ME206 Statics and Dynamics

Group 9, Experiment 4

# PLANAR FOUR-BAR MECHNAISM

Faayza Vora (23110109), Deepak Gadhave (23110110)

Gadiparthi Abhinav (23110111), Gaurav Kumar (23110113).

# TABLE OF CONTENTS

# INTRODUCTION

In mechanical engineering the design and analysis of various linkages helps us to create efficient mechanisms for various applications like cars, bicycles, and robots. One such mechanism is called a planar four-bar linkage. This system is made up of four solid bars (called links) that are connected by rotating joints to form a closed loop. That enables controlled motion transfer.

In a four-bar linkage, each bar has a specific job:

1. One bar is fixed in place, acting as the base (this is called the *frame*).
2. Another bar, called the crank, is turned to input motion.
3. The third bar, called the coupler, connects the crank to the fourth bar.
4. The fourth bar, called the rocker, moves in response to the crank.

When the crank rotates at a steady speed, it drives the coupler and the rocker. The rocker moves back and forth (partially rotates), depending on how the linkage is designed. This ability to transform motion from one type to another makes the four-bar linkage very valuable in mechanical design. The aim of this experiment is to study the kinematics of a four-bar mechanism particularly the velocity characteristics of the coupler link. We aim to determine velocity, angular velocity and the centrode for at least twelve different angles $(0, \frac{\pi}{6}, \frac{\pi}{3}, \dots.)$ analytically by solving the equations involved, experimentally by building a model and using stimulation software ADAMS.

This experiment focuses on understanding the motion of a four-bar linkage by analyzing the velocity and angular velocity of the coupler link at various crank angles. By combining analytical methods, experimental testing, and computer simulations using ADAMS, we aim to understand the kinematic behavior of this mechanism.

# AIM

The aim of this experiment is to:

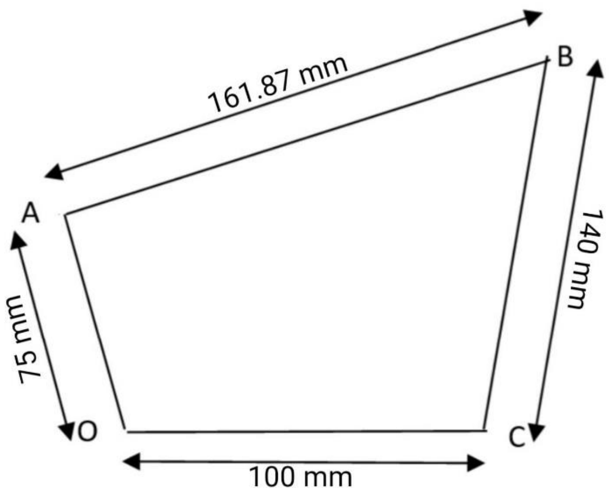
1) Model a planar four-bar mechanism with one link (crank) that can rotate (at a constant angular velocity) completely and the opposite link (rocker) rotates partially.

2) Find the velocity (analytically, experimentally and in ADAMS) of the centre of mass of the coupler link (that connects crank and the rocker), the angular velocity of the coupler and its centrode for at least twelve different angles $(0, \frac{\pi}{6}, \frac{\pi}{3}, \dots)$ of the crank.

# EXPERIMENTAL DESIGNS AND ENGINEERING DRAWINGS

The dimensions chosen are as follows:

1) Coupler link length (AB) = 161.87 mm
2) Rocker length (OC) = 100 mm
3) Crank OA length = 75mm
4) Crank CB length = 140 mm




Width of each link is 25mm. We have used 3mm MDF sheets from tinkerer's lab to make this model. In order to do these, we first made a DXF files of the links on Laser CAD software and then used the laser-cut technology available in tinkerer's lab to cut out the links.

The motor is attached at the point O (in the figure) and a coupler is at other points (A, B, C). The couplers are made out of joining 2 discs of 5mm acrylic and 1 disc of 10mm acrylic. These discs were also made using the Laser cut technology.

# MATERIAL DATA

In order to make the four-bar mechanism following material was used

1) 3mm thick MDF: The MDF was cut into the required shapes for the four main links of the mechanism (frame, crank, coupler, and rocker).
2) 10mm and 5mm acrylic: was used to make couplers due to its durability and smooth surface, allowing for efficient motion transfer between the crank and the rocker.
3) 20 rpm motor: Its steady rotation drives the entire four-bar mechanism, allowing the rocker and coupler to move according to the designed motion.

# FABRICATION DETAILS

In order to model this mechanism, we cut out its four links according to the above-mentioned measures using laser cut technology. MDF was used to make these link due to its light weight and compatibility with laser cut technology.

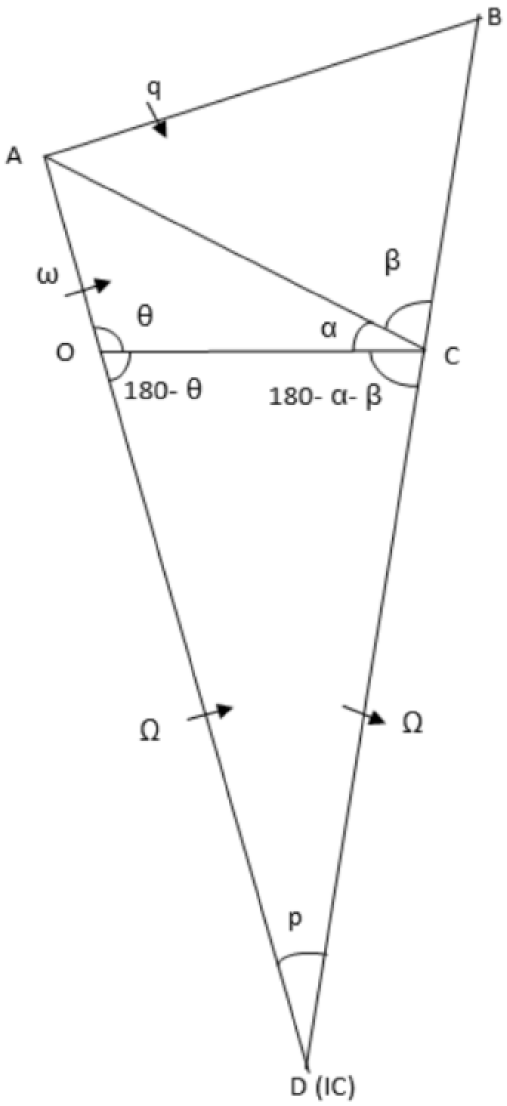
We then used 10 mm and 5mm acrylic to make a coupler. The inner and the outer diameter of the couplers is 16mm and 20mm respectively. Acrylic was chosen as a material for coupler due to its lightweight nature and also due to its compatibility with laser cut technology.  A 20rpm motor was used for moving point O.

For determining the velocity and angular velocity for different angles we recorded a video and then imported it in a video analysis software ‘The Tracker’.

# THEORITICAL ANALYSIS

OABC is the four-link structure.
We need to find out the velocity of the center of mass of the coupler link(AB),i.e. $v_G$ and the angular velocity of the coupler for different angles($\theta$) of the crank. We know:

$$p = \theta + \alpha + \beta - 180°$$




Using the cosine formula in triangle OAC,

$$\cos\theta = \frac{OC^2 + OA^2 - AC^2}{2(OC)(OA)}$$

From this, we get the length of AC.

Using the cosine formula in triangle OAC,

$$\cos\alpha = \frac{OC^2 + AC^2 - OA^2}{2(OC)(AC)}$$

From this, we get angle $\alpha$.

Using the cosine formula in triangle ABC,

$$\cos\beta = \frac{AC^2 + BC^2 - AB^2}{2(AC)(BC)}$$

From this, we get angle $\beta$.

Using the sine rule in triangle ABD,

$$\frac{\sin(180° - \theta)}{CD} = \frac{\sin(180° - \alpha - \beta)}{OD} = \frac{\sin(p)}{OC}$$

From here, we get BD and AD.

Now, for velocity at A,

$\vec{V_A} = \vec{\omega} \times \vec{AO}$

Where $\omega$ is the angular velocity of link OA.

Also, since point D is the instantaneous center of rotation, then

$\vec{V_A} = \vec{\Omega} \times \vec{AD}$

Where $\vec{\Omega}$ is the angular velocity about the IC.

Hence, solving the above two equations, we get the magnitude of $\vec{\Omega}$,

$|\vec{\Omega}| = \dfrac{|\vec{V_A}|}{|\vec{BD}|}$

We already know the direction of $\vec{\Omega}$, which is in the positive $z$-direction.

Again, since D is the IC, hence,

$\vec{V_B} = \vec{\Omega} \times \vec{BD}$

As $\vec{\Omega}$ is known, we get the velocity at point B.

We know, from Chasles's theorem,

$\vec{V_B} = \vec{V_A} + \vec{o} \times \vec{BA}$

Where $\vec{o}$ is the angular velocity of link AB.
From here, we get the magnitude of $\vec{o}$, the angular velocity of link AB, and the direction of $\vec{o}$ is already known.

To get the velocity of the center of mass of link AB, we use Chasles's theorem,

$\vec{V_G} = \vec{V_A} + \vec{o} \times \vec{GA}$

Now,to calculate $v_G$ and $o$ for different values of $\theta$,we use MATLAB

# MATLAB Code

Editor - C:\Users\Faayza Vora\Downloads\four_bar_link.m *

Qn1.m | Qn2.m | Qn4.m | Qn5.m | Qn6.m | four_bar_link.m *

```
clc;
clearvars;
% Given data
w = 2*pi/(3); % 20 rpm motor, angular velocity of crank

theta = 5*pi/6; % in radians

% Length in meters
OA = 75/1000;
OC = 100/1000;
BC = 140/1000;
AB = 161.87/1000;

% Calculations
AC = (OC^2 + OA^2 - cos(theta)*2*OC*OA)^(1/2);
alpha = acos((OC^2 + AC^2 - OA^2)/(2*OC*AC)) * 180/pi;
beta = acos((AC^2 + BC^2 - AB^2)/(2*AC*BC)) * 180/pi;

theta = theta * 180/pi;
p = (theta + alpha + beta) - 180;

CD = sind(180-theta)*OC/sind(p);
OD = sind(180-alpha-beta)*OC/sind(p);

```

Editor - C:\Users\Faayza Vora\Downloads\four_bar_link.m

Qn1.m | Qn2.m | Qn4.m | Qn5.m | Qn6.m | four_bar_link.m

```
% Points A, B, C, D
A = [-OA*cos(180-theta) OA*sin(180-theta) 0];
O = [0 0 0];
C = [100/1000 0 0];
D = [OD*cos(180-theta) -OD*sin(180-theta) 0];
B = [BC*cos(180-alpha-beta)+OC BC*sin(180-alpha-beta) 0];

% Velocity calculations
AOv = A-O;
wv = [0 0 w];
vAv = cross(wv,AOv);

ADv = A-D;
AD = norm(ADv);
vA = norm(vAv);
omega = vA/ AD;

BDv = B-D;
omegav = [0 0 omega];
vBv = cross(omegav, BDv);
vB = norm(vBv);
BAv = B-A;
AB = norm(BAv);
```

Editor - C:\Users\Faayza Vora\Downloads\four_bar_link.m

Qn1.m | Qn2.m | Qn4.m | Qn5.m | Qn6.m | four_bar_link.m

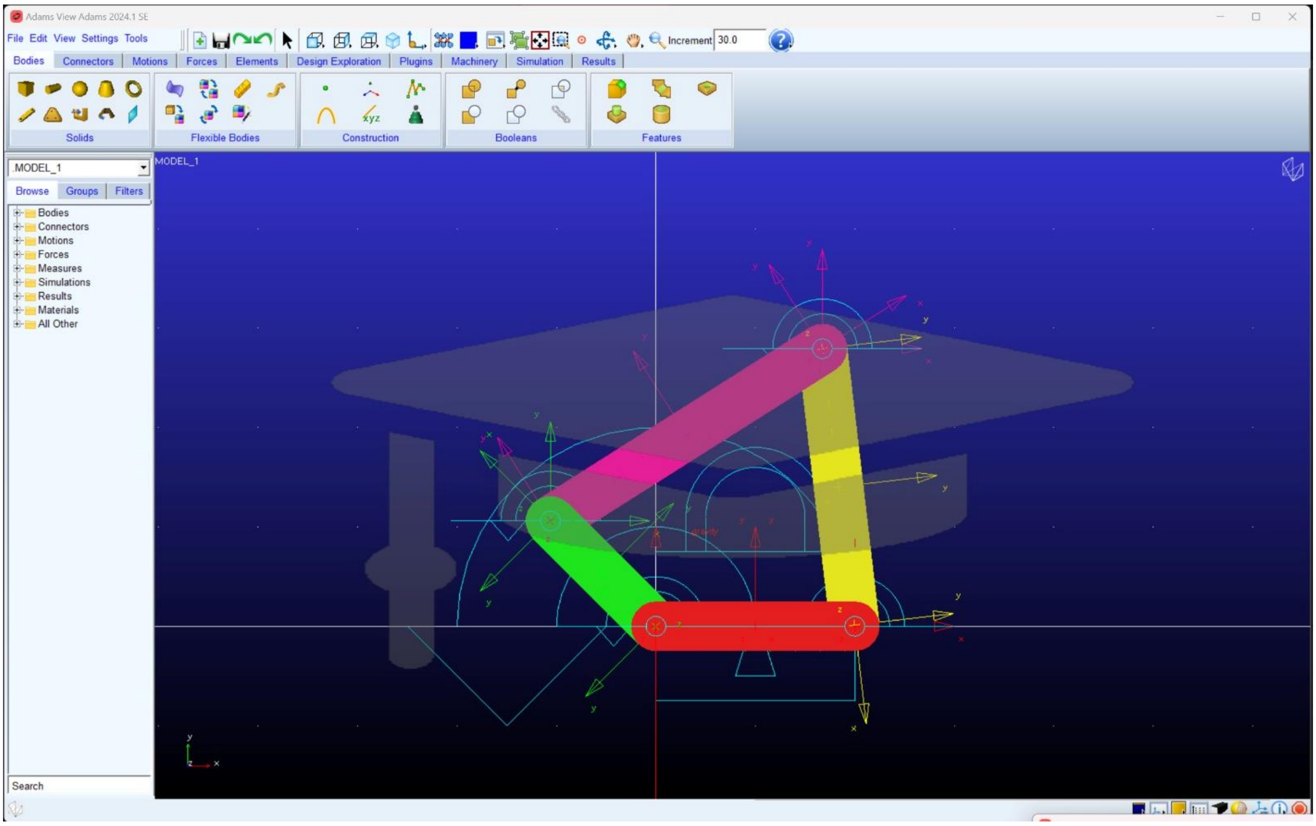
```
BDv = B-D;
omegav = [0 0 omega];
vBv = cross(omegav, BDv);
vB = norm(vBv);
BAv = B-A;
AB = norm(BAv);

q = (vB-vA) / AB;
qv = [0 0 q];
GAv = BAv / 2;
vGv = vAv + cross(qv, GAv);
vG = norm(vGv);

vB2v = vAv + cross(qv, BAv);
vB2v = norm(vB2v);

vA2v = cross(omegav,ADv);
vA2 = norm(vA2v);

q = norm(qv);

q= q*180/pi    %angular velocity of the coupler  (in deg/s)
vG = vG*1000   %velocity of cm of coupler link (in mm/s)
```

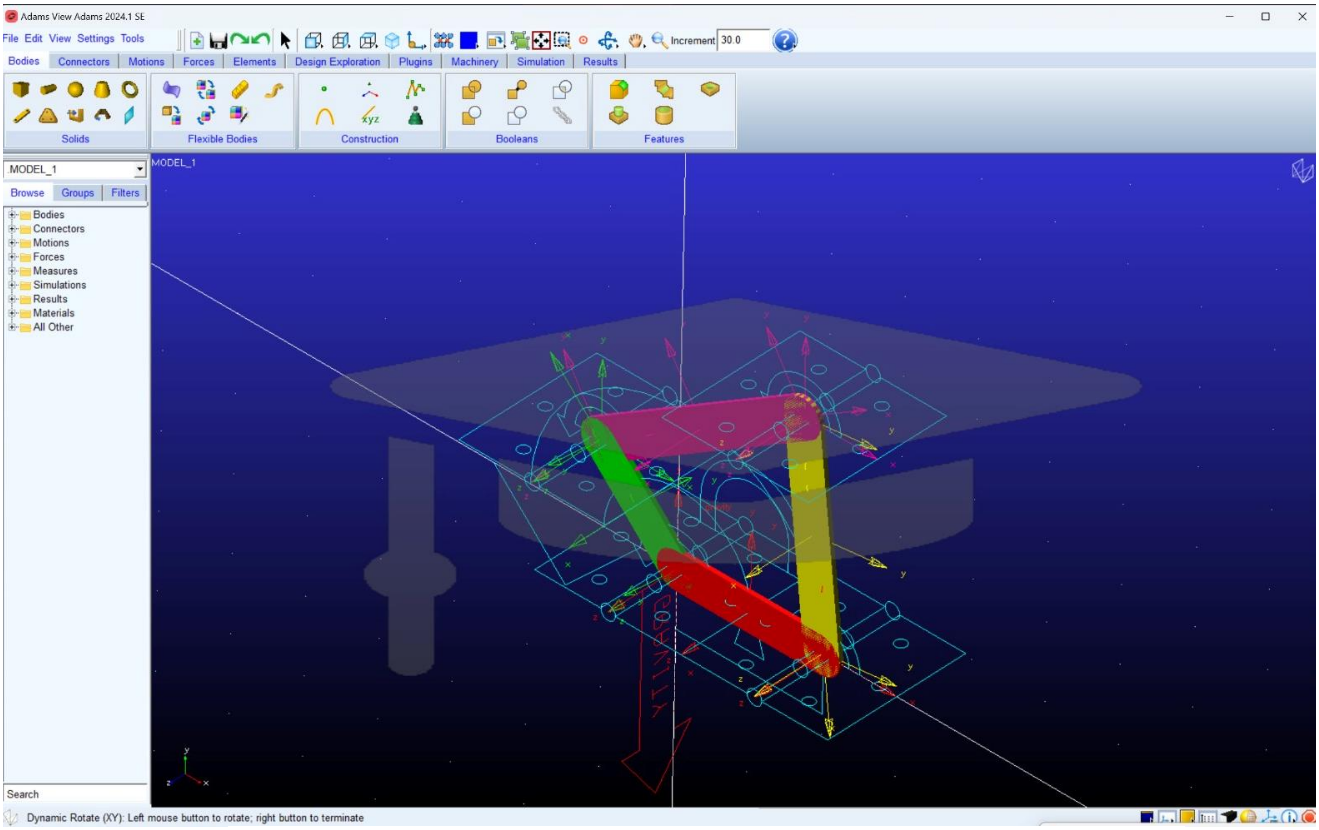
Results of this code on next page

Now varying theta in the above code, we get the following results:

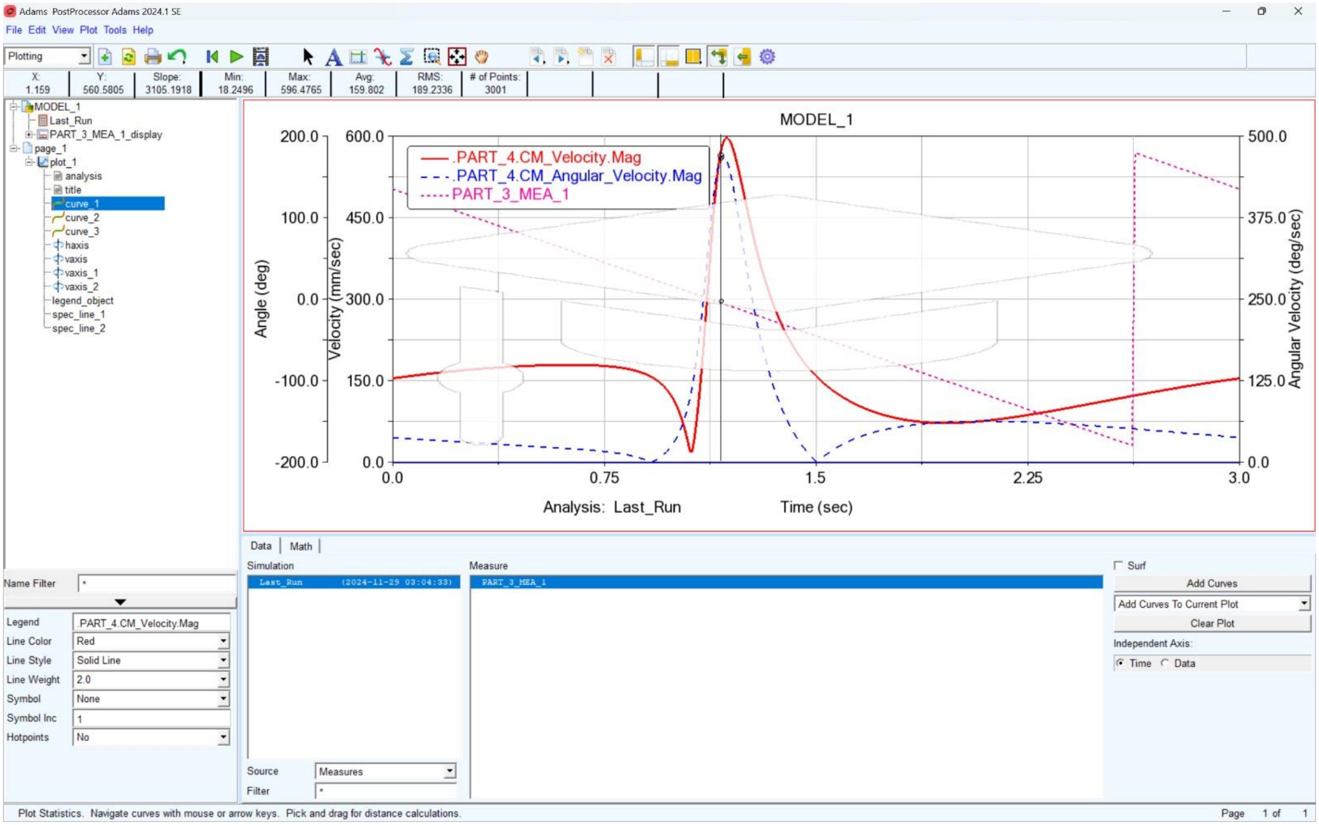
| Theta (in rad) | $v_G$(in mm/s) (magnitude) | o (in deg/s) (magnitude) |
|---|---|---|
| 0 | 607.4292 | 301.6797 |
| $\frac{\pi}{6}$ | 173.4475 | 9.0297 |
| $\frac{\pi}{3}$ | 166.6270 | 16.7420 |
| $\frac{\pi}{2}$ | 159.8802 | 2.9077 |
| $\frac{2\pi}{3}$ | 204.9376 | 34.3555 |
| $\frac{5\pi}{6}$ | 169.4995 | 28.2417 |
| $\pi$ | 172.7869 | 5.7146 |
| $\frac{7\pi}{6}$ | 213.6035 | 30.3958 |
| $\frac{4\pi}{3}$ | 186.4927 | 25.1592 |
| $\frac{3\pi}{2}$ | 139.0547 | 19.9528 |
| $\frac{5\pi}{3}$ | 879.1167 | 4343.2000 |
| $\frac{11\pi}{6}$ | 423.2271 | 360.1780 |
| $2\pi$ | 615.8497 | 305.4296 |

# Analysis of four bar mechanism using ADAMS:

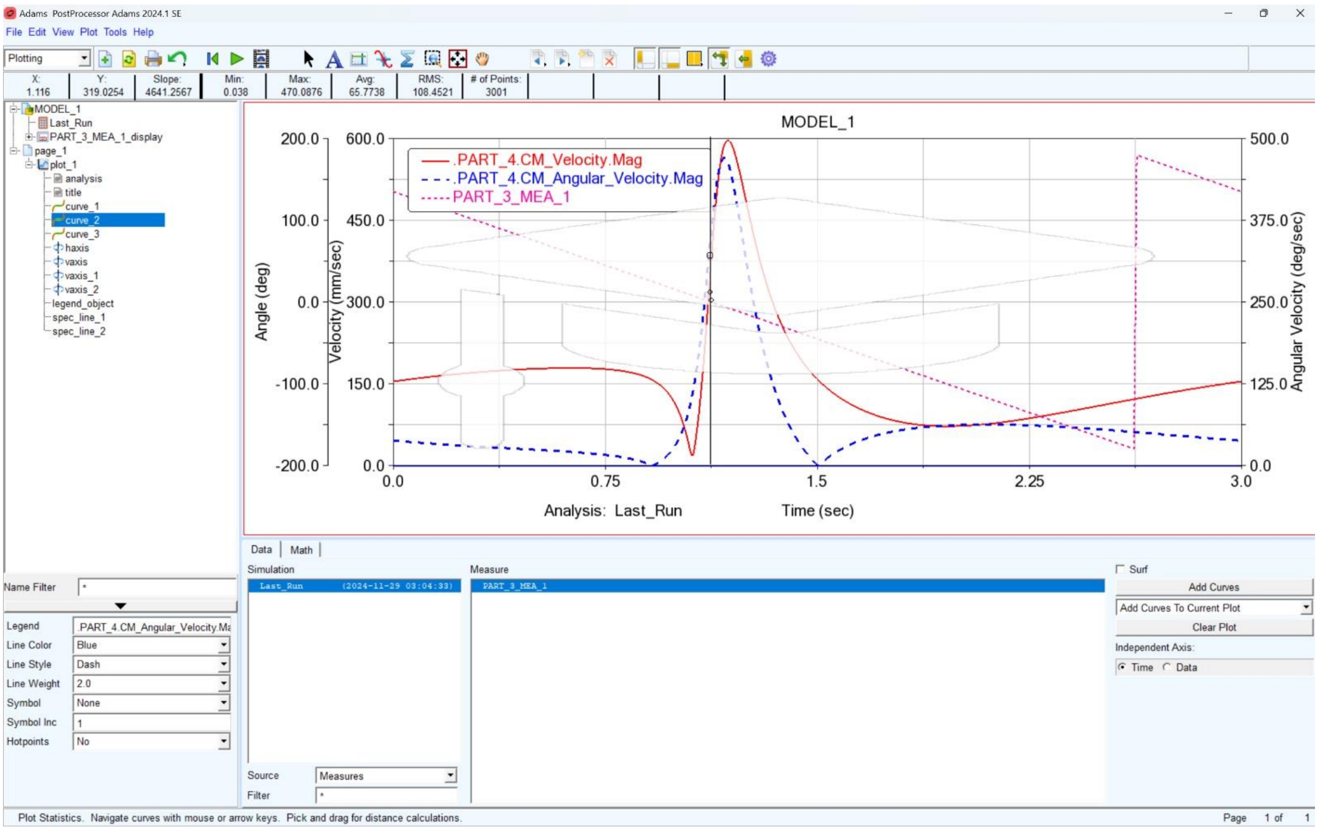


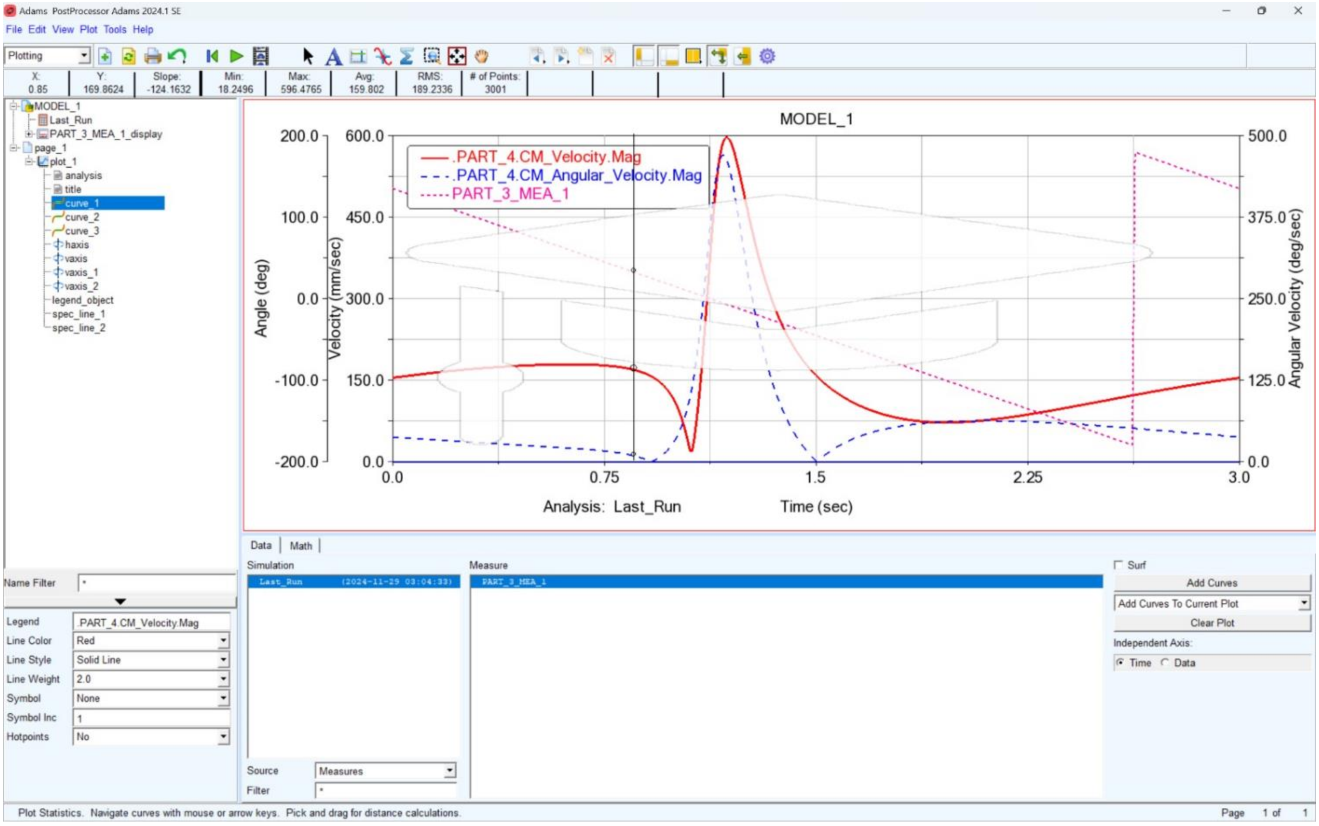

Four-bar mechanism model in ADAMS



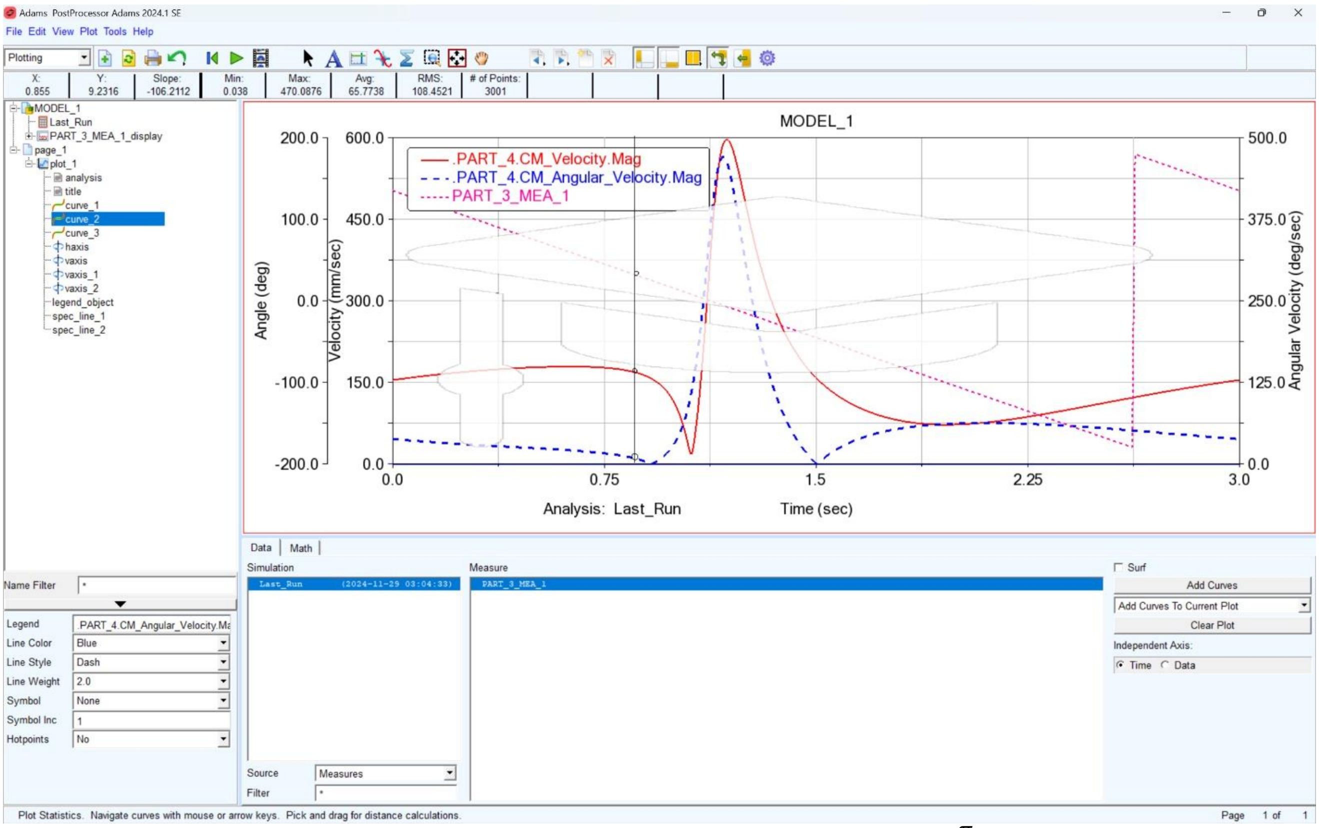
Velocity of centre of mass of coupler at theta = 0 rad



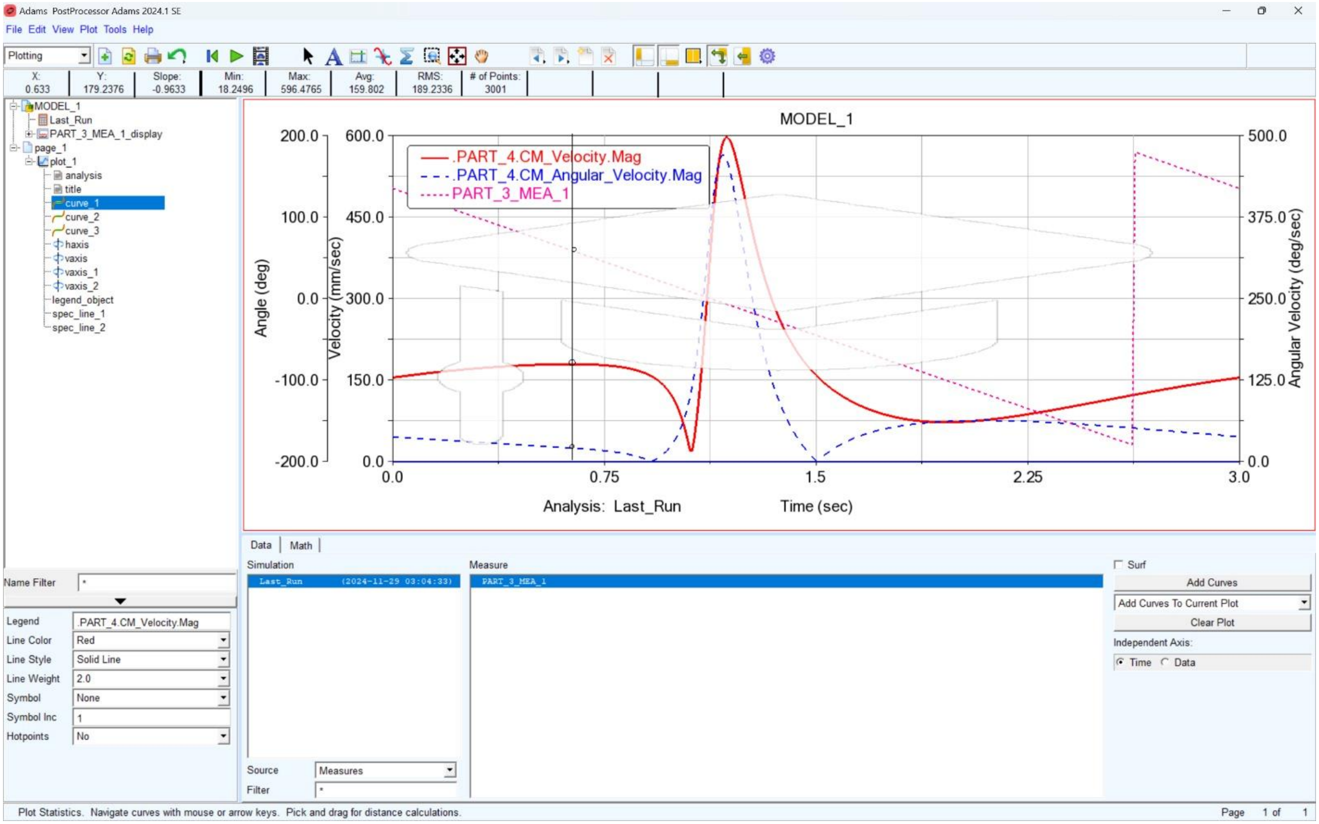
Angular velocity of centre of mass of coupler at theta = 0 rad



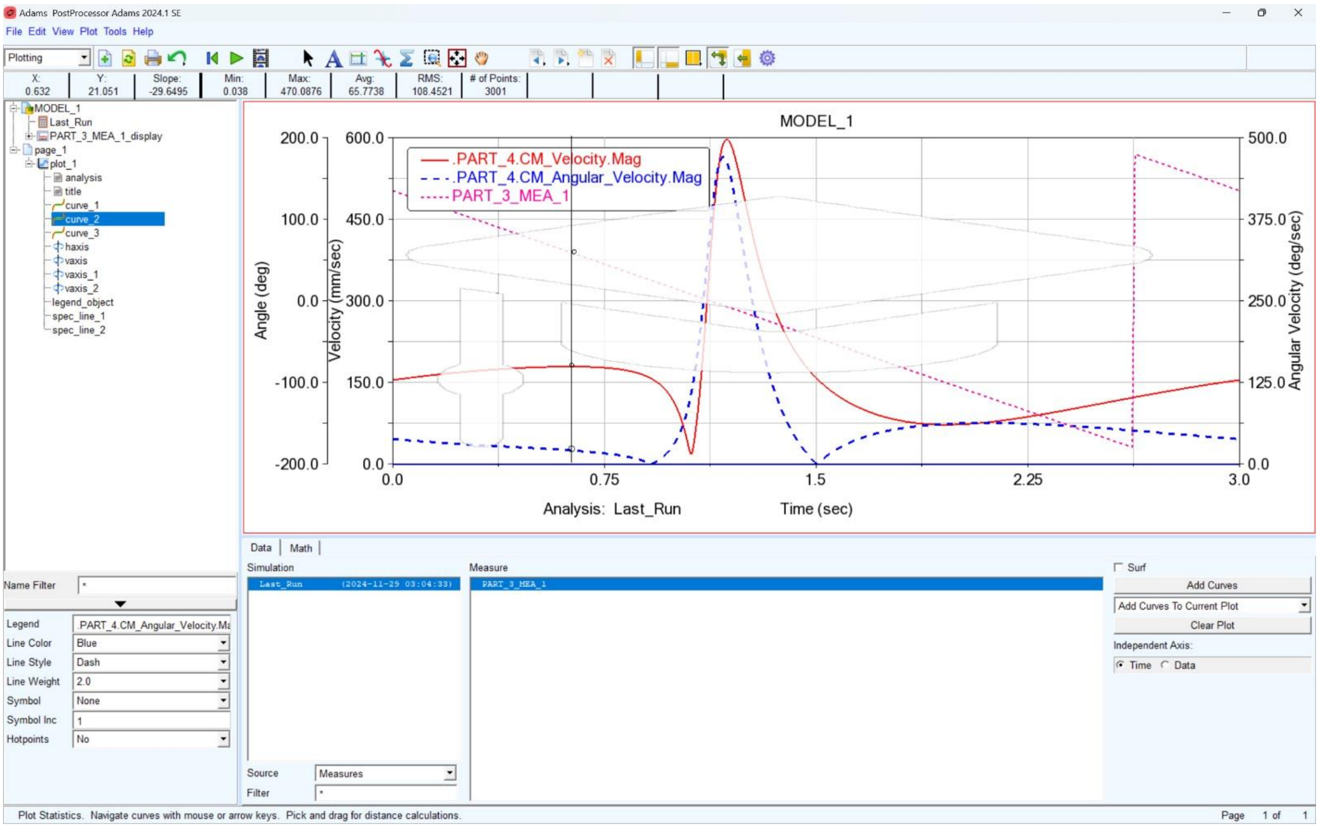

Velocity of centre of mass of coupler at theta = $\frac{\pi}{6}$ rad




Angular velocity of centre of mass of coupler at theta = $\frac{\pi}{6}$ rad




Velocity of centre of mass of coupler at theta = $\frac{\pi}{3}$ rad




Angular velocity of centre of mass of coupler at theta = $\frac{\pi}{3}$ rad

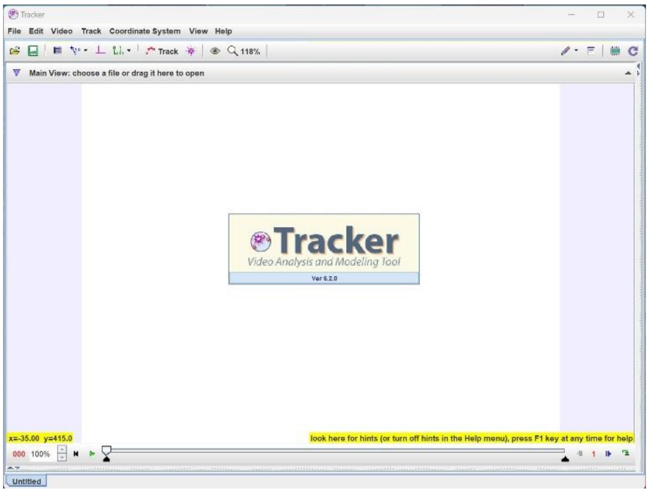
The graphs for other theta can be found in the repository given in the Result section.

Here are the approximate values from the graphs

| Theta (in rad) | $v_G$(in mm/s) (magnitude) | o (in deg/s) (magnitude) |
|---|---|---|
| 0 | 560.0000 | 319.0254 |
| $\frac{\pi}{6}$ | 169.8624 | 9.2316 |
| $\frac{\pi}{3}$ | 179.2376 | 21.0510 |
| $\frac{\pi}{2}$ | 174.1164 | 27.7860 |
| $\frac{2\pi}{3}$ | 162.8705 | 34.8069 |
| $\frac{5\pi}{6}$ | 141.7946 | 42.2207 |
| $\pi$ | 122.1913 | 26.9320 |
| $\frac{7\pi}{6}$ | 157.2840 | 37.9160 |
| $\frac{4\pi}{3}$ | 137.6910 | 32.4820 |
| $\frac{3\pi}{2}$ | 117.9840 | 26.2790 |
| $\frac{5\pi}{3}$ | 624.4670 | 3421.0000 |
| $\frac{11\pi}{6}$ | 297.1830 | 418.6170 |
| $2\pi$ | 426.5810 | 396.1720 |

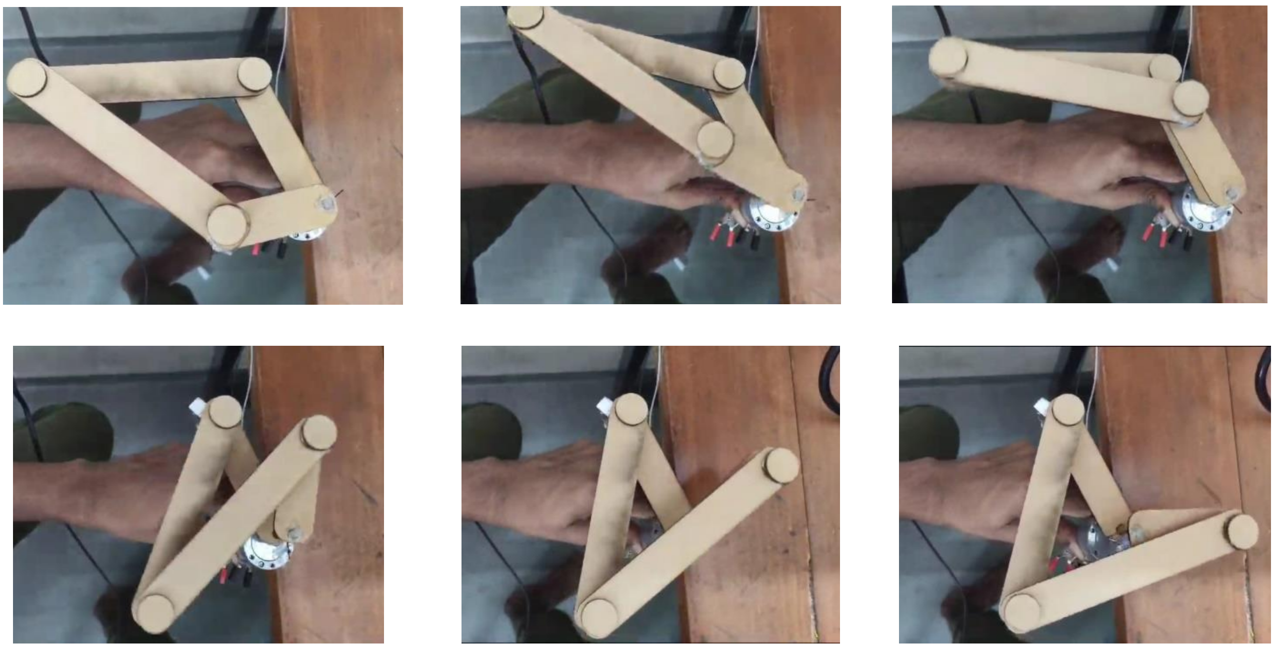
# MEASUREMENT TECHNIQUES

In order to accurately measure the velocity and the angular velocity of the centre of mass of the coupler link we recorded a video of the four-bar mechanism in motion. We the imported it in a video analysis software named 'Tracker' which helped us determine the velocity and the angular velocity of the link for varying theta. However, we don't really know how accurate this is. Such inaccuracies could have led to deviations of the experimental value from the theoretical ones.



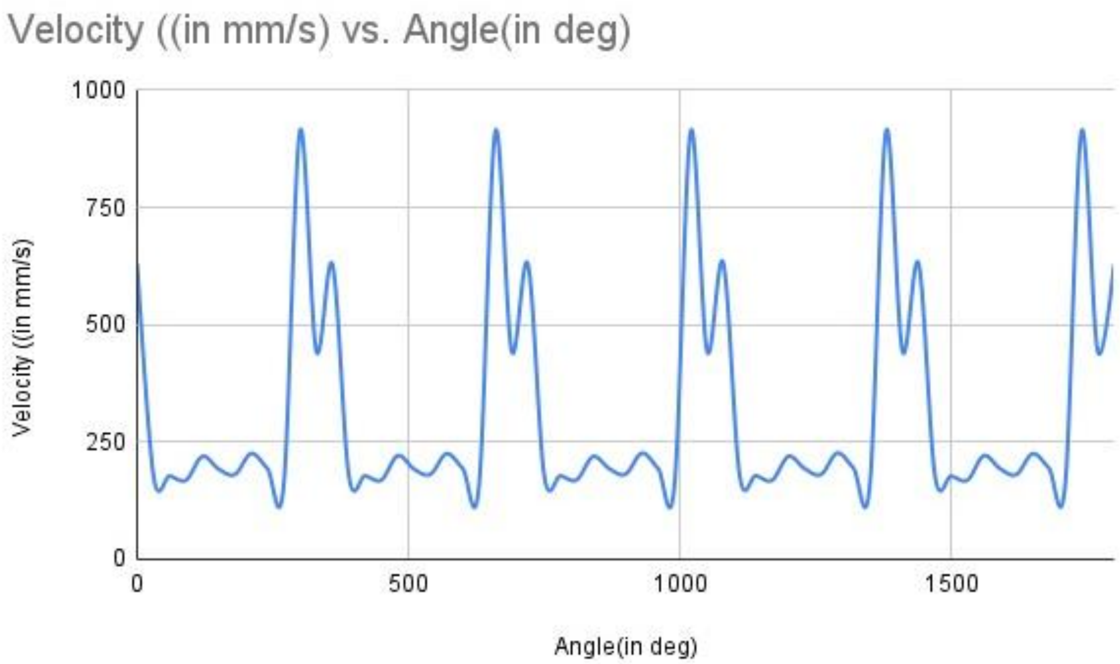

We could have also used a pocket lab sensor a data acquisition device which should be placed on the centre of mass of the four-bar mechanism to capture real time data during motion. In order, to measure angular velocity the sensor should be attached to a joint of the four-bar mechanism. This would directly give the angular velocity data due to its built-in gyroscope. In order to measure the linear velocity, accelerometer function of the sensor can be used which will give us the linear acceleration. This can then be integrated to get linear velocity.

<u>Experimental setup in motion.</u>

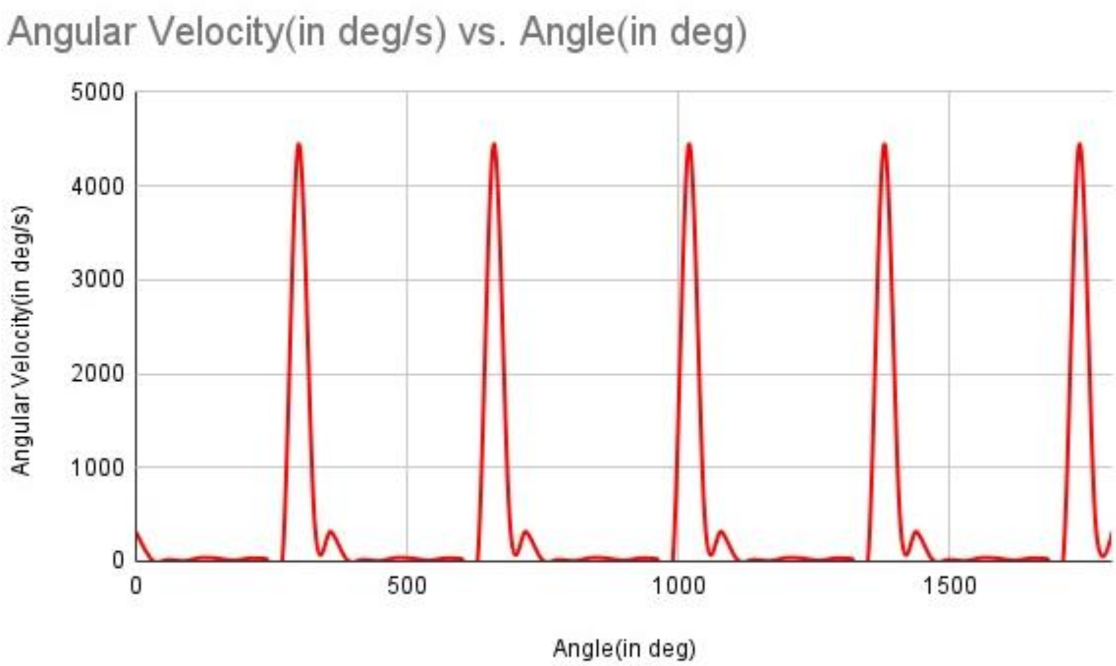


# RESULTS

The graphs obtained by video analysis software 'Tracker' are as follows




Linear velocity of centre of mass of coupler vs angle




Angular velocity of centre of mass of coupler vs angle

The data obtained is as follows:

| Theta (in rad) | $v_G$(in mm/s) (magnitude) | o (in deg/s) (magnitude) |
|---|---|---|
| 0 | 629.2300 | 313.5600 |
| $\frac{\pi}{6}$ | 178.4320 | 10.8900 |
| $\frac{\pi}{3}$ | 177.5800 | 17.2100 |
| $\frac{\pi}{2}$ | 168.5900 | 4.1200 |
| $\frac{2\pi}{3}$ | 219.3600 | 38.5500 |
| $\frac{5\pi}{6}$ | 192.3500 | 31.2600 |
| $\pi$ | 180.5800 | 7.5200 |
| $\frac{7\pi}{6}$ | 225.5900 | 33.0900 |
| $\frac{4\pi}{3}$ | 193.3600 | 28.2200 |
| $\frac{3\pi}{2}$ | 158.2300 | 22.3500 |
| $\frac{5\pi}{3}$ | 913.2883 | 4445.8700 |
| $\frac{11\pi}{6}$ | 444.2300 | 375.3500 |
| $2\pi$ | 627.4150 | 313.5600 |

The results obtained from the experimental analysis match that obtained by the ADAMS simulation and the theoretical analysis done in MATLAB. However, some deviations are obtained among these three which can be due to software errors and real-life problems like friction, manufacturing defects and material properties.

The detailed graphs, videos and pictures of ADAMS simulation, MATLAB files and video of the mechanism in motion can be found here:

https://iitgnacinmy.sharepoint.com/:f:/g/personal/23110110_iitgn_ac_in/EhqSht9SKYtEhayco7elydIBEDaP7KGGuqUOjHbxWXtouw?e=lHL99g

It can also be accessed through the given QR code.

# DISCUSSIONS

This experiment helped us understand how a planar four-bar mechanism works in real life. Using software like the Tracker MATLAB and ADAMS we measured how the coupler link moved and how fast it rotated at different crank angles.

While the theory and Adams simulations gave us good predictions, the real-life measurements showed some differences.

The results from the ADAMS simulations were very close to that calculated by MATLAB theoretically, which means the theory works well in a perfect, digital environment. However, when we tested the mechanism in real life, we noticed some differences. These differences happened because of real-world factors like friction between the joints and moving parts material deformation or manufacturing errors.

# SHORTCOMINGS

1) Tracing the instantaneous centers of rotation requires precise tracking equipment and a high frame rate camera setup, which we did not have access to during this phase of the project.

2) Time Constraints: Given the complexity of deriving the centrode analytically, combined with our focus on accurately determining the coupler's velocities, we had to prioritize other parts of the project.

# SCOPE FOR IMPROVEMENT

To improve the accuracy of the results, we need to address imperfections in the manufacturing process. Issues like uneven link lengths and joint misalignments cause differences between theoretical predictions and experimental results. Friction at the joints is another major factor that affects the experimental results.

Moreover, real materials do not always behave ideally. Variations in properties like stiffness and elasticity can change how the system moves. In future analyses, we should include the effects of friction and material behaviour to make our models more accurate.

The accuracy of video analysis tool like Tracker cannot be guaranteed. Exploring alternative software with better algorithms could help improve measurement accuracy.

Adams simulations can also be improved by adding more realistic conditions to the simulations—such as accounting for joint clearance, non-linear material properties, and dynamic changes—can make them more reliable. By fine-tuning the simulation parameters based on experimental observations, we can bridge the gap between theory and real-world behaviour.

Overall, combining improved manufacturing processes, better experimental tools, and refined simulations will help us achieve more accurate and reliable results in future studies.

# REFERENCES

1) Meriam, J.L., Bolton, Jeffrey N., & Kraige, L. Engineering Mechanics: Dynamics (Edition 7)
2) https://www.cs.cmu.edu/~rapidproto/mechanisms/chpt5.html
3) https://www.youtube.com/watch?v=knS124jlM14
4) https://dynref.engr.illinois.edu/aml.html
5) Lecture Slides.

# ACKNOWLEDGEMENT

We would like to thank Professor K.R. Jayaprakash for giving us this project and guiding us throughout. His teaching and support helped us a lot in completing this work.

We also want to thank our Teaching Assistants, Vaibhav Kumar Tandel and for their help during the project. We would also like to extend our gratitude to The Tikerer's lab which allowed us to use the Laser Cut technology and guided us through the process. We are also thankful to our seniors who helped us out whenever we were not able to proceed with the experiment.

Finally, we thank our friends and classmates for their support. Working and discussing ideas with them helped us solve challenges and finish the project on time. This project would not have been possible without the support and help of all these people. We are very thankful to each one of them.