Microgrid

From: D. Masti, T. Pippia, A. Bemporad, and B. De Schutter. "Learning Approximate Semi-Explicit Hybrid MPC with an Application to Microgrids". In: *IFAC-PapersOnLine* 53.2 (2020). 21st IFAC World Congress, pp. 5207–5212. ISSN: 2405–8963

Energy storage systems (ESSs)

$$x_{\mathrm{b}}(k+1) = \begin{cases} x_{\mathrm{b}}(k) + \dfrac{T_{\mathrm{s}}}{\eta_{\mathrm{d}}} P_{\mathrm{b}}(k) & \text{if } P_{\mathrm{b}}(k) < 0 \\ x_{\mathrm{b}}(k) + T_{\mathrm{s}} \eta_{\mathrm{c}} P_{\mathrm{b}}(k) & \text{if } P_{\mathrm{b}}(k) \geq 0 \end{cases}$$

- $x_{\mathrm{b}}(k)$ energy stored in the ESS
- η_{c} and η_{d} are the charging and discharging efficiencies
- $P_{\mathrm{b}}(k)$ power exchanged with the ESS
- T_{s} sampling interval

By introducing the continuous auxiliary variable z_b, the system can be rewritten as

$$x_{\mathrm{b}}(k+1) = x_{\mathrm{b}}(k) + T_{\mathrm{s}}(\eta_{\mathrm{c}} - \frac{1}{\eta_{\mathrm{d}}}) z_{\mathrm{b}}(k) + \frac{T_{\mathrm{s}}}{\eta_{\mathrm{d}}} P_{\mathrm{b}}(k)$$

where

- $z_{\mathrm{b}}(k) = \delta_{\mathrm{b}}(k) P_{\mathrm{b}}(k)$
- $\delta_b(k) = 1 \iff P_{\mathrm{b}}(k) \geq 0$

Constraints

$$\delta = \begin{bmatrix} \delta_{\mathrm{b}} \\ \delta_{\mathrm{grid}} \\ \delta_1^{\mathrm{on}} \\ \delta_2^{\mathrm{on}} \\ \delta_3^{\mathrm{on}} \end{bmatrix}, \qquad z = \begin{bmatrix} z_{\mathrm{b}} \\ z_{\mathrm{grid}} \end{bmatrix}, \qquad u = \begin{bmatrix} P_{\mathrm{b}} \\ P_{\mathrm{grid}} \\ P_1^{\mathrm{dis}} \\ P_2^{\mathrm{dis}} \\ P_3^{\mathrm{dis}} \end{bmatrix}, \qquad x = x_{\mathrm{b}}$$

Discrete:

$$\begin{aligned} &[\delta_{\mathrm{b}}(k) = 1] \iff [P_{\mathrm{b}}(k) \geq 0] \\ &- m\delta_{\mathrm{b}}(k) \leq P_{\mathrm{b}}(k) - m \\ &- (M + \epsilon)\delta_{\mathrm{b}}(k) \leq -P_{\mathrm{b}}(k) - \epsilon \end{aligned}$$

Continuous $z_{\mathrm{b}}(k) = \delta_{\mathrm{b}}(k) P_{\mathrm{b}}(k)$:

$$\begin{aligned} z_{\mathrm{b}}(k) &\leq M\delta_{\mathrm{b}}(k) \\ z_{\mathrm{b}}(k) &\geq m\delta_{\mathrm{b}}(k) \\ z_{\mathrm{b}}(k) &\leq P_{\mathrm{b}}(k) - m(1 - \delta_{\mathrm{b}}(k)) \\ z_{\mathrm{b}}(k) &\geq P_{\mathrm{b}}(k) - M(1 - \delta_{\mathrm{b}}(k)) \end{aligned}$$

Alternatively,

$$\begin{aligned} z_{\mathrm{b}}(k) &\leq M\delta_{\mathrm{b}}(k) \\ -z_{\mathrm{b}}(k) &\leq -m\delta_{\mathrm{b}}(k) \\ z_{\mathrm{b}}(k) &\leq P_{\mathrm{b}}(k) - m(1 - \delta_{\mathrm{b}}(k)) \\ -z_{\mathrm{b}}(k) &\leq -P_{\mathrm{b}}(k) + M(1 - \delta_{\mathrm{b}}(k)) \end{aligned}$$

where $m = -100\ kW$ and $M = 100\ kW$.

Dispatchable generators

- $P_{\text{dis}}(k) = [P_1^{\text{dis}}(k),\ \dots,\ P_{N_{\text{gen}}}^{\text{dis}}(k)]$
- $\delta_i^{\text{on}}(k)$ if dispatchable generator i is active at time k
- Constraint: $\delta_i^{\text{on}}(k)\underline{P}_i^{\text{dis}}(k) \leq P_i^{\text{dis}}(k) \leq \delta_i^{\text{on}}(k)\bar{P}_i^{\text{dis}}(k)$

Constraints

Discrete:

$$\begin{aligned}
&[\delta_i^{\text{on}}(k) = 1] \iff [P_i^{\text{dis}}(k) \geq 0] \\
&- m\delta_i^{\text{on}}(k) \leq P_i^{\text{dis}}(k) - m \\
&- (M + \epsilon)\delta_i^{\text{on}}(k) \leq -P_i^{\text{dis}}(k) - \epsilon
\end{aligned}$$

where $m = 6kW$ and $M = 150kW$.

Main grid

$$\begin{cases}
\delta_{\text{grid}}(k) = 0 & \iff P_{\text{grid}}(k) < 0 \text{ (exporting case)} \\
\delta_{\text{grid}}(k) = 1 & \iff P_{\text{grid}}(k) \geq 0 \text{ (importing case)}
\end{cases}$$

$$\begin{cases}
C_{\text{grid}}(k) = c_{\text{sell}}(k)P_{\text{grid}} & \iff P_{\text{grid}} < 0 \\
C_{\text{grid}}(k) = c_{\text{buy}}(k)P_{\text{grid}} & \iff P_{\text{grid}} \geq 0
\end{cases}$$

Constraints

Discrete:

$$\begin{aligned}
&[\delta_{\text{grid}}(k) = 1] \iff [P_{\text{grid}}(k) \geq 0] \\
&- m\delta_{\text{grid}}(k) \leq P_{\text{grid}}(k) - m \\
&- (M + \epsilon)\delta_{\text{grid}}(k) \leq -P_{\text{grid}}(k) - \epsilon
\end{aligned}$$

Continuous $z_{\text{grid}}(k) = \delta_{\text{grid}}(k)P_{\text{grid}}(k)$:

$$\begin{aligned}
z_{\text{grid}}(k) &\leq M\delta_{\text{grid}}(k) \\
-z_{\text{grid}}(k) &\leq -m\delta_{\text{grid}}(k) \\
z_{\text{grid}}(k) &\leq P_{\text{grid}}(k) - m(1 - \delta_{\text{grid}}(k)) \\
-z_{\text{grid}}(k) &\leq -P_{\text{grid}}(k) + M(1 - \delta_{\text{grid}}(k))
\end{aligned}$$

where $m = -1000\ kW$ and $M = 1000\ kW$.

Then

$$\begin{aligned}
C_{\text{grid}}(k) &= \delta_{\text{grid}}(k)c_{\text{buy}}(k)P_{\text{grid}}(k) + (1 - \delta_{\text{grid}}(k))c_{\text{sell}}(k)P_{\text{grid}}(k) \\
C_{\text{grid}}(k) &= c_{\text{buy}}(k)z_{\text{grid}}(k) - c_{\text{sell}}(k)z_{\text{grid}}(k) + c_{\text{sell}}(k)P_{\text{grid}}(k)
\end{aligned}$$

List of variables

Decision variables

$$\delta = \begin{bmatrix} \delta_{\text{b}} \\ \delta_{\text{grid}} \\ \delta_1^{\text{on}} \\ \delta_2^{\text{on}} \\ \delta_3^{\text{on}} \end{bmatrix}, \qquad z = \begin{bmatrix} z_{\text{b}} \\ z_{\text{grid}} \end{bmatrix}, \qquad u = \begin{bmatrix} P_{\text{b}} \\ P_{\text{grid}} \\ P_1^{\text{dis}} \\ P_2^{\text{dis}} \\ P_3^{\text{dis}} \end{bmatrix}, \qquad x = x_{\text{b}}$$

discrete aux	$\delta_{\mathrm{b}},\ \delta_{\mathrm{grid}}$
discrete	$\delta_1^{\mathrm{on}},\ \delta_2^{\mathrm{on}},\ \delta_3^{\mathrm{on}}$
continuous aux	$z_{\mathrm{b}},\ z_{\mathrm{grid}}$
continuous	$P_{\mathrm{b}},\ P_{\mathrm{grid}},\ P_1^{\mathrm{dis}},\ P_2^{\mathrm{dis}},\ P_3^{\mathrm{dis}}$

Table 1: Decision variables

Assumptions

Knowledge of:

- Energy prices: $c_{\mathrm{buy}}(k),\ c_{\mathrm{sell}}(k),\ c_{\mathrm{prod}}(k)$
- Power generated by renewable energy sources
- Load profile
- Not strong assumptions, since these values can be obtained by measurement and/or forecasts

Parameters

PARAMETER	VALUE
Maximum ultracapacitor energy level $\overline{x}_{\mathrm{uc}}$	50 [kWh]
Minimum ultracapacitor energy level $\underline{x}_{\mathrm{uc}}$	2 [kWh]
Maximum battery energy level $\overline{x}_{\mathrm{b}}$	250 [kWh]
Minimum battery energy level $\underline{x}_{\mathrm{b}}$	25 [kWh]
Battery charging efficiency $\eta_{\mathrm{c,b}}$	0.90
Battery discharging efficiency $\eta_{\mathrm{d,b}}$	0.90
Ultracapacitor charging efficiency $\eta_{\mathrm{c,uc}}$	0.99
Ultracapacitor discharging efficiency $\eta_{\mathrm{d,uc}}$	0.99
Maximum interconnection power flow limit $\overline{P}_{\mathrm{grid}}$	1000 [kW]
Minimum interconnection power flow limit $\underline{P}_{\mathrm{grid}}$	-1000 [kW]
Number of generators N_{gen}	3
Maximum power providable by the battery $\overline{P}_{\mathrm{b}}$	100 [kW]
Maximum power injectable to the battery $\underline{P}_{\mathrm{b}}$	-100 [kW]
Maximum power providable by the ultracapacitor $\overline{P}_{\mathrm{uc}}$	25 [kW]
Maximum power injectable to the ultracapacitor $\underline{P}_{\mathrm{uc}}$	-25 [kW]
Maximum power level of the dispatchable generators $\overline{P}_{\mathrm{dis}}$	150 [kW]
Minimum power level of the dispatchable generators $\underline{P}_{\mathrm{dis}}$	6 [kW]

Figure 1: Microgrid parameters

MLD formulas

MLD form:

$$E_2\delta_k + E_3 z_k \leq E_1 u_k + E_4 x_k + E_5$$

Discrete auxiliary variable (for ON and OFF constraints):

$$\begin{aligned}
&[\delta = 1] \iff [x \geq 0] \\
&-m\delta \leq x - m \\
&-(M+\epsilon)\delta \leq -x - \epsilon
\end{aligned}$$

Continuous auxiliary variable: $z = \delta x$

$$\begin{aligned}
&z \leq M\delta \\
&z \geq m\delta \\
&z \leq x - m(1-\delta) \\
&z \geq x - M(1-\delta)
\end{aligned}$$

Alternatively,

$$\begin{aligned} z &\leq M\delta \\ -z &\leq -m\delta \\ z &\leq x - m(1-\delta) \\ -z &\leq -x + M(1-\delta) \end{aligned}$$

MLD constraints

Constraints (MLD form: $E_2\delta_k + E_3 z_k \leq E_1 u_k + E_4 x_k + E_5$)

$\delta \in \mathbb{Z}^5,\ z \in \mathbb{R}^2,\ u \in \mathbb{R}^5,\ x \in \mathbb{R}^1$

- Initial condition

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} x_{\mathrm{b}} \leq \begin{bmatrix} x_0 \\ x_0 \end{bmatrix}$$

$$E_2 = 0_{2,5},\ E_3 = 0_{2,2},\ E_1 = 0_{2,5},\ E_4 = \begin{bmatrix} 1 \\ -1 \end{bmatrix},\ E_5 = \begin{bmatrix} x_0 \\ x_0 \end{bmatrix}$$

- State

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} [x_b] \leq \begin{bmatrix} 250 \\ -25 \end{bmatrix}$$

$$E_2 = 0_{2,5},\ E_3 = 0_{2,2},\ E_1 = 0_{2,5},\ E_4 = \begin{bmatrix} -1 \\ 1 \end{bmatrix},\ E_5 = \begin{bmatrix} 250 \\ -25 \end{bmatrix}$$

- Input

$$\begin{aligned} -100 \leq& P_b \leq 100 \\ -1000 \leq& P_{\mathrm{grid}} \leq 1000 \\ 6 \leq& P_i^{\mathrm{dis}} \leq 150 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} P_{\mathrm{b}} \\ P_{\mathrm{grid}} \\ P_1^{\mathrm{dis}} \\ P_2^{\mathrm{dis}} \\ P_3^{\mathrm{dis}} \end{bmatrix} \leq \begin{bmatrix} 100 \\ 100 \\ 1000 \\ 1000 \\ 150 \\ -6 \\ 150 \\ -6 \\ 150 \\ -6 \end{bmatrix}$$

$$E_2 = 0_{10,5},\ E_3 = 0_{10,2},\ E_1 = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},\ E_4 = 0_{10,1},\ E_5 = \begin{bmatrix} 100 \\ 100 \\ 1000 \\ 1000 \\ 150 \\ -6 \\ 150 \\ -6 \\ 150 \\ -6 \end{bmatrix}$$

- Continuous auxiliary variables

  $z_{\text{b}}(k) = \delta_{\text{b}}(k)P_{\text{b}}(k)$

$$\begin{aligned} z_{\text{b}}(k) &\leq M\delta_{\text{b}}(k) \\ -z_{\text{b}}(k) &\leq -m\delta_{\text{b}}(k) \\ z_{\text{b}}(k) &\leq P_{\text{b}}(k) - m(1-\delta_{\text{b}}(k)) \\ -z_{\text{b}}(k) &\leq -P_{\text{b}}(k) + M(1-\delta_{\text{b}}(k)) \end{aligned}$$

  where $m = -100\ kW$ and $M = 100\ kW$.

$$\begin{bmatrix} -M & 0 & 0 & 0 & 0 \\ m & 0 & 0 & 0 & 0 \\ -m & 0 & 0 & 0 & 0 \\ M & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{\text{b}} \\ \delta_{\text{grid}} \\ \delta_1^{\text{on}} \\ \delta_2^{\text{on}} \\ \delta_3^{\text{on}} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 1 & 0 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} z_{\text{b}} \\ z_{\text{grid}} \end{bmatrix} \leq \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_{\text{b}} \\ P_{\text{grid}} \\ P_1^{\text{dis}} \\ P_2^{\text{dis}} \\ P_3^{\text{dis}} \end{bmatrix} + 0_{4,1}x_{\text{b}} + \begin{bmatrix} 0 \\ 0 \\ -m \\ M \end{bmatrix}$$

  $z_{\text{grid}}(k) = \delta_{\text{grid}}(k)P_{\text{grid}}(k)$

$$\begin{aligned} z_{\text{grid}}(k) &\leq M\delta_{\text{grid}}(k) \\ -z_{\text{grid}}(k) &\leq -m\delta_{\text{grid}}(k) \\ z_{\text{grid}}(k) &\leq P_{\text{grid}}(k) - m(1-\delta_{\text{grid}}(k)) \\ -z_{\text{grid}}(k) &\leq -P_{\text{grid}}(k) + M(1-\delta_{\text{grid}}(k)) \end{aligned}$$

  where $m = -1000\ kW$ and $M = 1000\ kW$.

$$\begin{bmatrix} 0 & -M & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & -m & 0 & 0 & 0 \\ 0 & M & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{\text{b}} \\ \delta_{\text{grid}} \\ \delta_1^{\text{on}} \\ \delta_2^{\text{on}} \\ \delta_3^{\text{on}} \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & -1 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} z_{\text{b}} \\ z_{\text{grid}} \end{bmatrix} \leq \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_{\text{b}} \\ P_{\text{grid}} \\ P_1^{\text{dis}} \\ P_2^{\text{dis}} \\ P_3^{\text{dis}} \end{bmatrix} + 0_{4,1}x_{\text{b}} + \begin{bmatrix} 0 \\ 0 \\ -m \\ M \end{bmatrix}$$

- Discrete variables

  ESS:

$$\begin{aligned} &[\delta_{\text{b}}(k) = 1] \iff [P_{\text{b}}(k) \geq 0] \\ &-m\delta_{\text{b}}(k) \leq P_{\text{b}}(k) - m \\ &-(M+\epsilon)\delta_{\text{b}}(k) \leq -P_{\text{b}}(k) - \epsilon \end{aligned}$$

$$\begin{bmatrix} -m & 0 & 0 & 0 & 0 \\ -(M+\epsilon) & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{\text{b}} \\ \delta_{\text{grid}} \\ \delta_1^{\text{on}} \\ \delta_2^{\text{on}} \\ \delta_3^{\text{on}} \end{bmatrix} + 0_{2,2}z \leq \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_{\text{b}} \\ P_{\text{grid}} \\ P_1^{\text{dis}} \\ P_2^{\text{dis}} \\ P_3^{\text{dis}} \end{bmatrix} + 0_{2,1}x_{\text{b}} + \begin{bmatrix} -m \\ -\epsilon \end{bmatrix}$$

  where $m = -100\ kW$ and $M = 100\ kW$.

  Grid:

$$\begin{aligned} &[\delta_{\text{grid}}(k) = 1] \iff [P_{\text{grid}}(k) \geq 0] \\ &-m\delta_{\text{grid}}(k) \leq P_{\text{grid}}(k) - m \\ &-(M+\epsilon)\delta_{\text{grid}}(k) \leq -P_{\text{grid}}(k) - \epsilon \end{aligned}$$

  where $m = -1000\ kW$ and $M = 1000\ kW$.

$$\begin{bmatrix} 0 & -m & 0 & 0 & 0 \\ 0 & -(M+\epsilon) & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{\text{b}} \\ \delta_{\text{grid}} \\ \delta_1^{\text{on}} \\ \delta_2^{\text{on}} \\ \delta_3^{\text{on}} \end{bmatrix} + 0_{2,2} z \leq \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_{\text{b}} \\ P_{\text{grid}} \\ P_1^{\text{dis}} \\ P_2^{\text{dis}} \\ P_3^{\text{dis}} \end{bmatrix} + 0_{2,1} x_{\text{b}} + \begin{bmatrix} -m \\ -\epsilon \end{bmatrix}$$

Generators:

$$\begin{aligned} &[\delta_i^{\text{on}}(k) = 1] \iff [P_i^{\text{dis}}(k) \geq 0] \\ &-m\delta_i^{\text{on}}(k) \leq P_i^{\text{dis}}(k) - m \\ &-(M+\epsilon)\delta_i^{\text{on}}(k) \leq -P_i^{\text{dis}}(k) - \epsilon \end{aligned}$$

where $m = 6kW$ and $M = 150kW$.

1:

$$\begin{bmatrix} 0 & 0 & -m & 0 & 0 \\ 0 & 0 & -(M+\epsilon) & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{\text{b}} \\ \delta_{\text{grid}} \\ \delta_1^{\text{on}} \\ \delta_2^{\text{on}} \\ \delta_3^{\text{on}} \end{bmatrix} + 0_{2,2} z \leq \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_{\text{b}} \\ P_{\text{grid}} \\ P_1^{\text{dis}} \\ P_2^{\text{dis}} \\ P_3^{\text{dis}} \end{bmatrix} + 0_{2,1} x_{\text{b}} + \begin{bmatrix} -m \\ -\epsilon \end{bmatrix}$$

2:

$$\begin{bmatrix} 0 & 0 & 0 & -m & 0 \\ 0 & 0 & 0 & -(M+\epsilon) & 0 \end{bmatrix} \begin{bmatrix} \delta_{\text{b}} \\ \delta_{\text{grid}} \\ \delta_1^{\text{on}} \\ \delta_2^{\text{on}} \\ \delta_3^{\text{on}} \end{bmatrix} + 0_{2,2} z \leq \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} P_{\text{b}} \\ P_{\text{grid}} \\ P_1^{\text{dis}} \\ P_2^{\text{dis}} \\ P_3^{\text{dis}} \end{bmatrix} + 0_{2,1} x_{\text{b}} + \begin{bmatrix} -m \\ -\epsilon \end{bmatrix}$$

3:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & -m \\ 0 & 0 & 0 & 0 & -(M+\epsilon) \end{bmatrix} \begin{bmatrix} \delta_{\text{b}} \\ \delta_{\text{grid}} \\ \delta_1^{\text{on}} \\ \delta_2^{\text{on}} \\ \delta_3^{\text{on}} \end{bmatrix} + 0_{2,2} z \leq \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} P_{\text{b}} \\ P_{\text{grid}} \\ P_1^{\text{dis}} \\ P_2^{\text{dis}} \\ P_3^{\text{dis}} \end{bmatrix} + 0_{2,1} x_{\text{b}} + \begin{bmatrix} -m \\ -\epsilon \end{bmatrix}$$

- Power balance: $P_b(k) - P_1^{\text{dis}}(k) - P_2^{\text{dis}}(k) - P_3^{\text{dis}}(k) - P_{\text{res}}(k) - P_{\text{grid}}(k) + P_{\text{load}}(k) = 0$

$$\begin{bmatrix} 1 & -1 & -1 & -1 & -1 \\ -1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_{\text{b}} \\ P_{\text{grid}} \\ P_1^{\text{dis}} \\ P_2^{\text{dis}} \\ P_3^{\text{dis}} \end{bmatrix} + \begin{bmatrix} P_{\text{load}} - P_{\text{res}} \\ P_{\text{res}} - P_{\text{load}} \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$E_2 = 0_{2,5},\ E_3 = 0_{2,2},\ E_1 = \begin{bmatrix} -1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 & -1 \end{bmatrix},\ E_4 = 0_{2,1},\ E_5 = \begin{bmatrix} P_{\text{res}} - P_{\text{load}} \\ P_{\text{load}} - P_{\text{res}} \end{bmatrix}$

- Generator constraints:

Constraints (MLD form: $E_2\delta_k + E_3 z_k \leq E_1 u_k + E_4 x_k + E_5$)

$$\delta = \begin{bmatrix} \delta_{\text{b}} \\ \delta_{\text{grid}} \\ \delta_1^{\text{on}} \\ \delta_2^{\text{on}} \\ \delta_3^{\text{on}} \end{bmatrix}, \qquad z = \begin{bmatrix} z_{\text{b}} \\ z_{\text{grid}} \end{bmatrix}, \qquad u = \begin{bmatrix} P_{\text{b}} \\ P_{\text{grid}} \\ P_1^{\text{dis}} \\ P_2^{\text{dis}} \\ P_3^{\text{dis}} \end{bmatrix}, \qquad x = x_{\text{b}}$$

$$\delta_i^{\text{on}}(k)\underline{P}_i^{\text{dis}}(k) \leq P_i^{\text{dis}}(k) \leq \delta_i^{\text{on}}(k)\bar{P}_i^{\text{dis}}(k)$$

$$
6 \cdot \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_{\mathrm{b}} \\ \delta_{\mathrm{grid}} \\ \delta_1^{\mathrm{on}} \\ \delta_2^{\mathrm{on}} \\ \delta_3^{\mathrm{on}} \end{bmatrix} + 0_{3,2} z \leq \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_{\mathrm{b}} \\ P_{\mathrm{grid}} \\ P_1^{\mathrm{dis}} \\ P_2^{\mathrm{dis}} \\ P_3^{\mathrm{dis}} \end{bmatrix} + 0_{3,1} x_{\mathrm{b}} + 0_{3,1}
$$

$$
-150 \cdot \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_{\mathrm{b}} \\ \delta_{\mathrm{grid}} \\ \delta_1^{\mathrm{on}} \\ \delta_2^{\mathrm{on}} \\ \delta_3^{\mathrm{on}} \end{bmatrix} + 0_{3,2} z \leq - \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_{\mathrm{b}} \\ P_{\mathrm{grid}} \\ P_1^{\mathrm{dis}} \\ P_2^{\mathrm{dis}} \\ P_3^{\mathrm{dis}} \end{bmatrix} + 0_{3,1} x_{\mathrm{b}} + 0_{3,1}
$$

Misc

$$
\begin{aligned}
e(x) &= (y(x) - \hat{y}(x))^2 \\
\frac{de(x)}{d\theta} &= -2(y(x) - \hat{y}(x)) \frac{d\hat{y}(x)}{d\theta} = -2c(y(x) - \hat{y}(x)) \sigma'(Wx+b) \frac{d(Wx+b)}{d\theta} \\
\frac{de}{dW} &= -2c(y(x) - \hat{y}(x)) \sigma'(Wx+b) x \\
\frac{de}{db} &= -2c(y(x) - \hat{y}(x)) \sigma'(Wx+b) \\
\frac{de}{dc} &= -2cy(x) - \hat{y}(x)) \sigma'(Wx+b)
\end{aligned}
$$

where $\hat{y}(x) = c\sigma(Wx+b)$ and $\sigma(x)$ is the sigmoid function.

$$
\begin{aligned}
\frac{d\sigma(x)}{dx} &= \sigma(x)(1 - \sigma(x)) \\
\frac{d\sigma(Wx+b)}{dx} &= W\sigma(Wx+b)(1 - \sigma(Wx+b))
\end{aligned}
$$

Generators:

$$
\begin{aligned}
&[\delta_i^{\mathrm{on}}(k) = 1] \iff [P_i^{\mathrm{dis}}(k) \geq 0] \\
&- m\delta_i^{\mathrm{on}}(k) \leq P_i^{\mathrm{dis}}(k) - m \\
&- (M + \epsilon)\delta_i^{\mathrm{on}}(k) \leq -P_i^{\mathrm{dis}}(k) - \epsilon
\end{aligned}
$$

where $m = 6kW$ and $M = 150kW$.

$$
\delta_i^{\mathrm{on}} = 1 \implies \begin{cases} -m \leq P_i^{\mathrm{dis}} - m & \implies P_i^{\mathrm{dis}} \geq 0 \\ -(M+\epsilon) \leq -P_i^{\mathrm{dis}} - \epsilon & \implies P_i^{\mathrm{dis}} \leq M \end{cases}
$$

$$
\delta_i^{\mathrm{on}} = 0 \implies \begin{cases} & \implies P_i^{\mathrm{dis}} \geq m \\ \epsilon \leq -P_i^{\mathrm{dis}} & \implies P_i^{\mathrm{dis}} \leq -\epsilon \end{cases}
$$

From Bemporad99 paper:

$$
f(x) \leq 0 \iff \delta = 1 \text{ is true iff } \begin{cases} f(x) \leq M(1-\delta) \\ f(x) \geq \epsilon + (m - \epsilon)\delta \end{cases}
$$

$$
\delta = 1 \implies \begin{cases} f(x) \leq 0 \\ f(x) \geq m \end{cases}
$$

$$\delta = 0 \implies \begin{cases} f(x) \leq M \\ f(x) \geq \epsilon \end{cases}$$

Changing to $-f(x)$

$$-f(x) \leq 0 \iff \delta = 1 \text{ is true iff } \begin{cases} -f(x) \leq M(1-\delta) \\ -f(x) \geq \epsilon + (m-\epsilon)\delta \end{cases}$$

$$f(x) \geq 0 \iff \delta = 1 \text{ is true iff } \begin{cases} f(x) \geq -M(1-\delta) \\ f(x) \leq -\epsilon - (m-\epsilon)\delta \end{cases}$$

$$\delta = 1 \implies \begin{cases} f(x) \geq 0 \\ f(x) \leq -m \end{cases}$$

$$\delta = 0 \implies \begin{cases} f(x) \geq -M \\ f(x) \leq -\epsilon \end{cases}$$