14.02 Principles of Macroeconomics
Problem Set 9

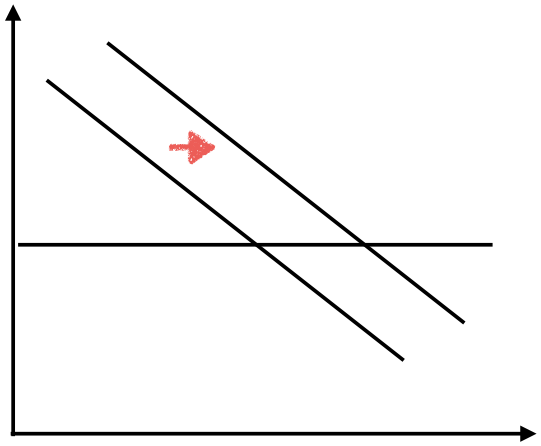
Fall 2017

General Advice in Writing Answers

I have reviewed some of your quiz and problem sets answers, and I realized that so many of you are loosing points just because you are not writing down answers appropriately. Bare in your mind the followings, which will help you getting appropriate scores in the coming quiz 3.

- When you draw figure, it is necessarily to indicate (i) what the x axis is, (ii) what the y axis is, (iii) what each line represents and (iv) what the intersections of lines are. The following Figure 1 shows the good and the bad examples that I have seen.

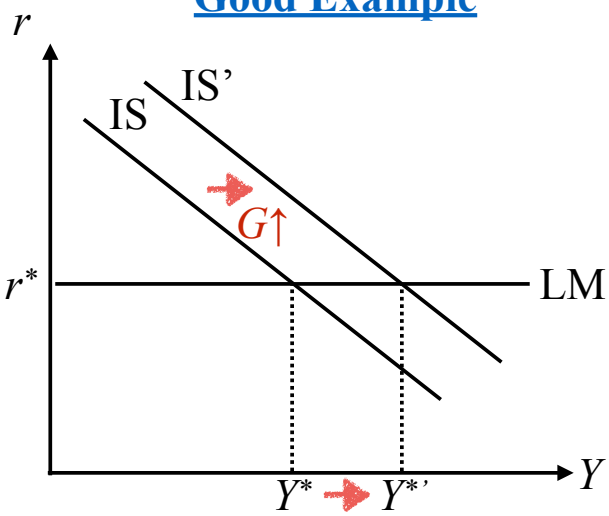
Bad Example



- What is y axis?
- What is x axis?
- What are these lines?
- What does the arrow mean?
- What is the intersection?

⇒ We have no way to give you points

Good Example



The above figure shows how the output responds in response to an increase in G. As G increases, IS curve shifts to the right, and this will increase output.

⇒ Perfect score

Figure 1: Good and Bad Examples in Drawing Figures

- You have to make it clear where your answers are. Write down all the calculations or the diagram that are not directly related to your answers on the back of answer sheets. We, graders, will be confused if you write down something unrelated to your answers. If the location of answers doesn't appear clearly, you should underline or square your answers to emphasize that they are your answers.

- You have to answer with concise and complete English sentence. For example, when you are asked "how does the investment respond?", we are not expecting you to write down "I $\downarrow$" (we, graders, will not understand this), "Investment goes down" (we are also asking you to explain the reasoning), or "Investment is affected by an increase in interest rate by monetary policy" (we are asking how it is affected). Your answers should be "Investment goes down because monetary policy raises interest rate."
- One question typically asks multiple things. For example, "How does the monetary policy respond? What is the effect on investment?". Make sure you answer to all of them.

Keep in mind that we are grading more than 200 exam sheets. We cannot spend an hour trying to understand what you intended in each of your answer sheet. It is **not** our responsibility to figure out your intention. **It is your responsibility to make your answers crystal clear to anyone who reads your answers**.

1 Question 1: Technology Growth (Chapter 12)

Suppose that the aggregate production function is

$$Y = \sqrt{K}\sqrt{AN}.$$

The savings rate is $s = 20\%$, and the rate of depreciation is $\delta = 10\%$ per year. Suppose further that the number of workers grows at $n = 2\%$ per year and that the rate of technological change is $g = 4\%$ per year.

Steady State Analysis

(a) Find the steady state values of (i) The capital stock per effective worker (ii) Output per effective worker (iii) The growth rate of output per effective worker (iv) The growth rate of output per worker (v) The growth rate of output.

(b) Re-do (a) if $g = 8\%$.

(c) Re-do (a) if $g = 4\%$ but $n = 6\%$. Are people better off in (a) or in (c)? Explain.

Transition Dynamics

Assume $g = 4\%$ and $n = 2\%$. We are now interested in the dynamics of the economy off the steady state. Recall the capital stock evolves

$$K_{t+1} = (1 - \delta)K_t + sY_t.$$

(d) Write down the evolution of capital per effective unit of worker, $k_t \equiv \frac{K_t}{A_t N_t}$. That is, express k_{t+1} as a function of k_t and other parameters.

(e) Suppose the economy is at its steady state at $t - 1$ with the saving rate 20%. However, saving rate suddenly drops to 10%. Plot the evolution of (i) output per effective worker, $\frac{Y_t}{A_t N_t}$, (ii) total output, Y_t (iii) consumption per worker, $\frac{(1-s)Y_t}{N_t}$ (iv) consumption per effective worker, $\frac{(1-s)Y_t}{A_t N_t}$, all in log-scale. (You do not need to compute the exact numbers, but make sure you have the right qualitative properties. We are asking you to draw something like Figure 12-4 in the textbook).

(f) Now suppose the economy is at its steady state at $t-1$ with the saving rate 60%, but the saving rate drops to 50%. Re-do the same thing as above.

2 Question 2: Productivity and the Aggregate Supply Curve (Chapter 13)

Consider an economy in which production is given by $Y = AN$. Assume that the price setting and wage setting are described in the following equations

$$P = (1+m)\frac{W}{A}$$
$$W = A^e P^e (1-u).$$

Recall that the relation between employment N, the labor force L and the unemployment rate u is

$$N = (1-u)L.$$

(a) Draw the aggregate supply curve. That is, plot a curve relating the price level P and aggregate output Y so that P is on the vertical axis on Y is on the horizontal axis.

(b) How does the aggregate supply curve shift when A increases to $2A$ and A^e increases to $2A^e$, but the ratio $\frac{A}{A^e} = \frac{2A}{2A^e}$ does not change?

(c) How does the aggregate supply curve shift when A increases to $2A$ but A^e does not change? Compare your results to the results in part (b). Explain the difference.

3 Question 3: Solow Model With Technology Growth (Chapter 12)

Consider the Solow growth model with technological change. Specifically, suppose that the total output produced at time t, Y_t, is governed by a Cobb-Douglas production function:

$$Y_t = F(K_t, A_t \cdot N_t) = K_t^{\alpha}(A_t \cdot N_t)^{1-\alpha},$$

where K_t is the amount of capital used, N_t is the amount of labor and A_t is the level of technology at time t. Suppose that the population grows at a constant rate, $n \geq 0$, the capital depreciates at rate $\delta \in (0,1)$, and the saving rate satisfies $s \in (0,1)$. Also assume that the technology grows at a constant rate. That is,

$$A_{t+1} = (1+g_A)A_t,$$

where $g_A \geq 0$.

(a) Denote capital per effective worker as $k_t = \frac{K_t}{A_t N_t}$. Derive the law of motion of capital per effective worker.

(b) Find the steady state level of capital per effective worker and the level of output per effective worker. Does it depend on A_0? How does it depend on g_A? What is the intuition?

(c) Suppose two countries only differ in their initial level of technology $A_0^H > A_0^L$. For both of them, technology grows at the same rate g_A. Let k^{H*} and k^{L*} denote the steady-state values of capital per effective worker in country H and L, respectively. Is output per effective worker, $y_t = \frac{Y_t}{A_t N_t}$, different across two countries?

(d) How does the ratio $\frac{Y_t^H}{Y_t^L}$ change over time?