14.02 Principles of Macroeconomics

Problem Set 6 Solutions

Fall 2017

Question 1 (Chapter 14)

In finance, an arbitrage opportunity in a world without uncertainty can occur in the two following ways:[1]

1. A transaction in which you receive money today and promise (pay) nothing in the future.
2. A transaction where you pay nothing now and receive money in the future.

In what follows, assume that the future is known with certainty.

(a)

Consider a zero coupon bond (ZCB) issued today, where the issuer pays \$100 after one year. The current annual interest rate is 2%. Suppose the price of asset is \$95. Assume that you do not have any money, but that you can borrow any amount, as long as you repay it the following year at the given interest rate. Construct an arbitrage opportunity based on purchasing one unit of the financial asset. Be specific on why that strategy constitutes an arbitrage. Given that you can borrow any amount, how many units of this "investment strategy" would you want to implement?

Solution: The arbitrage strategy consists of the following steps:

1. *Borrow* \$95 *today.*
2. *Buy one unit of the asset today using the borrowed* \$95*.*
3. *Receive a payoff of* \$100 *from the asset tomorrow.*
4. *Pay back* $\$95 \times (1 + 2\%) = \$96.9$ *tomorrow.*
5. *Total profit* $\$100 - \$96.9 = \$3.1$*.*

This is an arbitrage opportunity because we paid nothing today (we borrowed 95 and used it to buy the asset), but received a sure payoff of \$3.1 *tomorrow. If prices do not change (a very big assumption), we could multiply the amounts we borrow and buy so as to get an arbitrarily large payoff tomorrow without investing anything today.*

[1] This is a slightly different, and more formal concept than the one we use for arbitrage conditions. Indeed, in the latter case we require that arbitrage opportunities are absent in expectations, whereas in the formal definition this should hold "with certainty", that is, for every possible realization of that uncertainty. We will not concern ourselves with this distinction neither here, nor in the rest of the course. In particular, this question assumes away uncertainty so the two definitions are equivalent in this case.

(b)

Suppose that the price of the asset today is \$99 instead. Repeat part (a), but this time construct an arbitrage opportunity based on short selling one unit of the financial asset,[2] and assume that you may lend any amount and get it back with certainty after one year together with the current annual interest rate payment. If there were no restriction on how much people would be willing to borrow from you, how many units of this "investment strategy" would you want to implement?

[Hint: what must be the price of the asset tomorrow?]

Solution: The arbitrage strategy consists of the following steps:

1. *Selling short the asset today for* \$99
2. *Lending the* \$99 *obtained from selling short today.*
3. *Obtaining* $\$99 \times (1 + 2\%) = \$101$ *tomorrow when the loan is repaid.*
4. *Buying back the asset at price* \$100*.*
5. *Total profit* $\$101 - \$100 = \$1$

This is an arbitrage opportunity because we paid nothing today (we sold the asset short and lent the money we received from selling the asset), but receive a certain payoff tomorrow.

(c)

Argue why, if there are no arbitrage opportunities, then the price of asset must equal the present discounted value of future payments.

[Hint: consider the cases $P < P^\star$ and $P > P^\star$, where $P^\star$ is the PDV of future payments for the asset.]

Solution: The price of the asset should be

$$P^\star = \frac{\$100}{1 + 2\%} \approx 98.04$$

If $P < P^\star$, then we can create an arbitrage opportunity by borrowing $P^\star$ dollars and using them to buy one unit of the asset. This strategy costs nothing today and its payoff tomorrow is:

$$\$100 - (1 + 2\%) \times \$P > \$100 - P^\star (1 + 2\%) = 0$$

If $P > P^\star$, then we can create an arbitrage opportunity by shorting the asset and lending the money obtained from shorting. This costs nothing today and its payoff tomorrow is

$$(1 + 2\%) \times \$P - \$100 > P^\star (1 + 2\%) - \$100 = 0$$

Thus, if we assume no arbitrage, then we must have $P = P^\star$.

[2] Short selling is the practice of borrowing an asset from a broker and selling the asset on the financial market. The investor who is short selling the asset receives a payoff of P today (where P is the price of the asset), but must return the asset to the broker in the future. If the asset gains value, the investor will face a loss (since she has to buy the asset at a higher price than the one she sold it for). If the asset loses value, the investor will profit.

(d)

Consider a two-year asset that pays a coupon of \$5 at the end of the first and second years, as well as a principal of \$100 at the end of the second year. Assume that the interest rate will stay indefinitely at 2%. What is the current price of asset, $P^\star$, if we assume no arbitrage?

Solution: The price must equal the PDV of future payments, thus:

$$P^\star = \frac{\$5}{1+2\%} + \frac{\$105}{(1+2\%)^2} \approx \$105.82$$

(e)

Assume instead that the interest rate will be 4% from next year on, but it is still 2% this year.[3] What is the price of asset in (d), if we assume no arbitrage?

Solution: The price must equal the PDV of future payments, thus:

$$P^\star = \frac{\$5}{1+2\%} + \frac{\$105}{(1+2\%)\,(1+4\%)} \approx \$103.88$$

[3] This means that 2% is the return on a bond paying one year from now, while 4% is the return on a bond issued next year paying two years from now.

Question 2 (Chapter 14)

(a)

Consider a ZCB with face value of \$1000,[4] a maturity of three years, and a price of \$800. What is the yield to maturity of such asset?

Solution: The YTM is

$$\sqrt[3]{\frac{\$1000}{\$800}} - 1 \approx 7.72\%$$

(b)

Suppose the interest rate this year is 5, and the financial markets expect the interest rate to increase to 5.5 next year, to 6 two years from now. Determine the yield to maturity of a one-year ZCB, a two-year ZCB, and a three-year ZCB.

Solution:

- *1-year is simply the interest rate, i.e., 5%*
- *2-year*

$$\sqrt{(1+5\%)\,(1+5.5\%)} - 1 \approx 5.24\%$$

- *3year*

$$\sqrt[3]{(1+5\%)\,(1+5.5\%)\,(1+6\%)} - 1 \approx 5.49\%$$

(c)

Explain why a downward-sloping yield curve may indicate a recession is expected.

Solution: An downward-sloping yield curve (in the contemporary U.S.) means that investors expect future interest rates to be lower.[5] One explanation is that investors foresee a recession, and expect the Central Bank to lower interest rates in the future, leading to an inverted yield curve.

(d)

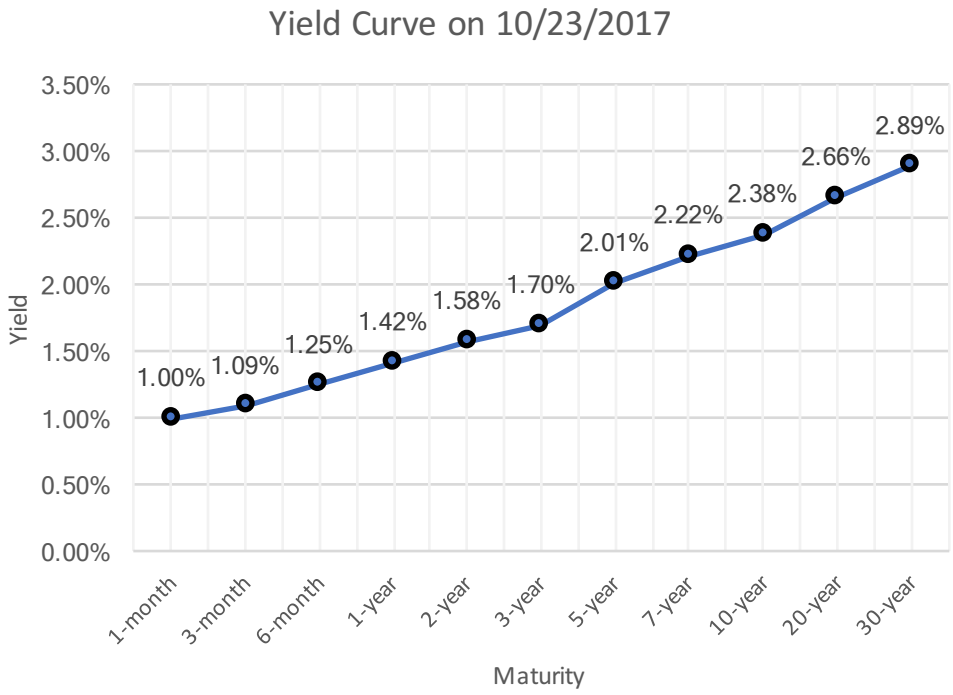
The federal reserve regularly releases selected interest rates value.[6] Pick any recent date and plot the US Treasury (constant maturities) yield curve at all available maturities between 1-month and 30-year. What does the slope of the yield curve imply about agents' expectations about the state of the economy?

Solution: See the plot below for the yield curve on 10/23/2017. It is clearly upwards sloping, so that we might conclude that the economy is expected to improve in the future.

[4] The face value of a bond is the amount it pays at maturity.

[5] *To be precise, low enough to compensate for the higher risk premium that long-term bonds must pay.*

[6] This data is available at: `http://www.federalreserve.gov/releases/h15/`.



(e)

Treasury Inflation Protected Securities are particular kinds of Government issued bonds that adjust coupons to compensate for inflation. In this respect, their yield can be thought of as being expressed in real terms. The spread between US Treasuries and TIPS is informative of inflation expectations. Suppose that people expected that inflation and the nominal rate were constant in each of the next 5 years, that is, $i_t = i$, $\pi^e_t = \pi^e$, and obviously $r_t = r$. Use the 5-year yield on the US Treasuries and the 5-year yield on the TIPS to compute the expected annual rate of inflation. Comment on how inflation expectations might change the interpretation of the slope of the yield curve.

[Hint: remember that $1 + r_t = {}^{1+i_t}/_{1+\pi^e_t}$, where r_t is the real rate paid on TIPS, i_t is the nominal rate paid on US Treasuries, and π^e_t are inflation expectations. You must solve for π^e.]

Solution: since everything is constant and the rates on the FED website are annualized, $\pi^e = {}^{1+i}/_{1+r} - 1 \approx 1.69\%$ *is the expected inflation in this case where it is believed to be constant for the next five years. In general, the profile of inflation expectations may induce a tilt in the nominal yield curve. Given that the real rate is the one that influences economic activity, this should be kept in mind when looking at the data.*

Question 3 (Chapter 15)

Using the information on the slides and in Chapter 15, mark each of the following statements as true, false, or uncertain and briefly explain why in no more than 3 lines.

(a)

For a typical college student, human wealth and nonhuman wealth are approximately equal.

Solution: FALSE

Apart from extreme cases, young people tend not to own financial or housing assets in excess of their debt, so the largest share of their wealth is "human".

(b)

Expectations of future income generally do not matter for individuals, because they don't save enough for retirement.

Solution: FALSE

Most people save enough for retirement according to Poterba Venti and Wise (JPE, 2011). However, aggregate data may conceal some individual variability and some people might be below target.

(c)

Following the financial crisis, expected future income growth fell.

Solution: TRUE

CFR Figure 15-1 on the textbook. Income growth expectations fell to nearly zero for 5 years after 2009.

(d)

Buildings and factories depreciate much faster than machines.

Solution: TRUE

CFR data from U.S. statisticians page 319, depreciation is 2.5% p.a. for office buildings, 15% p.a. for communication equipment, 55% p.a. for software.

(e)

A high value for the Tobin's q indicates that the stock market believes that capital is overvalued and thus investment should be lower.

Solution: FALSE

Investment should comove with Tobin's q. The idea is that capital replacement cost is lower than the implied market value of capital, so people should invest more.

(f)

Unless current profit affects expectations of future profit, it should have no impact on investment according to the theory.[7]

Solution: TRUE

[7] We are not concerning ourselves with what the data reveals, only the theory matters for this sentence.

Investment only depends on future expected profits and not current profits directly, because, logically, it is a forward looking variable.

(g)

Data from the past three decades in the United States suggest that corporate profits are closely tied to the business cycle.

Solution: TRUE

CFR Figure 15-4 on the textbook. Clearly, profits seem to be procyclical to a large extent. In other words, they tend to comove with GDP.

(h)

Changes in consumption and investment typically occur in the same direction and are roughly of the same magnitude.

Solution: TRUE

CFR Figure 15-5 on the textbook and the associated explanation. While they tend to comove, changes in investment are of a much larger relatve size than changes in consumption in percent terms. However, when considered in absolute terms ($Bn), their magnitudes are roughly comparable.