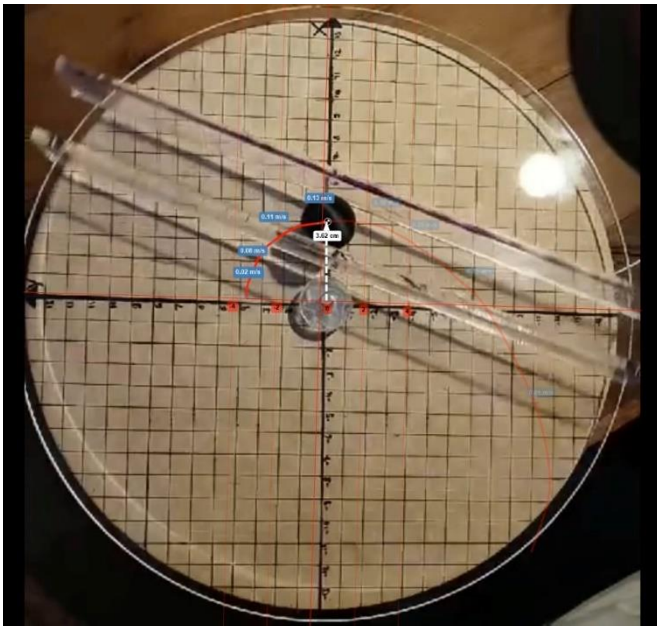
At t = 0.3 s



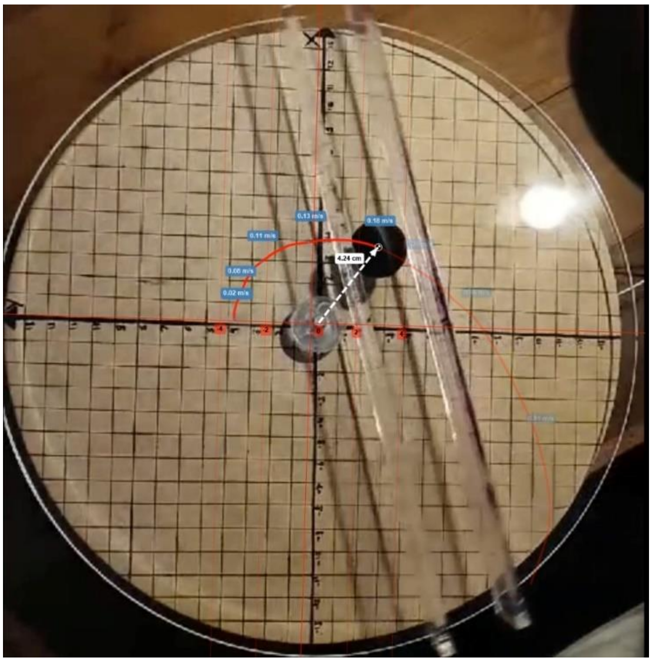
At t = 0.5 s



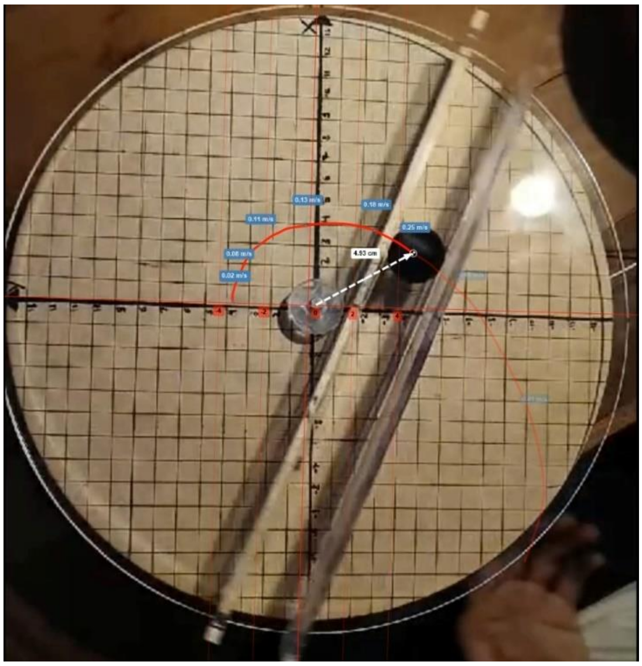
At t = 0.7 s



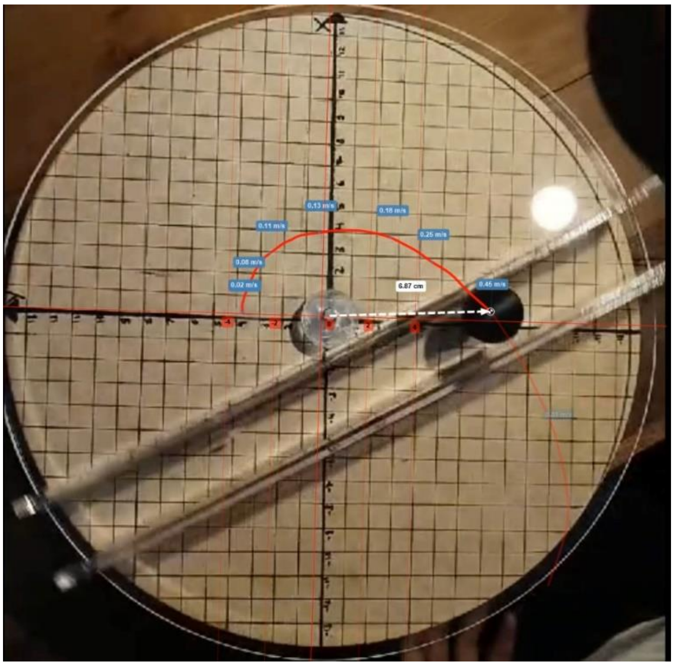
At t = 0.8 s



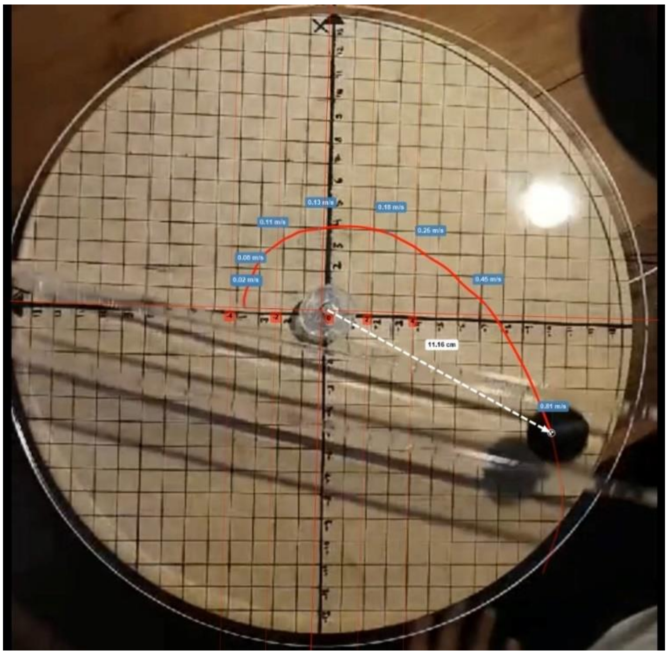
At t = 1.0 s



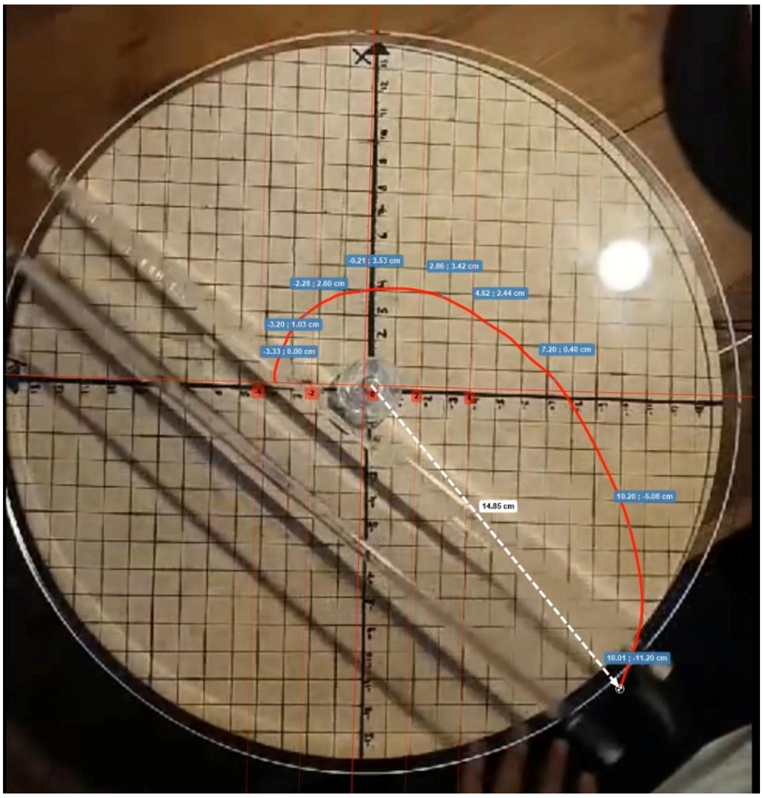
At t = 1.1 s



At t = 1.2 s



Final Trajectory



The values at other intermediate timestamps and the analysis video can be accessed from https://tinyurl.com/Group9SnDE2 or from the QR.

Calculations

Sr no. (n)	Time (t) (in s	x_n(in cm)	y_n(in cm)
1	0.1	-3.45	0.05
2	0.2	-3.47	0.39
3	0.3	-3.27	1.03
4	0.4	-2.98	1.80

Now, for $\Delta t = 0.2$

$$\left(\frac{d\vec{r}_{AB}}{dt}\right)_{Bxyz} = \left(\frac{x_3 - x_1}{\Delta t}\right)\hat{\imath} + \left(\frac{y_3 - y_1}{\Delta t}\right)\hat{\jmath}$$

Substituting the values, we get,

$$\left(\frac{d\vec{r}_{AB}}{dt}\right)_{Bxyz} = \left(\frac{-3.27 + 3.45}{0.2}\right)\hat{\imath} + \left(\frac{1.03 - 0.05}{0.2}\right)\hat{\jmath}$$

$$\left(\frac{d\vec{r}_{AB}}{dt}\right)_{Bxyz} = (0.9\,\hat{\imath} + 4.9\,\hat{\jmath}\,)cm/s \tag{1}$$

$$\vec{\omega} \times \vec{r}_{PB} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & -6.28 \\ -3.47 & 0.39 & 0 \end{vmatrix}$$

$$\vec{\omega} \times \vec{r}_{PB} = (2.45\,\hat{\imath} + 21.79\,\hat{\jmath})\ cm/s \tag{2}$$

Velocity of the particle in the fixed frame of reference is:

$$\left(\frac{d\vec{r}_{AB}}{dt}\right) = \left(\frac{d\vec{r}_{AB}}{dt}\right)_{Bxyz} + \vec{\omega} \times \vec{r}_{PB}$$

From (1) and (2),

$$\begin{aligned}\left(\frac{d\vec{r}_{AB}}{dt}\right) &= 0.9\,\hat{\imath} + 4.9\,\hat{\jmath} + 2.45\,\hat{\imath} + 21.79\,\hat{\jmath} \\ &= (3.35\,\hat{\imath} + 26.69\,\hat{\jmath})cm/s\end{aligned}$$

$$\therefore \left(\frac{d\vec{r}_{AB}}{dt}\right) = (0.0335\,\hat{\imath} + 0.2669\,\hat{\jmath})m/s$$

Now,

$$\left(\frac{d^2\vec{r}_{AB}}{dt^2}\right) = \frac{\left(\frac{x_3 - x_2}{\Delta t}\right) - \left(\frac{x_2 - x_1}{\Delta t}\right)}{\Delta t}\,\hat{\imath} + \frac{\left(\frac{y_3 - y_2}{\Delta t}\right) - \left(\frac{y_2 - y_1}{\Delta t}\right)}{\Delta t}\,\hat{\jmath}$$

Substituting the values we get,

$$\left(\frac{d^2\vec{r}_{AB}}{dt^2}\right) = \frac{\left(\frac{-3.27 + 3.47}{0.1}\right) - \left(\frac{-3.47 + 3.45}{0.1}\right)}{0.1}\hat{\imath} + \frac{\left(\frac{1.03 - 0.39}{0.1}\right) - \left(\frac{0.39 - 0.05}{0.1}\right)}{0.1}\hat{\jmath}$$

$$\left(\frac{d^2\vec{r}_{AB}}{dt^2}\right) = (22\,\hat{\imath} + 98\,\hat{\jmath})\; cm/s^2 \tag{3}$$

Now,

$$\vec{\omega} \times \left(\frac{d\vec{r}_{AB}}{dt}\right) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & -6.28 \\ 0.9 & 4.9 & 0 \end{vmatrix} \tag{from 1}$$

$$\therefore \vec{\omega} \times \left(\frac{d\vec{r}_{AB}}{dt}\right) = (30.77\,\hat{\imath} - 5.65\,\hat{\jmath})\; cm/s^2 \tag{4}$$

Now,

$$\vec{\omega} \times \{(\vec{\omega} \times \vec{r}_{AB})\} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & -6.28 \\ 2.45 & 21.79 & 0 \end{vmatrix} \tag{from 2}$$

$$\therefore \vec{\omega} \times \{(\vec{\omega} \times \vec{r}_{AB})\} = (136.84\,\hat{\imath} - 15.38\,\hat{\jmath})\; cm/s^2 \tag{5}$$

Now $\vec{\omega}$ is constant.Thus,

$$\dot{\vec{\omega}} = 0 \tag{6}$$

Acceleration of the particle in the fixed frame of reference is:

$$\therefore \frac{d}{dt}\left(\frac{d\vec{r}_{AB}}{dt}\right) = \left(\frac{d^2\vec{r}_{AB}}{dt^2}\right) + \dot{\vec{\omega}} \times \vec{r}_{AB} + 2\vec{\omega} \times \left(\frac{d\vec{r}_{AB}}{dt}\right) + \vec{\omega} \times \{(\vec{\omega} \times \vec{r}_{AB})\}$$

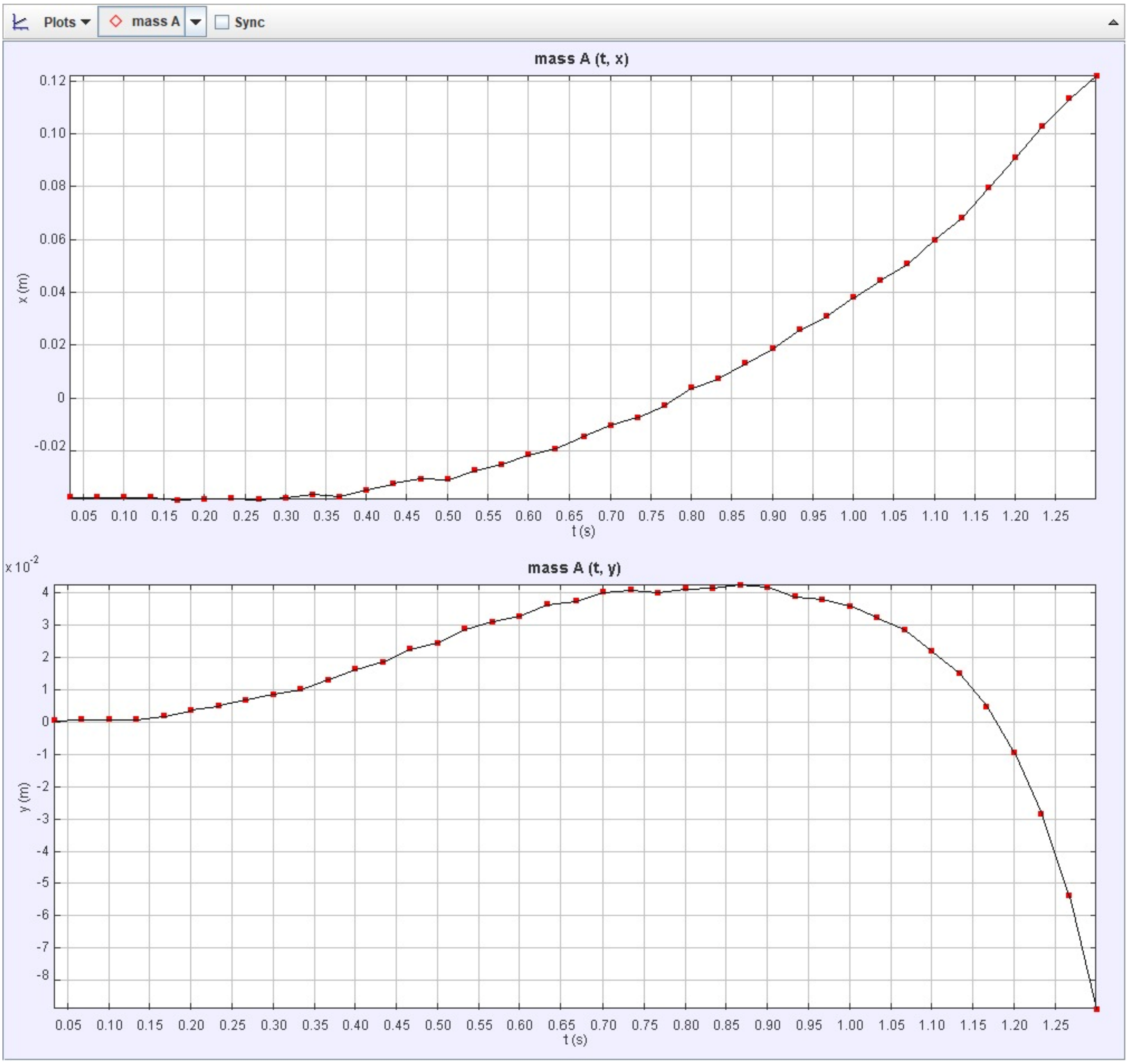
From 3,4,5 and 6, we get

$$\therefore \frac{d}{dt}\left(\frac{d\vec{r}_{AB}}{dt}\right) = 22\,\hat{\imath} + 98\,\hat{\jmath} + 0 + 2(30.77\,\hat{\imath} - 5.65\,\hat{\jmath}) + 136.84\,\hat{\imath} - 15.38\,\hat{\jmath}$$

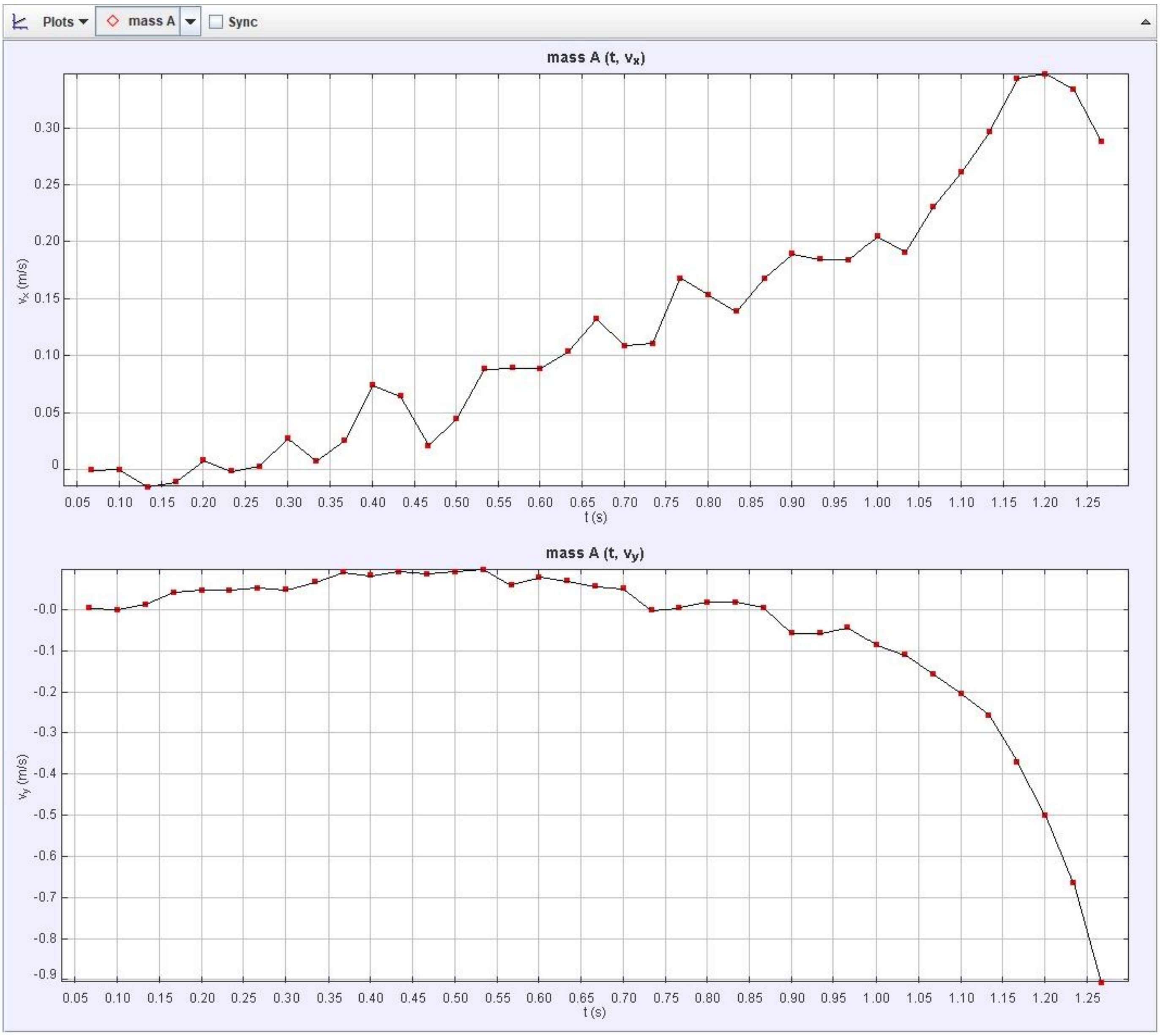
$$\therefore \frac{d}{dt}\left(\frac{d\vec{r}_{AB}}{dt}\right) = (220.38\,\hat{\imath} + 71.32\,\hat{\jmath})\; cm/s^2$$

$$\therefore \frac{d}{dt}\left(\frac{d\vec{r}_{AB}}{dt}\right) = (2.2038\,\hat{\imath} + 0.7132\,\hat{\jmath})\; m/s^2$$

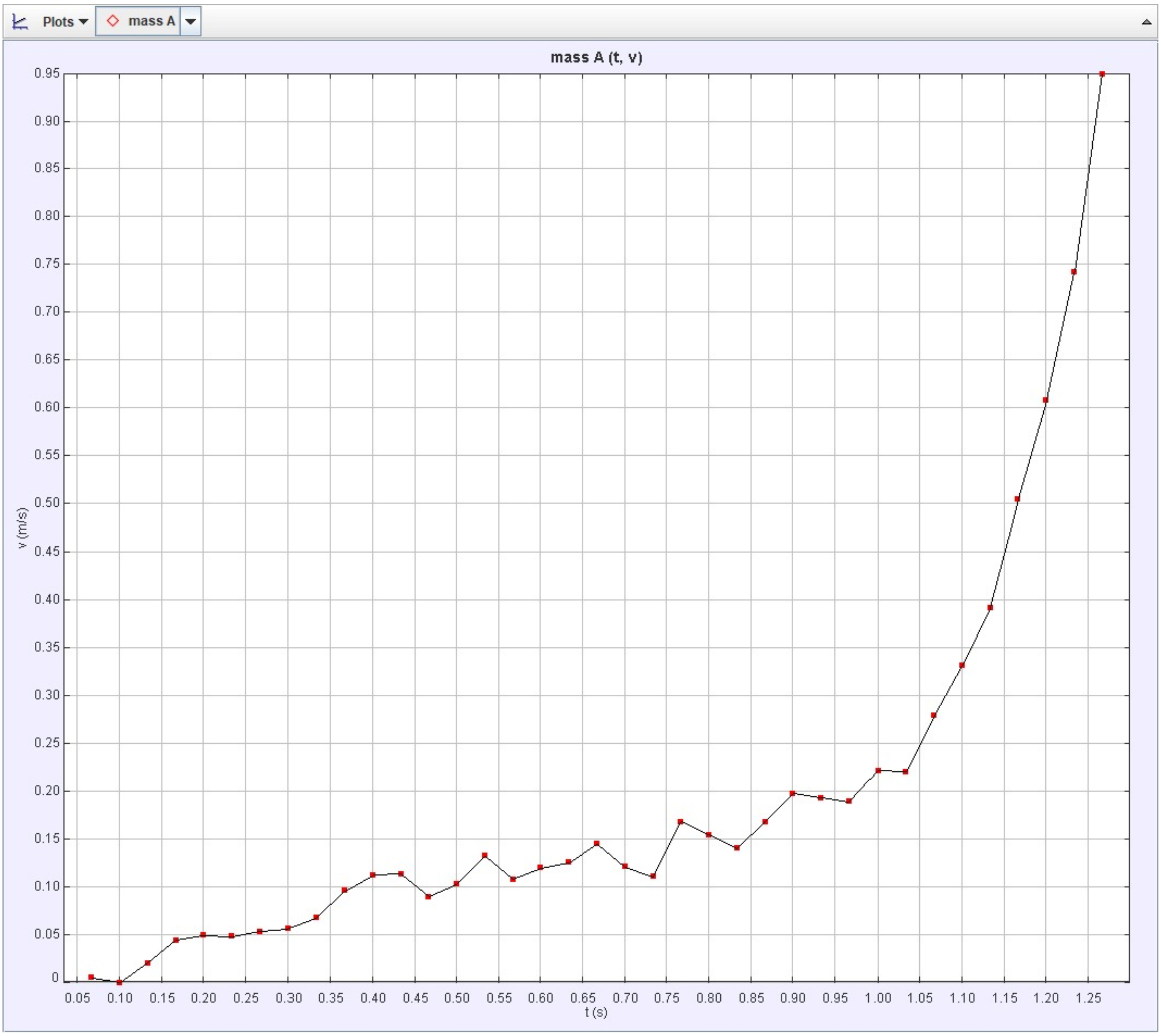
Plots



Horizontal and vertical position vs time

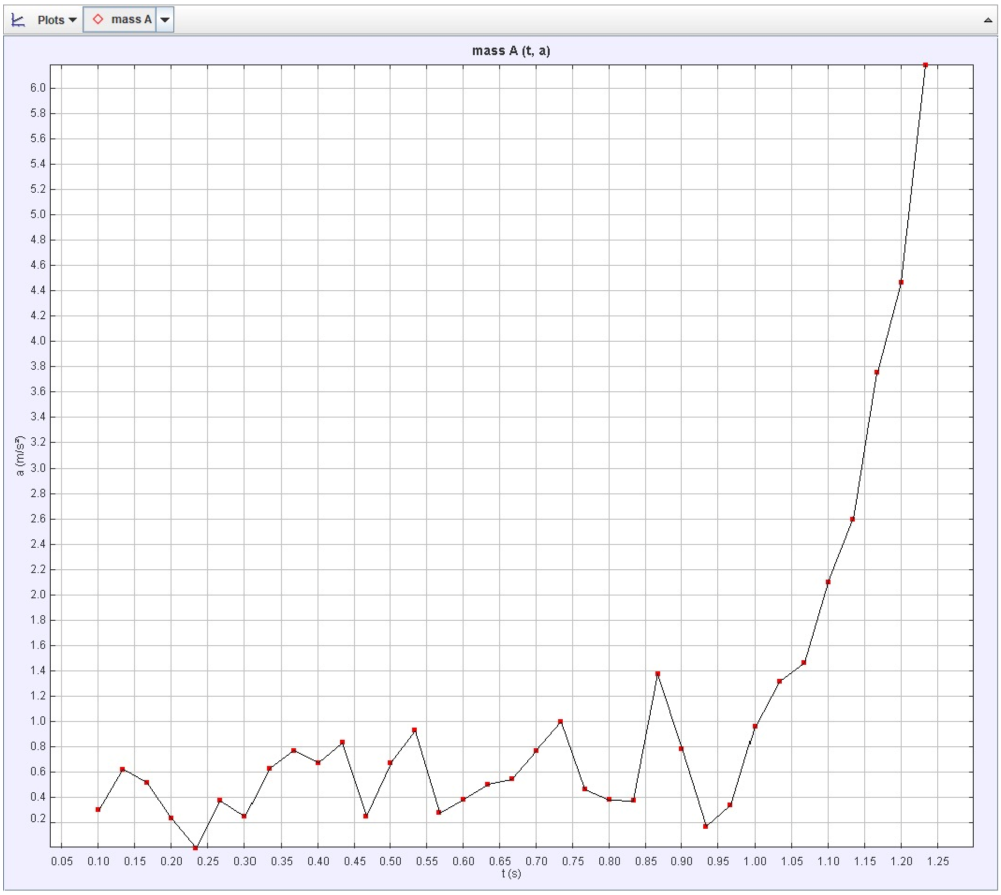


Horizontal and vertical velocity vs time



Velocity vs time

Horizontal and vertical acceleration vs time



Acceleration vs time

DISCUSSIONS

From this experiment we get a clear understanding of how to calculate rate of change of position and velocity vector with respect to time. It enables us to analyse the motion of a particle in rotating frame with respect to the stationary ones. The trajectory of the particle is curved indicating presence of Coriolis and centrifugal forces.

However, we obtain a deviation of experimental results from the theoretical ones. This can be due to various factors like friction, air resistance and imperfections in the setups. However, One of the main reason for this deviation could be the 12V DC motor with a rated speed of 60 RPM which will generally have a slight delay in reaching its full speed when it starts. This delay occurs because the motor needs to overcome its initial inertia and build up to its operating speed. The time it takes to reach the rated speed depends on factors like:

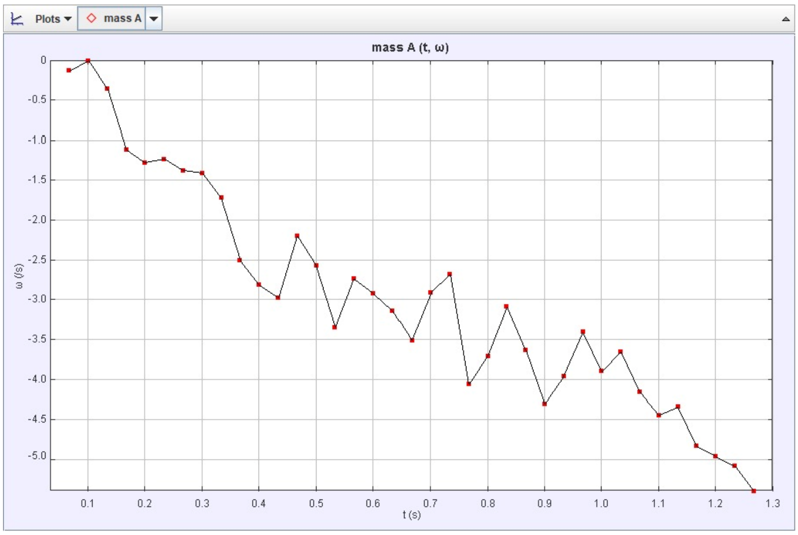
1) Load: If there’s a significant load attached, it will take longer to reach the desired speed compared to running with no load.

2) Motor Characteristics: The motor’s internal resistance, inductance, and design characteristics can affect how quickly it accelerates.

3) Power Supply: A steady and sufficient voltage supply will help the motor reach its full speed more consistently. If the voltage fluctuates, the motor speed may also fluctuate.

However, once the motor reaches its rated speed under constant voltage and load conditions, it should maintain that speed consistently. The speed could vary slightly if there are changes in load or supply voltage, but generally, DC motors tend to have a fairly stable speed once they are up and running.

The following graph shows variation of angular velocity of motor



SCOPE FOR IMPROVEMENT

To Improve the accuracy and reliability of this experiment, several key areas for improvement should be considered:

1. Improved Measurement Techniques: We don't really know how precise is the 'Tracker' software that we have used. Thus we can shift towards using more precise sensors.

2. Enhanced Data Accuracy :By achieving higher accuracy in capturing the particle's motion, we can gain more precise inputs into the experiment's dynamics.

3. Refinement of Grid Coordinates: Increasing the precision of the grid coordinates will improve the accuracy of position measurements.

4. Advanced Imaging Systems: Using more advanced cameras or imaging systems with higher resolution and higher frame rates will allow for better capture of fine details. This will improve the overall clarity of the data and help to visualize and analyse the particle's movement more effectively.

5. Minimizing External Interference: Reducing external factors like air resistance and friction.

These improvements can help us to get better results and minimise the deviation of the experimental results from the theoretical ones.

REFERENCES

1. Meriam, J.L., Bolton, Jeffrey N., & Kraige, L. Engineering Mechanics: Dynamics (Edition 7).
2. https://www.youtube.com/watch?v=IOcrHOc23N4
3. Lecture 3 and 4 slides

ACKNOWLEDGEMENT

We sincerely thank everyone who contributed to the successful completion of our experiment on particle motion within a rotating frame for the Statics and Dynamics course.

A special thanks to Professor K.R. Jayapraksh for assigning this project and guiding us through the concepts of reference frames and the Coriolis effect. His proficiency is key to our understanding and completion of the work.

We also appreciate the support from our Teaching Assistants, whose feedback and assistance were invaluable.