14.02 Principles of Macroeconomics
Problem Set 8

Fall 2017

Question 1

Suppose that a firm's production function is given by:

$$Y = F(K, N) = AK^{\alpha}N^{1-\alpha}$$

for $\alpha \in (0, 1)$. The firm must rent capital from the owners of capital, and must hire workers. It pays the owners of capital a constant rental rate r for each unit of capital, and it pays workers a wage w for each unit of labor. As it is standard in economics, let's normalize the price of the good produced by the firm to $P = 1$, so that the wage and rental rates are expressed relative to this price. The firms' profits are given by:

$$\Pi(K, N) = F(K, N) - rK - wN$$

In what follows, assume that r and w are taken as given, so they are parameters.

(a)

Let

$$y = \frac{Y}{N} \qquad k = \frac{K}{N}$$

Using the production function, express y as a function of k.

(b)

Assume the firm chooses $k \geq 0$ to maximize:

$$\frac{\Pi}{N} = y(k) - rk - w$$

What is the value $k^{\star}$ that maximizes profits per worker? Is it unique?

(c)

Take the partial derivative of Π with respect to N and set it equal to zero. This is the necessary and sufficient condition for the firm when it chooses N optimally. Express w as a function of k and parameters.

(d)

Show that the firm's profits, when $k = k^{\star}$, are such that $\Pi = 0$.

[Hint: use the equation you derived in (b) to substitute out r, and the expression for w from (c) to substitute out w, when $k = k^{\star}$. Also, note that $\Pi = {}^{\Pi}/_{N} N$.]

(e)

Now assume that the firm chooses K and N optimally, that is, to maximize Π. What condition should K/N satisfy for K and N to be optimal?

(f)

In subpoint (d), we showed that profits are zero. This implies that of the revenues of the firms, a certain share will accrue to capital owners, while the rest will accrue to workers. Compute these shares.

Question 2

Suppose that the economy's production function is given by:

$$Y = AK^{\alpha}N^{1-\alpha}$$

Assume that $A = 1$, and $\alpha = 1/3$. Population N is constant.

(a)

Is this production function characterized by constant returns to scale? Explain.

(b)

Are there decreasing returns to capital? Are there decreasing returns to labor? Justify your answers.

(c)

Transform the production function into a relation between output per worker and capital per worker.

(d)

For a given saving rate, s, and depreciation rate δ, give an expression for capital per worker in the steady state

(e)

Give an expression for output per worker in the steady state.

(f)

Solve for the steady state level of output per worker when $s = 0.32$ and $\delta = 0.08$.

(g)

Suppose that the depreciation rate remains constant at $\delta = 0.08$, while the saving rate is reduced by half, to $s = 0.16$. What is the new steady-sate output per worker?

(h)

Suppose now that both the saving rate, s, and the depreciation rate δ are equal to 0.10. What is the steady state level of capital per worker? What is the steady state level of output per worker?

(i)

Suppose now that the economy is in steady state and that, at the beginning of period t, the depreciation rate increases permanently from 0.10 to 0.20. Assume here that $s = 0.10$. What will be the new steady state levels of capital per worker and output per worker?

(j)

Consider the situation in (i). Compute the path of capital per worker and output per worker over the first three periods t, $t+1$, and $t+2$ after the change in the depreciation rate.

[Hint: capital does not change immediately, that is, k_t is inherited from the previous period!]

Question 3

Consider the following Solow growth model

$$y = Ak^{\alpha}$$

Assume that $A = 6$, $\alpha = 0.5$, and $\delta = 0.1$. We set the population growth rate, n, equal to 0.02.

(a)

If the savings rate is equal to 0.4, find the steady state levels of capital per worker, $k^{\star}$, output per worker, $y^{\star}$, and consumption per worker, $c^{\star}$.

(b)

Define and find the golden rule level of (per capita) capital stock and explain why adding capital in excess of that amount will not increase per capita living standards.

(c)

Suppose that the saving rate is at the golden rule level. What happens if the economy is initially in steady state, but then loses half of its (per capita) capital in a war and all other parameters, such as δ, n, A and α stay the same? Determine in particular how consumption and investment react, and how the growth rate of the economy changes. How does this affect the golden rule saving rate? What about the rate of return earned by capital in the short-run and long-run?