14.02 Principles of Macroeconomics

Problem Set 2 Solutions

Fall 2017

1 Question 1: Money Demand (Lecture 4)

(a). Central Bank Controlling Money Supply

1. Money are useful because it can be used for transactions, while bonds cannot. Holding money comes at the cost of given up the interest rate that bonds would have paid. As the interest rate goes up, we have to give up more interest payments for a given amount of money holdings. Therefore people have less incentive to hold money, and they tend to put more of their wealth into bonds.

2. The equilibrium requires money supply to be equal to $M^s = M^d$. Using this, we can solve for interest rate as a function of money supply:

$$i = a - \frac{M^s}{\$Y}.$$

Plugging the numbers, we get $i = 0.05$ or $i = 5\%$.

3. Let $\$P_B$ denote the price of the bond. The interest rate on bond is

$$1 + i = \frac{\$240}{\$P_B},$$

which we can rewrite

$$\$P_B = \frac{\$240}{1+i}.$$

If $i = 0.2$, then $\$P_B = \$200$. So the price of bond is \$200.

(b). Central Bank Controlling Interest Rate

Now suppose the central bank controls the interest rate.

4. The money demand is

$$\begin{aligned} M^d &= \$10,000 \times (0.2 - 0.08) \\ &= \$1,200. \end{aligned}$$

The equilibrium in money market requires $M^s = M^d$. Therefore the central bank needs to supply \$1,200 amount of money.

2 Question 2: IS-LM Model (Lecture 5)

(a) Equilibrium in Goods Market

1. In order to invest in new factory or machines, firms typically needs to borrow. The higher the interest rate, the more costly it is to borrow, and thus to invest. Therefore higher interest rate makes the borrowing and investment less attractive, which decreases the investment by firms.

2. The demand for goods in this economy is

$$\begin{aligned} Z &= C + I + G \\ &= c_0 + c_1(Y - T) + b_0 + b_1 Y - b_2 i + G \\ &= c_0 + b_0 + G - c_1 T + (c_1 + b_1) Y - b_2 i. \end{aligned}$$

Using the equilibrium condition, $Y = Z$, we can solve for Y:

$$Y = \frac{1}{1 - (c_1 + b_1)} \left(c_0 + b_0 + G - c_1 T - b_2 i \right). \tag{1}$$

Figure 1 shows the determination of goods market equilibrium graphically.

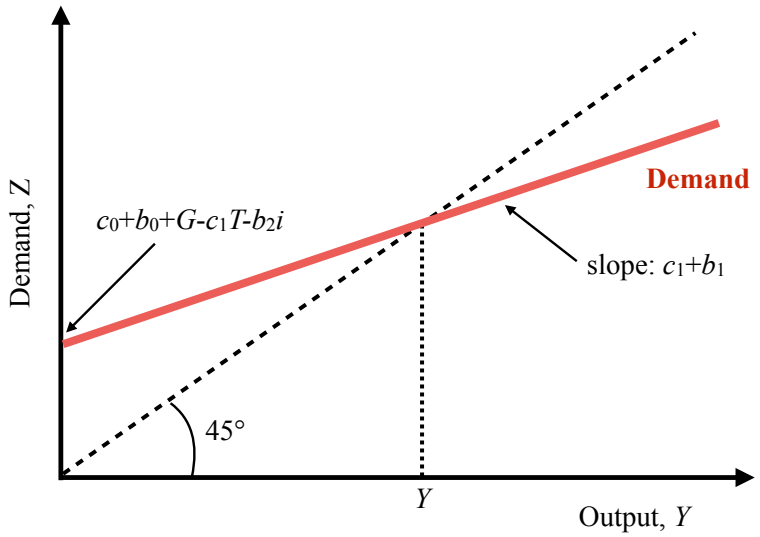

Figure 1: Determination of Goods Market Equilibrium

3. From equation (1), we can see that the government spending multiplier is $\frac{1}{1-(c_1+b_1)}$. Recall from the previous problem set that the government spending multiplier was $\frac{1}{1-c_1}$ when investment was exogenous. Therefore the government spending multiplier with endogenous investment is larger than the economy with exogenous investment. The reason is that in an economy where investment responds to output, when the government spending increases, this increases output, which in turn increases **both consumption and investment**. This further increases demand, and thus output. In an economy with exogenous investment, this additional increase coming from investment was absent.

The following Figure 2 describes how the economy responds.

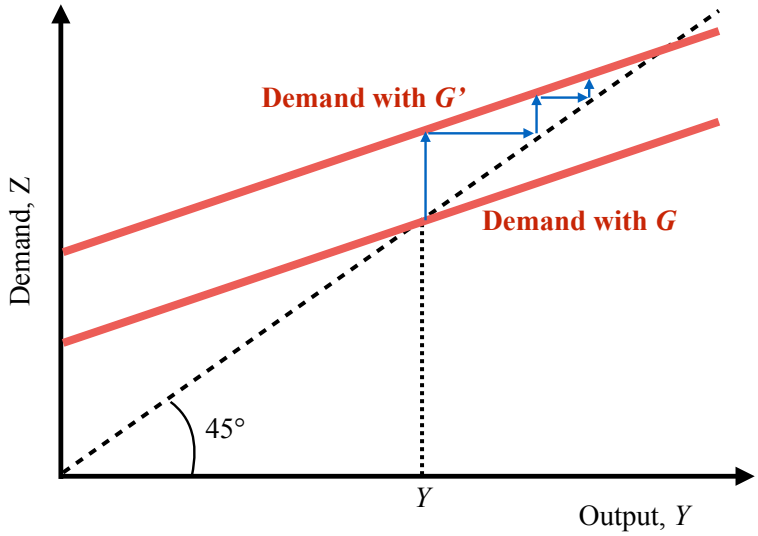

Figure 2: Response of the Goods Market Equilibrium when G increases to G'

(b) IS-LM Analysis

4. The IS curve is given by equation (1), which can be written as

$$i = \frac{1}{b_2}\left[(c_0 + b_0 + G - c_1 T) - (1 - (c_1 + b_1))Y\right].$$

As is clear from the above expression, the slope of IS-curve is $-\frac{1}{b_2}(1 - (c_1 + b_1))$. Since the LM-curve is $i = \bar{i}$, IS-LM diagram can be written as in Figure 3.

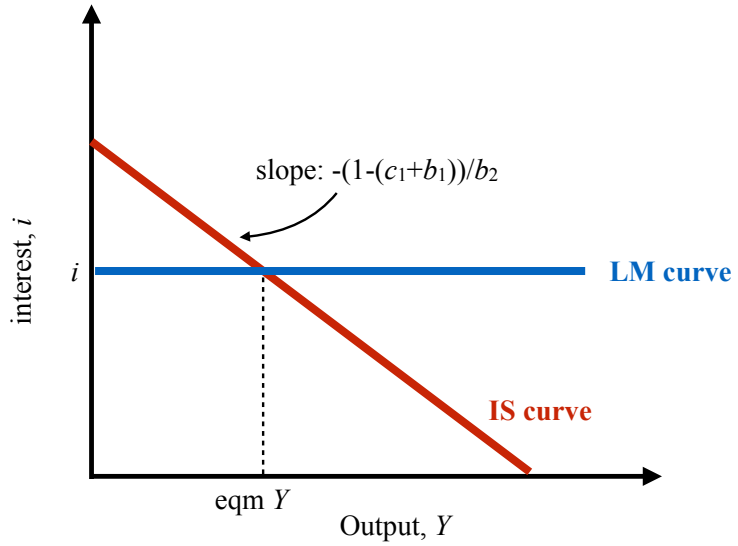

Figure 3: IS-LM Diagram

5. Combining equation (1) and LM-curve, $i = \bar{i}$, we have the expression for equilibrium output:

$$Y = \frac{1}{1-(c_1+b_1)}\left(c_0 + b_0 + G - c_1 T - b_2 \bar{i}\right). \tag{2}$$

From this expression, the output decreases by $-\frac{b_2}{1-(c_1+b_1)}$ in response to an increase in $\bar{i}$. Figure 4 shows this using IS-LM diagram.

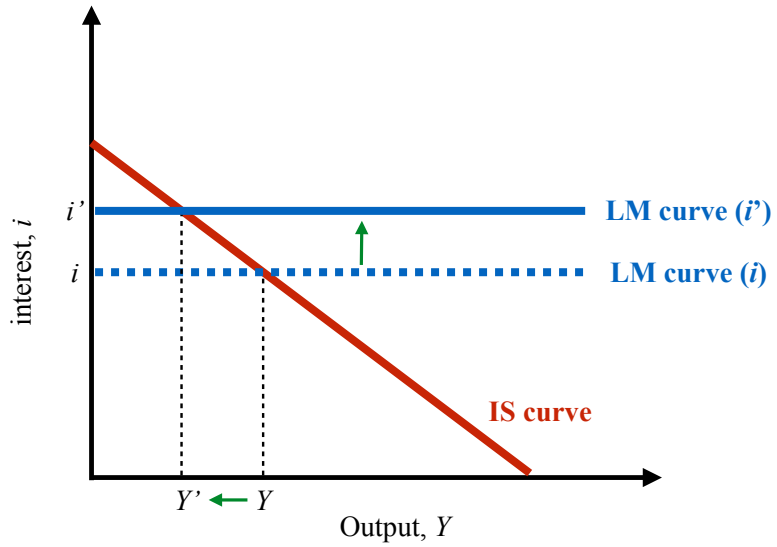

Figure 4: Response of the economy when i increases to i'

6. From equation (2), we can see that equilibrium output falls by $\frac{1}{1-(c_1+b_1)}$ in response to a decrease in c_0. Figure 5 describes this situation using IS-LM diagram. The expression for private saving is

$$\begin{aligned}
S &= Y - T - C \\
&= Y - T - c_0 - c_1(Y - T) \\
&= (1 - c_1)(Y - T) - c_0 \\
&= \frac{(1-c_1)}{1-(c_1+b_1)}\left(c_0 + b_0 + G - c_1 T - b_2 \bar{i}\right) + (1-c_1)T - c_0 \\
&= \frac{b_1}{1-(c_1+b_1)} c_0 + \frac{(1-c_1)}{1-(c_1+b_1)}\left(b_0 + G - b_2 \bar{i} - c_1 T\right) + (1-c_1)T.
\end{aligned}$$

From the above expression, the private saving **decreases** by an amount $\frac{b_1}{1-(c_1+b_1)}$ in response to a decrease in c_0. That is, despite the fact that all the consumers decide to save more, they end up saving less in equilibrium ("**the paradox of saving**").

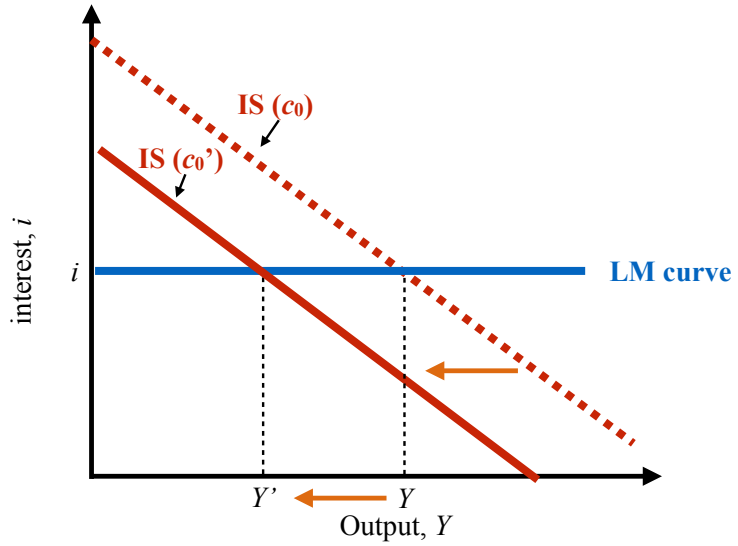

Figure 5: Response of the economy when c_0 falls to c_0'

7. (LOOK at Equation) From equation (2), the central bank needs to decrease the interest rate by $\frac{1}{b_2}$ in response to a unit decrease in c_0. Figure 6 describes the policy response using IS-LM diagram. The private saving is

$$\begin{aligned} S &= Y - T - C \\ &= (1 - c_1)(Y - T) - c_0. \end{aligned}$$

Because Y doesn't change, and T is exogenous, the private saving increases by one in response to a unit decrease in c_0. Therefore the paradox of saving is avoided: consumers end up saving the same amount that they decided to save.

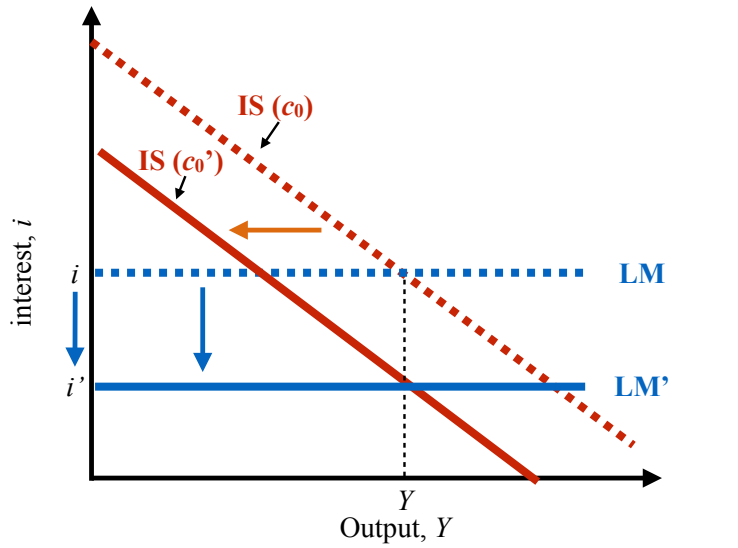

Figure 6: Monetary Policy Response to a fall in c_0

8. From equation (2), the central bank needs to increase the government spending by one in response to a unit decrease in c_0. Figure 7 explains this using IS-LM diagram.

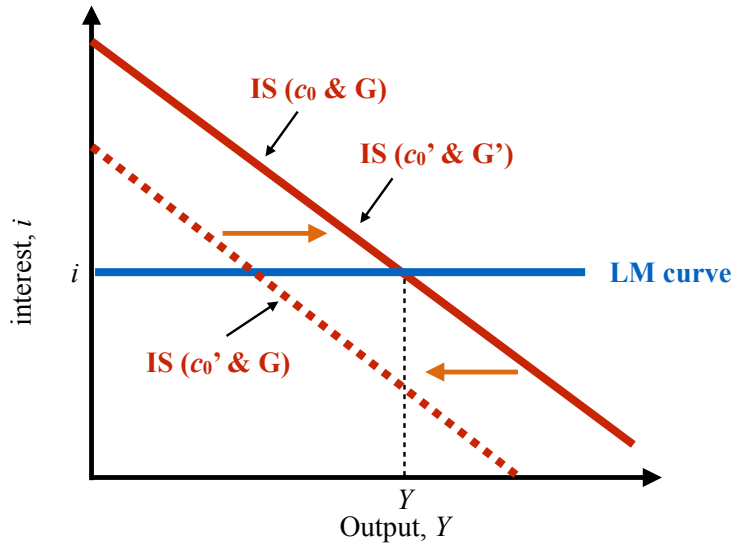

Figure 7: Fiscal Policy Response to a fall in c_0

Note: The government increases government spending from G to G' in response to c_0 falling to c_0'.

9. From equation (2), Y decreases by $\frac{1}{1-(b_1+c_1)}$. Therefore the real money supply decreases by $\frac{1}{1-(b_1+c_1)}(a_0 - a_1\bar{i})$.

(c) IS-LM Model and Facts

10. The central bank increases interest rate precisely the time when the economy is in booms, and it decreases interest rate precisely the time when the economy is in recessions. Therefore the fact that the output tends to be higher when the central bank increases the interest rate does not imply that the higher interest rate causes an increase in output (it's the other way around), and thus does not invalidate the prediction of IS-LM model.

3 Question 3: Extended IS-LM Model (Lecture 6)

(a) Nominal and Real Interest Rate and Risk Premium

1. The real interest rate can be computed as

$$\begin{aligned} 1 + r &= (1 + i)\frac{P_t}{P^e_{t+1}} \\ &= 1.2 \times \frac{\$200}{\$240} \\ &= 1. \end{aligned}$$

Hence the real interest rate, r, is zero.

2. The risk premium has to satisfy

$$1 + i = (1 - p)(1 + i + x) + p \times 0.$$

Therefore the expression for x is

$$\begin{aligned} x &= \frac{1 + i}{1 - p} - (1 + i) \\ &= \frac{1.05}{0.75} - 1.05 \\ &= 0.35. \end{aligned}$$

Therefore risk premium has to be 35%.

(b) Extended IS-LM Model

3. The IS relation is

$$\begin{aligned} & Y = c_0 + c_1 Y + b_0 + b_1 Y - b_2(r + x). \\ \Leftrightarrow \quad & Y = \frac{1}{(1 - (c_1 + b_1))}(c_0 - b_0 + b_2(r + x)). \end{aligned}$$

Combining with LM curve, $r = \bar{r}$, the equilibrium output is

$$Y = \frac{1}{(1 - (c_1 + b_1))}(c_0 + b_0 - b_2(\bar{r} + x)).$$

In the original situation, the output is $Y = \frac{1}{1-0.5}(100 - 100(0.1 + 0.04)) = 172$. If the risk premium, x, increases to 0.08, the output becomes $Y = \frac{1}{1-0.5}(100 - 100(0.1 + 0.08)) = 164$. So the output decreases by 8. Figure 8 describes the situation using IS-LM diagram.

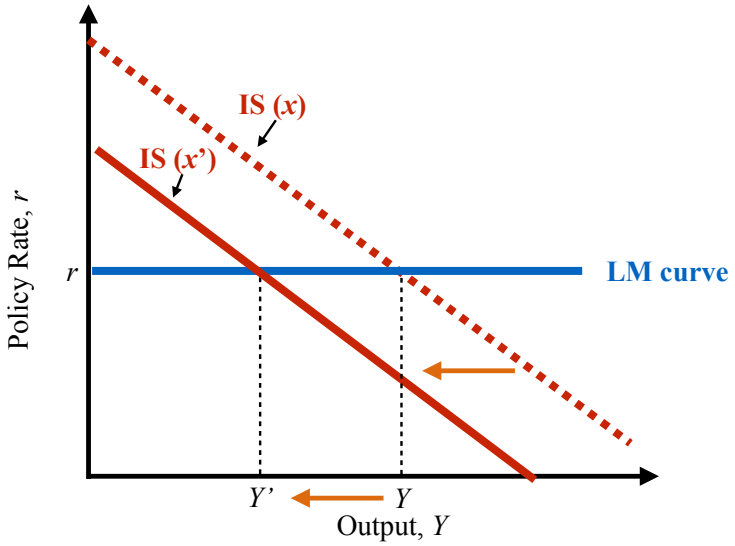

Figure 8: The Effect of an Increase in Risk Premium from x to x'

4. If the central bank lowers the interest rate by the same amount as increases in x, then the central bank is able to undo this effect. In this context, the central bank should lower the interest rate from 10% to 6% because risk premium has increased by 4%. Figure 8 describes the situation using IS-LM diagram.

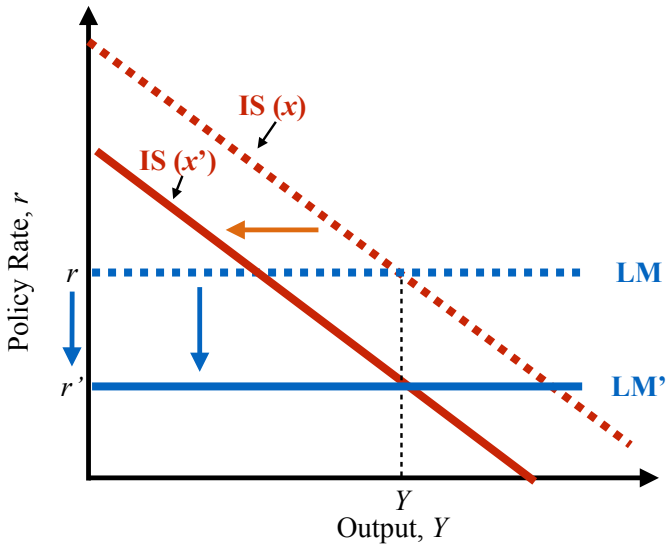

Figure 9: Monetary Policy Response to Risk Premium Increase

5. Since inflation rate is 2%, the central bank cannot lower real interest rate below -2%. Because the current policy rate is 10%, if the risk premium increases by more than 12%, the central bank is unable to further lower the interest rate. In this context, if $x' = 16\%$ is the level of risk premium that triggers the zero lower bound.

(c) Financial Intermediaries

6. Capital ratio is $120/600 = 20\%$. Leverage ratio is $600/120 = 5$.