Natural Gas Storage Valuation

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

SUMMARY

In this thesis, one methodology for natural gas storage valuation is developed and two methodologies are improved. Then all of the three methodologies are applied to a storage contract.

The first methodology is called “intrinsic rolling with spot and forward”, which takes both the spot and forward prices into account in the valuation. This method is based on the trading strategy by which a trader locks the spot and forward positions by solving an optimization problem based on the market information on the first day. In the following days, the trader can obtain added value by adjusting the positions based on new market information. The storage value is the sum of the first day’s value and the added values in the following days.

The problem can be expressed by a Bellman equation and solved recursively. A crucial issue in the implementation is how to compute the expected value in the next period conditioned on the information in current period. One way to compute the expected value is Monte Carlo simulation with ordinary least square regression. However, if all of the state variables, spot, and forward prices are incorporated in the regression there are too many terms, and the regression becomes uncontrollable. To solve this issue, three risk factors are chosen by performing principle component

analysis. Dimension of the regression is greatly reduced by only incorporating the three risk factors.

Both the second methodology and the third methodology only consider the spot price in the valuation. The second methodology uses Monte Carlo simulation with ordinary least square regression, which is based on the work of Boogert and Jong (2006). The third methodology uses stochastic dual dynamic programming, which is based on the work of Bringedal (2003). However, both methodologies are improved to incorporate bid and ask prices.

Price models are crucial for the valuation. Forward prices of each month are assumed to follow geometric Brownian motions. Future spot price is also assumed to follow a geometric Brownian motion but for a specific month its expectation is set to the corresponding forward price on the valuation date. Since the simulation of spot and forward prices is separated from the storage optimization, alternative spot and forward models can be used when necessary.

The results show that the value of the storage contract estimated by the first methodology is close to the market value and the value estimated by the Financial Engineering Associates (FEA) provided function. A much higher value is obtained when only spot price is considered, since the high volatility of the spot curve makes frequent position change profitable. However in the reality traders adjust their positions less frequently.

CHAPTER 1

INTRODUCTION

Natural gas storage valuation is a complicated topic in the field of asset and derivative valuation. One can think of natural gas storage as a dynamic basket of calendar spreads, including not only the spreads among forwards, but also the spreads between spot and forwards. On the one hand, the operation of natural gas storage is subject to many constraints, which make the valuation of the storage more complicated than a pure financial instrument. On the other hand, it’s difficult to simulate the spot and forward curves of natural gas, especially when one wants to take both spot and forwards into account at the same time.

Therefore, only the spot price or forward prices instead of both are used in most of the literature on natural gas storage valuation. There is more literature using the spot price only, such as Boogert and Jong (2006), Chen and Forsyth (2006), Jong and Walet (2004), Thompson, Davison, and Rasmussen (2003), Bringedal (2003), and Weston (2002). Although it is challenging to develop a model that can capture the short and long-term dynamics (such as mean-reverting and jumps) of natural gas spot price, the spot price method has some advantages. There are very limited state variables in the stochastic control problem, usually only the spot price and storage level, therefore it is relatively easy to find a decision rule (inject or withdraw) and estimate the value of a storage. In comparison to forward prices, spot prices are more volatile, therefore some authors, including Boogert and Jong (2006), argue that the value priced by spot prices is the true value, which is usually greater than the value priced by forward prices.

Some authors use forward prices in natural gas storage valuation, such as Eydeland and Wolyniec (2002), Gray and Khandelwal (2004), and Blanco, Soronow and Stefiszyn (2002). An interesting point to note is all of the authors are working or have experience in the energy trading industry. All of them apply the "intrinsic rolling" strategy, where a trader locks the positions of natural gas forward contracts and achieves the intrinsic value of the storage given the forward curve and constraints on the first day. During the following days, the trader can adjust the positions based on new forward curve and constraints to obtain more values. This method seems more realistic, since calendar spreads are frequently traded when storage is traded. Another advantage of forward price method is that the monthly forward price curve is relatively easy to simulate. However, it usually requires more computer resources to conduct the optimization in every stage. For example, even if we only consider 12 forward contracts, there are 66 spreads we need to consider in the optimization. If we consider 24 contracts, then the number of spreads increases to 276.

The reasons that most authors use spot only or forwards only are not only because the stochastic control problem becomes more complicated when both are considered, but also due to the fact that it is difficult to make the spot price and forward prices consistent. In other words, it is a challenge to develop spot and forward models and satisfy the no-arbitrage condition. Theoretically, one can develop spot models and calibrate them by the forward prices to make the spot price consistent with the forward prices. Chen and Forsyth (2006) calibrated 3 different natural gas spot models by the forward prices. However, the stability of the parameters is unclear since the significant levels are not provided in their paper. Even for the best model (the regime-switch GBM model), its ability of predicting forwards of the model need further verification, since only one observation of the market prices is compared with the predicted prices. Another problem in calibrating spot models by forwards is that usually different results for the parameters are obtained when the spot model is calibrated by the spot prices.

Since a storage trader can trade both forwards and spot at the same time in reality, we will compare the valuation results by various methods. We combine the forwards and spot in a simple but practical way in the storage valuation in this paper.

This thesis is organized as follows: Chapter 2 provides an introduction on the basics of natural gas storage market in the US. Chapter 3 describes how to connect the spot price and forward prices and their simulation. Chapter 4 introduces the methodologies for storage valuation. Chapter 5 describes the data and parameter estimations and the results. Chapter 6 concludes.

CHAPTER 2

THE BASICS OF THE NATURAL GAS MARKET IN THE US

Natural gas market in the US has its own characteristics. Natural gas prices are location based prices. Henry Hub price is the benchmark and almost all of the prices are derived by adding adders to the Henry Hub price. Henry Hub is called the "backbone" and the adders are called the "bases". The spot market is traded every business day and settled on the next business day and non-business days if there are any before the next business day. For instance, natural gas traded on July 26, 2007 (Thursday) is settled on July 27, 2007 (Friday), and natural gas traded on July 27, 2007 is settled on July 28 (Saturday), July 29 (Sunday), and July 30, 2007 (Monday). The trading date is also called the transaction date, and the settlement date(s) is (are) called the flow date(s).

The forward contracts are widely traded on the New York Mercantile Exchange (NYMEX) and some other on-line markets, such as ICE. NYMEX contracts are based on Henry Hub price. The prices of 72 forward contracts are available on every business day, but there are only about 24 or less frequently traded contracts. The trading volume of natural gas forwards has a seasonal pattern. Most frequently traded contracts are prompt month (the nearest month from spot) and a few following months, October, January, March, and April. Another feature of forward contracts is that they expire on the third from the last business day of the previous month. For example, 2007 August contract expires on July 27, 2007 and 2007 September contract expires on August 29, 2007. Usually, a trader closes the positions before they expire, but a storage trader takes the physical natural gas. Natural gas forward contracts are settled in a special way in that a buyer receives the physical gas at a specific location in an equal amount on each day of the contract month. For example, 2007 August contract expires on July 27, 2007, but a buyer does not receive the gas in any day in July, instead the buyer receives gas every day in August, 2007. If a trader buys a contract, then the trader receives natural gas 1/31 contract every day in August. If the trader buys 2 contracts, then he or she receives gas 2/31 contract every day. A fraction of a contract, such as a quarter, half, or three quarters of a contract can be traded. A natural gas contract has an energy value of 10,000 MMBtu.

There are various kinds of storages, such as depleted oil reservoirs, aquifers, salt caverns, and LNG storages. These storages have different physical characteristics. For more detailed information, one can refer to the Energy Information Agency (EIA) website of the US government: www.eia.doe.gov (November 2, 2007).

A natural gas storage contract can have a term from a few months to a few years. The contract also covers some physical constraints and operational costs, such as initial working gas capacity, maximum working gas capacity, maximum daily injection and withdrawal rates, unit injection and withdrawal costs. Usually, a storage contract is also based on the natural gas price at a specific location, such as Henry Hub, Houston

Ship Channel etc. Finally, like all other financial contracts, one of the most important parameters is the premium of the contract. This is the target of our exercise.

Here is a typical natural gas storage contract:

- Term: 2/1/2008-6/30/2012
- Basis: Henry Hub
- Premium: $X/MMBtu-month
- Maximum working gas capacity: 1,000,000 MMBtu ~ 1 billion cubic feet
- Initial working gas: 0 MMBtu
- Maximum injection rate: 35,000 MMBtu/day
- Maximum withdrawal rate: 75,000 MMBtu/day
- Operating costs: 1.5% of the fuel cost on injection

As the contract indicated, it takes almost 2 weeks to withdraw and almost a month to inject the full amount of the storage. So even though one can lock the positions based on a forward curve in one period, usually it can't be implemented in one period, this is different from a pure financial instrument. Like other financial instruments, a trader has to pay the ask price when he or she wants to inject gas while the trader receives the bid price when he or she withdraws the gas. A storage trader can trade natural gas only on business days, but physical injection and withdrawal can happen on every day, including the weekends and holidays.

CHAPTER 3

SIMULATE FORWARD AND SPOT PRICES

If we simulate forward prices of various months by a single model, then many factors, such as mean-reverting and jumps, should be taken into account. For example, the magnitudes and volatilities of March and April contracts can be very different. In this thesis, we simulate each forward contract by a specific model. In other words, we treat the forward contracts for different months as different commodities. For example, we use one model to simulate the March contract and use another model for April's contract. Since each month has its own forward curve, mean-reverting and jumps can be ignored. In other words, we expect there is a significant drop from March prices to April prices, but we don't expect a spike or drop within March prices or April prices under the normal market conditions. Thus, relatively simple models can be created for each forward contract. This is one reason that we use different models for different contracts. Another reason is that the price for each forward contract is available in the market. In this exercise, we assume that all of the forward contracts follow a geometric Brownian motion process:

$$dF_{t,T_i} = F_{t,T_i}\sigma_i dW_{t,i} \tag{3.1}$$

Where i = 1, …, n (1 for January, …, 12 for December, 13 for next January contract, …),

T_i is the expiration date of the i^{th} contract,

F_{t,T_i} is the price of the i^{th} month forward contract at time t,

σ_i is the volatility of the i^{th} contract, and

$W_{t,i}$ is the Brownian motion associated with the i^{th} contract.

By Ito's lemma, we have

$$
\begin{aligned}
d\ln F_{t,T_i} &= \frac{1}{F_{t,T_i}} dF_{t,T_i} - \frac{1}{2(F_{t,T_i})^2}(dF_{t,T_i})^2 \\
&= \sigma_i dW_{t,i} - \frac{1}{2}\sigma_i^2 dt
\end{aligned}
$$

Therefore, we have

$$\ln F_{t,T_i} - \ln F_{0,T_i} = -\frac{1}{2}\sigma_i^2 t + \sigma_i W_{t,i}\text{, or}$$

$$F_{t,T_i} = F_{0,T_i} \exp(-\frac{1}{2}\sigma_i^2 t + \sigma_i W_{t,i})$$

Where F_{0,T_i} is the observed forward price of the i^{th} contract on the valuation date.

Since $W_{t,i} \sim N(0,t)$, we can rewrite the above equation as

$$F_{t,T_i} = F_{0,T_i} \exp(-\frac{1}{2}\sigma_i^2 t + \sigma_i \sqrt{t}\varepsilon) \qquad \varepsilon \sim N(0,1) \qquad (3.2)$$

As we mentioned earlier, theoretically one should be able to develop a spot price model so that its expected value equals the forward price for any term contract (from prompt month to the 72nd month) under a risk-neutral probability measure. However, it is not practicable in reality, since the natural gas market is very different from a pure financial market. First, the settlement of natural gas market is very special, i.e., a buyer does not receive the gas when a contract expires and does not receive the gas on the same day. Second, the demand of the natural gas market indicates a seasonal pattern that is very difficult to be captured by a mathematical model. Finally, the market becomes less liquid as the expiration of a contract increases. In other words, the market is not complete and we may not find a risk-neutral probability measure for spot and all of the forward contracts in the natural gas market, especially for the long-term forward contracts.

Therefore, we only connect spot price to the forward prices in the same month in this thesis. We assume that spot price also follows a piece-wise geometric Brownian motion. Specifically, since the spot price is available in the market for the valuation date, during the same month, we assume that the spot price follows a geometric Brownian motion where expected value equals the spot price from the market. During the following months, we set the expected value same as the forward prices that are available from the market on the valuation date. Following the assumptions, we have

$$S_t = \begin{cases} S_0 \exp(-\frac{1}{2}\sigma_s^2 t + \sigma_s \sqrt{t}\varepsilon) & \textit{for the valuation month} \quad (a) \\ F_{0,i} \exp(-\frac{1}{2}\sigma_s^2 t + \sigma_s \sqrt{t}\varepsilon) & \textit{for the } i^{th} \textit{ month contract} \quad (b) \end{cases} \qquad \varepsilon \sim N(0,1) \qquad (3.3)$$

Where S_t is the spot price at t,

S_0 is the spot price on the valuation date,

$F_{o,i}$ is the forward price for the i^{th} month contract on the valuation date, and

σ_s is the volatility of spot price.

Equation (3.2) and (3.3) look very similar, but there are differences. In equation (3.2), the forward volatility σ_i changes from month to month while the spot volatility σ_s in equation (3.3) is constant over the term of a storage contract. This is consistent with what we observed from the natural gas market. Namely, spot prices always have high volatilities, but the volatility of a forward contract decreases as the maturity increases. Note that even the same contract month has different volatilities if it is in different years. We can expect that the volatility for January 2008 is greater than the volatility of January 2009 when we are on a date before January 2008.

If the term of a storage contract is very long and only some of the forward prices are available from the market on the valuation date, then the forward prices for the remaining months are derived from the forward prices of the same months in the previous year assuming the growth rate is the same as the risk-free interest rate. For example, a storage contract has a term from September 1st, 2007 to August 31st, 2012 and the valuation date is August 1st, 2007. On the valuation date, assume that all of the prices we can obtain from the market are spot on August 1st, 2007 and the forward prices from September 2007 to August 2010, then the forward price of September 2010 equals the product of the forward price of September 2009 and (1 + risk-free interest rate), and the forward price of October 2010 equals the product of the forward price of October 2009 and (1 + risk-free interest rate). After we derive all of the forward prices, we can simulate forward and spot prices by equation (3.2) and (3.3) respectively.

Correlation is also very important for the simulation, since we can expect that the forward prices are highly correlated and also correlated to the spot price. Specifically, when we simulate the prices of different forward contracts and the spot price, we need to draw correlated samples for $dW_{t,i}$. Both volatilities and correlations can be calibrated by historic forward prices.

CHAPTER 4

VALUATION METHODOLOGIES

4.1 Problem Description

The valuation of natural gas storage is a stochastic control problem. The value of a storage contract is the maximum value of the sum of the discounted cash flows during the term of the contract, which can be expressed by the following equation:

$$V_0 = \max(\sum_{t=1}^{T} \beta_t E[\pi_t(v_t, P_t; \Delta v_t)] \qquad (4.1)$$

$$s.t.$$
$$v_{t+1} = v_t + \Delta v_t$$
$$0 \le v_t \le \overline{v}$$
$$-\Delta \underline{v} \le \Delta v_t \le \Delta \overline{v}$$

Where V_0 is the value of the storage at time t=0,

β_t is the discount factor at t,

v_t is the storage level at t,

P_t is the natural gas prices at t, which can be spot price, or the vector of forward prices, or both,

Δv_t is the injection or withdrawal amount during the period t,

π_t is the profit or loss at t,

E is the expectation operator,

$\bar{v}$ is the maximum working gas capacity,

$\Delta\underline{v}$ is the maximum withdrawal rate, and

$\Delta\bar{v}$ is the maximum injection rate.

Usually, this problem can be expressed by the Bellman equation:

$$V_t(v_t, P_t; \Delta v_t) = \max(\pi_t(v_t, P_t; \Delta v_t) + \beta E[V_{t+1}(v_{t+1}, P_{t+1}; \Delta v_{t+1}) \mid F_t] \qquad (4.2)$$

Where V_t is the storage value at t,

V_{t+1} is the storage value at t+1, also called the continuation value of the storage at t, and

F_t is the information filter at t.

Since there are too many state variables in the problem, including the storage level, spot price, and forward prices, it is very difficult to solve the problem by tree models or by solving a finite difference equation. Actually, the piece wise connection between spot and forwards prevents us from using the methodology of solving a finite difference equation. Therefore, we choose the Monte Carlo methodology to solve the problem. Next, we introduce the two Monte Carlo methodologies that can be applied in the natural gas storage valuation problem.

4.2 Monte Carlo with Stochastic Dual Dynamic Programming

Before introducing the stochastic dual dynamic programming, we give a review of the traditional stochastic dynamic programming by showing how it can be applied to the storage valuation problem. The procedure is as follows:

Initialize the continuation value at T: $V_{T+1} \leftarrow 0$ for all of the scenarios

For t = T, …, 1

For each storage level $v_t = (v_t^m, m = 1, ..., M)$

For each price scenario $P_t = (P_t^k, k = 1, ..., K)$, solve the one-stage problem

$$
\begin{aligned}
& V_t^k(v_t^m, P_t; \Delta v_t^k) = \max(\pi_t(v_t^m, P_t; \Delta v_t^k) + \beta E[V_{t+1}(v_{t+1}, P_{t+1}; \Delta v_{t+1}) \mid F_t] \\
& s.t. \\
& \quad v_{t+1} = v_t^m + \Delta v_t^k \\
& \quad 0 \le v_t \le \bar{v} \\
& \quad -\Delta \underline{v} \le \Delta v_t \le \Delta \bar{v}
\end{aligned}
$$

Next

$V_t(v_t^m)$ *is the average of* V_t^k *or the probability weighted* V_t^k

Next

Create a complete $V_t\ (v_t)$ curve for the previous stage by interpolating over the different storage levels

Next

The traditional methodology is resource consuming since it requires the optimization on each storage level and interpolation. To improve the traditional methodology, Pereira, Campodonico, and Kelman (1999) developed stochastic dual dynamic programming and applied it to the hydrothermal scheduling problem. In this methodology, it is assumed that the continuation value is linear for a specific storage level: $V_{t+1} = \varphi_{t+1}^{n} v_{t+1} + \delta_{t+1}^{n} \quad n = 1, ..., N$ Therefore, if a storage capacity is broken into N parts, then the valuation is subject to N linear constraints in each stage. The slope coefficients ($\varphi_{t+1}^{n} \quad n = 1, ..., N$) are the simplex multipliers that are associated with the constraints of storage level change. In other words, the slope of a storage level is the shadow price of the storage level, i.e, the increased value of the storage given there is one more unit gas in the storage. The method can be applied to the storage valuation problem by the following procedure:

Initialize the continuation value at T: $V_{T+1} \leftarrow 0$ (or set $\varphi_{T+1}^{n} = 0, \ \delta_{T+1}^{n} = 0$)

Set the number of segments N = the number of storage levels M

For t = T, …, 1

For each storage level $v_t = (v_t^m, m = 1, ..., M)$

For each price scenario $P_t = (P_t^k, k = 1, ..., K)$, solve the one-stage problem

$$
\begin{aligned}
& V_t^k(v_t^m, P_t^k; \Delta v_t^k) = \max(\pi_t(v_t^m, P_t^k; \Delta v_t^k) + \beta E[V_{t+1}(v_{t+1}, P_{t+1}; \Delta v_{t+1}) \mid F_t] \\
& s.t. \\
& \quad v_{t+1} = v_t^m + \Delta v_t^k \quad simplex\ multiplier\ \ \lambda_t^k \\
& \quad 0 \le v_{t+1} \le \bar{v} \\
& \quad -\Delta \underline{v} \le \Delta v_t \le \Delta \bar{v} \\
& \quad V_{t+1} \le \varphi_{t+1}^{n} v_{t+1} + \delta_{t+1}^{n} \quad n = 1, ..., N
\end{aligned} \tag{4.3}
$$

Next

$$\varphi_t^n = \sum p_t^k \lambda_t^k \qquad \delta_t^n = \sum p_t^k V_t^k - \varphi_t^n v_t^n$$

Next

Next

Bringedal (2003) applied this methodology to the valuation of natural gas storage, but only the spot price is considered and did not take into account the difference of bid and ask prices. The simplex multiplier can be approximated by the increment in the value of the storage when there is a small increase in the storage level.

4.3 Monte Carlo with Ordinary Least Square

Another methodology is the ordinary least square Monte Carlo simulation, which is developed by Longstaff and Schwartz (2001). One of the key issues in using Monte Carlo methodology for derivative and asset valuation is how to compute the expected continuation value at time t that is conditioned on the information at time t. Longstaff and Schwartz (2001) used a linear combination of basis functions to approximate the continuation value at t. The bases are usually the state variables and known at time t. There are a lot of choices for the basis functions, which can be Laguerre polynomials, Hermite polynomials and other polynomials. Actually, the simple powers of the state variables also work well. For example, when valuing an America spread option where S_t^1 and S_t^2 are the prices of the two underlying at t, then a set of basis functions can be S_t^1, S_t^2, $(S_t^1)^2$, $(S_t^2)^2$, and $(S_t^1 S_t^2)$, so the continuation value at t, V_{t+1}, can be expressed by the following linear regression model:

$$V_{t+1} = \gamma_0 + \gamma_1 S_t^1 + \gamma_2 S_t^2 + \gamma_3 (S_t^1)^2 + \gamma_4 (S_t^2)^2 + \gamma_5 (S_t^1 S_t^2) + \varepsilon \qquad \varepsilon \sim N(0,1)$$

The implementation procedure is: first, simulate N paths of underling prices of S_t^1 and S_t^2 by Monte Carlo simulation. Second, conduct the valuation by backwardation induction. In each stage, N values of V_{t+1} can be computed by the simulated prices, and a regression is conducted based on the above regression model. Then replace the conditional expected value by the regression model and make the decision of early exercise. Finally, get the value of the spread option by taking the average of discounted cash flow at time t= 0.

The ordinary least square Monte Carlo methodology can be applied to natural gas storage valuation by the following procedure:

1. Simulate N independent price paths $P_1^n, ..., P_T^n, n = 1, ..., N$

2. Initialize the continuation value at T: $V_{T+1} \leftarrow 0$

3. Conduct backward induction:

For t = T, …, 1,

For each storage level $v_t = (v_t^m, m = 1, ..., M)$

Carry out an ordinary least square regression and compute the conditional expected continuation value by the regression results,

Next

For each simulation n = 1, …, N

For each storage level $v_t = (v_t^m, m = 1, ..., M)$, solve the one-stage problem and find a decision rule,

$$\begin{aligned} &V_t^n(v_t^m, P_t^n; \Delta v_t^n) = \max(\pi_t(v_t^m, P_t^n; \Delta v_t^n) + \beta E[V_{t+1}(v_{t+1}, P_{t+1}; \Delta v_{t+1}) \mid F_t] \\ &s.t. \\ &\quad v_{t+1} = v_t^m + \Delta v_t^n \\ &\quad 0 \le v_{t+1} \le \bar{v} \\ &\quad -\Delta \underline{v} \le \Delta v_t \le \Delta \bar{v} \end{aligned} \tag{4.4}$$

Next

Next

Next

For n=1, …, N

Compute the present value of the storage by summing the discounted future cash flows following the decision rule

Next

4. Storage value is the average of the present values under n paths

Boogert and Jong (2006) applied a similar methodology to natural gas storage valuation, but only spot price is considered, so a simple regression model works well. They also ignored the difference of bid and ask prices. A natural idea is to incorporate the storage level to avoid the interpolation over various storage levels, but they found the results are not stable when storage level is in the regression model. In this thesis, we take into account both spot price and forward prices, so even if we exclude the storage level in the regression, there will be too many terms if the basis functions cover the spot price and all of the forward prices.

Since there may be many price curves to be considered in the valuation, if we incorporate all of the prices in the regression model, the model may become unstable. Even if we only take the prices, their squares, and cross products, there will be 91 (1 intercept + 12 prices + 12 squares of prices + 66 cross products) terms in the regression model if there are 12 price curves. Therefore we need to reduce the dimension of the regression. One way to achieve this goal is principal component analysis, which is described in following section.

4.4 Principal Component Analysis

Principal component analysis (PCA) is a way to find the patterns in completed data and reduce the dimension of the data. PCA is widely used in many fields, such as image analysis, simulation, etc. For our purpose, PCA not only can be used to reduced the time of Monte Carlo simulation for the spot and forward prices, but more importantly, we create basis functions based on the principal components obtained from PCA thus reduce the dimension of the regression. Here is a brief review of PCA.

Suppose Q is the covariance matrix of the log-returns of natural gas spot and 12 forward prices, then Q is square, symmetric, and positive semi-definite. Let λ_i (i = 1, …, n) be the eigenvalues of Q with $\lambda_1 > \lambda_2 > ...\lambda_n$ and U_i (i = 1, …, n) be the associated eigenvectors, then we have

$$QU = U\Lambda$$

Where $\Lambda = \begin{bmatrix} \lambda_1 \; 0 \; ... \; 0 \\ 0 \; \lambda_2 \; ... \; 0 \\ .... \\ 0 \; 0 \; ... \; \lambda_{13} \end{bmatrix}$

The eigenvectors are orthogonal, therefore, the transpose of the eigenvector matrix is the same of its inverse matrix and we have

$$Q = U\Lambda U^T$$

Let $X = U\sqrt{\Lambda}z$,

where $\sqrt{\Lambda} = \begin{bmatrix} \sqrt{\lambda_1} \; 0 \; ... \; 0 \\ 0 \; \sqrt{\lambda_2} \; ... \; 0 \\ .... \\ 0 \; 0 \; ... \; \sqrt{\lambda_{13}} \end{bmatrix}$, and $z = \begin{bmatrix} z_1 \\ z_2 \\ ... \\ z_{13} \end{bmatrix}$ is a vector with independent and standard normally distributed components

Then we can show that the covariance matrix of X is Q by the following derivation:

$$\begin{aligned} Q_X &= [X - E[X]][X - E[X]]^T \\ &= XX^T \qquad (E[X] = 0) \\ &= U\sqrt{\Lambda}z(U\sqrt{\Lambda}z)^T \\ &= U\Lambda U^T \quad (zz^T = I, \sqrt{\Lambda}^T = \sqrt{\Lambda}) \\ &= Q \end{aligned}$$

So we can rewrite X as

$$X = U_1\sqrt{\lambda_1}z_1 + U_2\sqrt{\lambda_2}z_2 + \dots + U_n\sqrt{\lambda_n}z_n$$

The representation is called the principal component expansion of X. The random variables $y_i = \sqrt{\lambda_i}z_i$ (i = 1, …, n) are called the principal components of the random variable X. Since the eigenvalues λ_i are ranked in decreasing order, we can approximate X by the first j terms in the principal component expansion:

$$X \approx U_1\sqrt{\lambda_1}z_1 + U_2\sqrt{\lambda_2}z_2 + \dots + U_j\sqrt{\lambda_j}z_j \qquad (4.5)$$

This approximation can be used in Monte Carlo simulation. By this approximation, we only need j samples to simulate all of the variables in X, and more importantly, these j samples are independent standard normally distributed, which can be easily implemented in many soft packages.

Usually, it is accurate enough to pick up the 3 biggest eigenvalues and the corresponding eigenvectors for financial and energy markets. These 3 risk factors can be explained as parallel shift, slope, and curvature of the price curves respectively. These explanations can be found from the work of Cortazar and Schwartz (1994), Schwartz (1997), Blanco, Soronow, and Stefiszyn (2002), and Lautier (2003).

As we discussed earlier, a more important application of PCA is to reduce the dimension of the regression. Equation (4.5) indicates that every state variable (price) is correlated to the principal components, thus we can use the principal components to replace state variables in the regression. In this case we choose 3 risk factors, a possible regression model can be

$$\begin{aligned} E[V_{t+1} \mid F_t] = \alpha_0 + \alpha_1 \exp(y_t^1) + \alpha_2 \exp(y_t^2) + \alpha_3 \exp(y_t^3) + \alpha_4 \exp(2y_t^1) \\ + \alpha_5 \exp(2y_t^2) + \alpha_6 \exp(2y_t^3) + \alpha_7 \exp(y_t^1 + y_t^2 + y_t^3) \end{aligned} \tag{4.6}$$

The reason that we use the exponential functions of the principal components in the regression instead of the principal components themselves is that usually prices instead of their log-returns are used in the regression. This is different from Chalamandaris (2007), who also used PCA to reduce the dimension in the regression to compute the expected value for multicallable range accruals.

What we have derived is based on the covariance matrix for a unit time, it can be a day or a year or some other time horizon. Since we need to simulate the whole price path, we need to derive corresponding equations of (4.5) and (4.6) for any time t. We know the covariance matrix at t is tQ, so after a simple derivation, we can rewrite the equations (4.5) at any time t as:

$$X_t \approx U_1 \sqrt{\lambda_1 t} z_1 + U_2 \sqrt{\lambda_2 t} z_2 + \ldots + U_j \sqrt{\lambda_j t} z_j \tag{4.7}$$

We don’t need to rewrite the equation (4.4), but now the principal components at t are

$y_{t,i} = \sqrt{\lambda_i t} z_i$ (i = 1, …, j)

4.5 Revised Implementation Procedure

All of the procedures mentioned above are only applicable if the spot price is the sole input. When both the spot and forwards are considered, we need to rewrite the problem at each stage. One valuation method is to take the storage as an American option, so the value of the storage at t is the max of the current value and discounted conditional expected continuation value. The one step problem is expressed by the following equation:

$$
\begin{aligned}
& V_t^n(P_t^n) = \max \ \pi_t(P_t^n) \\
& s.t. \\
& \quad -\bar{v} \le \Delta v_t^{T_i,n} \le \bar{v} \qquad i = 0,1,\ldots,I_t \\
& \quad -d_t^{T_0}\Delta \underline{v} \le \Delta v_t^{T_0,n} \le d_t^{T_0}\Delta \bar{v} \qquad \text{constraint for spot month} \\
& \quad -d^{T_i}\Delta \underline{v} \le \Delta v_t^{T_i,n} \le d^{T_i}\Delta \bar{v} \qquad i = 1,\ldots,I_t \ \text{constraints for forward months} \\
& \pi_t = -\sum_{i=0}^{I_t}(P_t^n + c)\Delta v_t^{T_i,n} = \begin{cases} -\sum_{i=0}^{I_t}(P_t^{i,b,n} - c_{wth})\Delta v_t^{T_i,n} & \text{if } \Delta v_t^{T_i,n} \le 0 \\ -\sum_{i=0}^{I_t}(P_t^{i,a,n} + c_{inj})\Delta v_t^{T_i,n} & \text{if } \Delta v_t^{T_i,n} > 0 \end{cases} \qquad i = 0,1,\ldots,I_t
\end{aligned}
\tag{4.8}
$$

$$
If \ V_t^n(P_t^n) \le \beta E[V_{t+1}(P_{t+1}) \mid F_t], \ then \ V_t^n(P_t^n) = \beta E[V_{t+1}(P_{t+1}) \mid F_t]
$$

Where I_t is the number of forward months available at t. If I_t =0, then only the spot market is available,

$d_t^{T_0}$ is the number of remaining days in current month, which changes over time,

d^{T_i} is the number of days in the i^{th} forward month,

$P_t^{i,b,n}$ is the i^{th} month bid price under scenario n at t. If i=0, then it is the spot price, otherwise, it is the i^{th} month forward price,

$P_t^{i,a,n}$ is the i^{th} month ask price under scenario n at t. If i=0, then it is the spot price, otherwise, it is the i^{th} month forward price,

c_{wth} is the withdrawal cost, and

c_{inj} is the injection cost.

One way to solve problem (4.8) is to rewrite it as

$$
\begin{aligned}
&V_t^n(P_t^n) = \max\ \pi_t(P_t^n) \\
&s.t. \\
&(1)\ 0 \le \Delta v_{t,i}^n \le \bar{v} \qquad i = 0,1,\ldots,I_t,\ I_t+1,\ \ldots,\ 2(I_t+1) \qquad \textit{constraints for} \\
&\qquad\qquad \textit{injection and withdrawl for spot and forward months} \\
&(2)\ 0 \le \Delta v_{t,0}^n \le d_t^{T_0} \Delta \bar{v} \qquad \textit{constraint for injection in the spot month} \\
&(3)\ 0 \le \Delta v_{t,i}^{a,n} \le d^{T_i} \Delta \bar{v} \quad i = 1,\ldots,I_t \qquad \textit{constraints for injection in forward months} \\
&(4)\ 0 \le \Delta v_{t,i}^{b,n} \le d^{T_i} \Delta \underline{v} \quad i = 1,\ldots,I_t \qquad \textit{constraints for withdrawal in forward months} \\
&(5)\ \sum_{k=0}^{i} \Delta v_{t,k}^{a,n} - \sum_{k=0}^{i-1} \Delta v_{t,k}^{b,n} \le \bar{v} \quad i = 1,\ldots,\ I_t \qquad \textit{constraints for injection in forward months} \\
&(6)\ \sum_{k=0}^{i} \Delta v_{t,k}^{b,n} - \sum_{k=0}^{i-1} \Delta v_{t,k}^{a,n} \le 0 \quad i = 1,\ldots,\ I_t \qquad \textit{constraints for withdrawl in forward months} \\
\\
&\pi_t = \pi_t^+ + \pi_t^- \\
&\quad = -\sum_{i=0}^{I_t} (P_t^{i,a,n} + c_{inj}) \Delta v_t^{a,T_i,n} + \sum_{i=0}^{I_t} (P_t^{i,b,n} - c_{wth}) \Delta v_t^{b,T_i,n} \qquad i = 0,1,\ldots,I_t \\
&\quad = P_t^n .* \Delta v_t \\
&\textit{where } P_t^n = [P_t^{a,n}\ \ P_t^{b,n}], \\
&\Delta v_t^n = [\Delta v_t^{a,n}\ \ \Delta v_t^{b,n}],\ \textit{and} \\
&.* \textit{ is the inner product}
\end{aligned}
\tag{4.9}
$$

In equation (4.9), injections and withdrawals are defined by separate variables and withdrawals are redefined as positive variables. Injections are associated with the ask prices and withdrawals are associated with the bid prices. Although the number of variables to be solved is doubled, it is easier to be implemented. Constraints defined by (5) are explained as: the amount that can be injected in the i^{th} month should be less than or equal to the total capacity minus the total injections in months from 0 to i-1 plus the total withdrawals in months from 0 to i-1. Constraints defined by (6) are explained as: the amount can be withdrawn in the i^{th} month should be less than or equal to the total injections minus the total withdrawals in months from 0 to i-1.

Note that the constraints change over t and the number of constraints changes from month to month. So it's necessary to determine the number of constraints to be used in the optimization in every time step.

Since the current value is the intrinsic value of the storage given the current market information, this method is very similar to the so-called "intrinsic rolling" valuation or the "forward dynamic optimization" method given by Eydeland and Wolyniec (2002). However, spot price is taken into account in our valuation. We name this method as "intrinsic rolling with spot and forward". This method can be implemented by the Monte Carlo with ordinary least square method.

The procedure is as follows:

1. Simulate N independent price paths $P_1^n, ..., P_T^n, n=1, ..., N$ using equations (3.2), (3.3) and (4.7)

2. Initialize the continuation value at T: $V_{T+1} \leftarrow 0$

3. Conduct backward induction:

For t = T, …, 1,

Carry out an ordinary least square regression and compute the conditional expected continuation value by equation (4.6),

Determine the number of constraints to be included in the optimization,

For n = 1, …, N

Solve the one-stage problem described by equation (4.9)

$If\ V_t^n(P_t^n) \le \beta E[V_{t+1}(P_{t+1}) \mid F_t], then\, V_t^n(P_t^n) = \beta E[V_{t+1}(P_{t+1}) \mid F_t]$

Next

Next

V_0^n, n=1, …, N is the storage value under the n^{th} path

4. Storage value is the average of the present values under n paths

To take into account the bid and ask price, both (4.3) and (4.4) need to be rewritten in a similar way as in (4.9).

CHAPTER 5

RESULTS AND DISCUSSIONS

5.1 Historic Data and Parameter Estimation

The price models are calibrated based on historic data. Those historic data are the spot and forward prices from December 1st, 2004 to June 29th, 2007. For each business day, the prices of 24 forward month contracts are collected.

Figure 5.1 shows the historic spot price at Henry Hub. The spot price has a seasonal pattern and is very volatile. There were two spikes. One was in September 2005, which was due to the hurricane Katrina. The other was in December 2005, which was due to the higher demand and lower supply in the winter. Figure 5.2 shows the NYMEX natural gas spot and forward prices on June 28 and June 29, 2007. The seasonal pattern of the forward price curve is more obvious compared to the spot price curve.

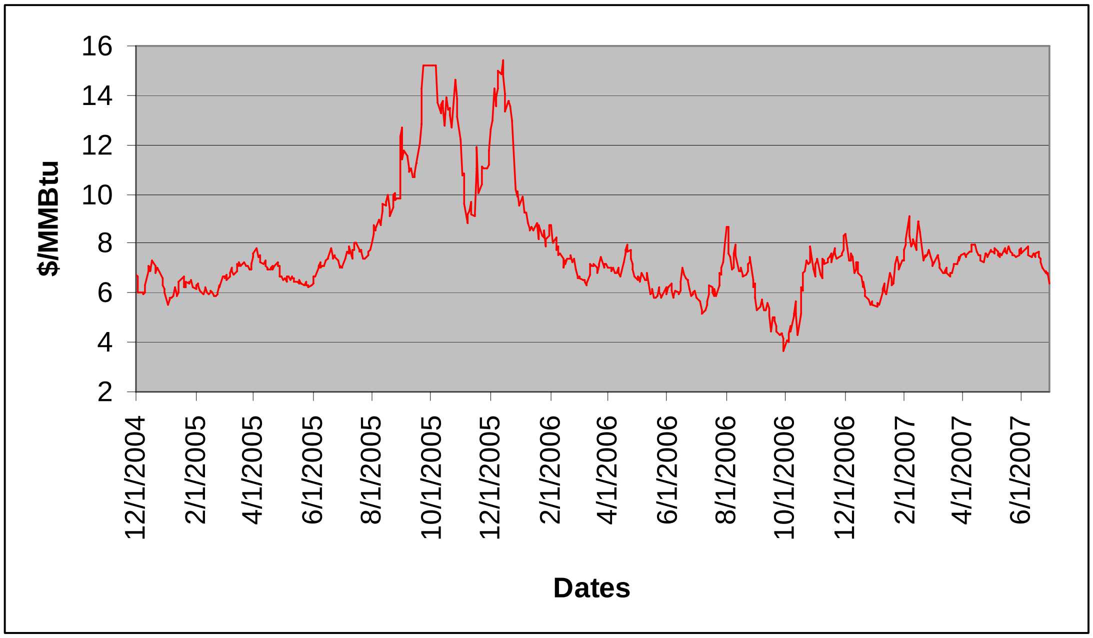

Figure 5.1: Historic Henry Hub Natural Gas Spot Price

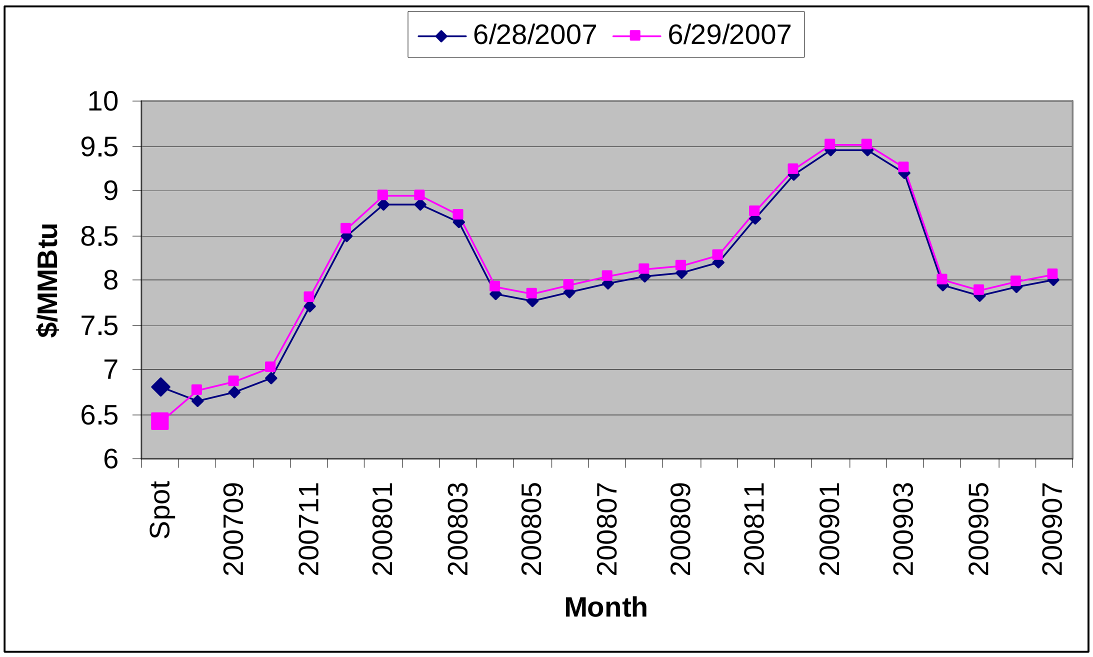

Figure 5.2: NYMEX Natural Gas Prices on June 28 and June 29, 2007

Tables 5.1 to 5.6 show the covariance, correlation, eigenvectors, and eigenvalues. For eigenvectors and eigenvalues, one is for the spot and forwards and the other is for the forwards only. Note that 1 represents January contract instead of the prompt month contract. When we take both spot and forwards into account, there are 13 eigenvalues and the ratio of the biggest three factors to the total is 79%. When only forwards are considered, there are 12 eignenvalues and the ratio of the biggest three factors to the total is 80%. Figure 5.3 is the 10 simulations of the spot price based on the covariance matrix and the spot and forward prices on June 29, 2007. The red curve is the average of the 10 simulations.

Table 5.1: Covariance Matrix of the Spot and Forward Prices

	Spot	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0.531	0.103	0.106	0.090	0.077	0.070	0.086	0.079	0.084	0.086	0.093	0.101	0.085
1	0.103	0.210	0.151	0.142	0.106	0.101	0.104	0.105	0.111	0.114	0.123	0.122	0.115
2	0.106	0.151	0.245	0.175	0.134	0.127	0.128	0.129	0.134	0.136	0.144	0.139	0.128
3	0.090	0.142	0.175	0.248	0.113	0.108	0.109	0.111	0.116	0.119	0.128	0.126	0.117
4	0.077	0.106	0.134	0.113	0.134	0.105	0.104	0.104	0.108	0.106	0.114	0.102	0.093
5	0.070	0.101	0.127	0.108	0.105	0.138	0.099	0.099	0.104	0.101	0.109	0.099	0.091
6	0.086	0.104	0.128	0.109	0.104	0.099	0.160	0.112	0.116	0.113	0.119	0.105	0.094
7	0.079	0.105	0.129	0.111	0.104	0.099	0.112	0.182	0.120	0.116	0.122	0.108	0.096
8	0.084	0.111	0.134	0.116	0.108	0.104	0.116	0.120	0.153	0.132	0.137	0.118	0.103
9	0.086	0.114	0.136	0.119	0.106	0.101	0.113	0.116	0.132	0.182	0.144	0.123	0.107
10	0.093	0.123	0.144	0.128	0.114	0.109	0.119	0.122	0.137	0.144	0.320	0.128	0.115
11	0.101	0.122	0.139	0.126	0.102	0.099	0.105	0.108	0.118	0.123	0.128	0.166	0.117
12	0.085	0.115	0.128	0.117	0.093	0.091	0.094	0.096	0.103	0.107	0.115	0.117	0.115

Table 5.2: Correlation Matrix of the Spot and Forward Prices

	Spot	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
	0	1	2	3	4	5	6	7	8	9	10	11	12
0	1.000	0.308	0.294	0.249	0.287	0.259	0.294	0.256	0.296	0.276	0.226	0.340	0.345
1	0.308	1.000	0.667	0.622	0.632	0.597	0.565	0.539	0.619	0.584	0.473	0.655	0.738
2	0.294	0.667	1.000	0.708	0.738	0.694	0.646	0.610	0.689	0.645	0.515	0.688	0.759
3	0.249	0.622	0.708	1.000	0.620	0.582	0.548	0.522	0.598	0.563	0.454	0.622	0.692
4	0.287	0.632	0.738	0.620	1.000	0.772	0.706	0.664	0.754	0.678	0.550	0.687	0.752
5	0.259	0.597	0.694	0.582	0.772	1.000	0.669	0.628	0.714	0.641	0.520	0.655	0.722
6	0.294	0.565	0.646	0.548	0.706	0.669	1.000	0.658	0.738	0.661	0.525	0.644	0.693
7	0.256	0.539	0.610	0.522	0.664	0.628	0.658	1.000	0.719	0.640	0.507	0.621	0.667
8	0.296	0.619	0.689	0.598	0.754	0.714	0.738	0.719	1.000	0.789	0.618	0.739	0.778
9	0.276	0.584	0.645	0.563	0.678	0.641	0.661	0.640	0.789	1.000	0.596	0.709	0.737
10	0.226	0.473	0.515	0.454	0.550	0.520	0.525	0.507	0.618	0.596	1.000	0.558	0.598
11	0.340	0.655	0.688	0.622	0.687	0.655	0.644	0.621	0.739	0.709	0.558	1.000	0.850
12	0.345	0.738	0.759	0.692	0.752	0.722	0.693	0.667	0.778	0.737	0.598	0.850	1.000

Table 5.3: Eigenvectors of the Covariance Matrix of the Spot and Forward Prices

0.003	-0.004	0.001	0.032	-0.013	-0.001	-0.008	-0.007	-0.048	-0.008	-0.036	-0.957	0.282
0.107	-0.019	0.003	0.079	-0.189	0.118	-0.321	-0.071	0.794	-0.243	0.230	0.046	0.284
0.042	-0.082	0.047	-0.209	-0.288	-0.491	0.274	-0.550	-0.170	-0.151	0.267	0.098	0.334
0.043	-0.003	0.002	0.080	0.019	0.201	-0.181	0.377	-0.472	-0.558	0.374	0.104	0.303
0.035	0.556	-0.623	0.279	0.171	0.160	0.070	-0.263	-0.062	0.145	0.065	0.077	0.246
0.058	-0.219	0.510	0.511	0.385	0.253	0.105	-0.326	-0.056	0.166	0.066	0.084	0.237
0.009	0.081	0.155	-0.571	-0.231	0.624	-0.042	-0.122	-0.117	0.319	0.004	0.063	0.254
0.014	0.084	0.102	0.008	0.047	-0.413	-0.712	0.083	-0.170	0.436	-0.011	0.087	0.260
0.039	-0.737	-0.509	0.086	-0.080	0.059	0.088	0.156	-0.016	0.250	-0.061	0.085	0.270
0.021	0.267	0.230	0.321	-0.501	-0.098	0.387	0.457	0.058	0.231	-0.099	0.091	0.279
0.018	0.020	0.020	-0.026	0.020	-0.001	-0.086	-0.114	-0.037	-0.385	-0.841	0.132	0.322
0.402	0.083	0.043	-0.390	0.578	-0.195	0.296	0.309	0.202	0.052	0.045	0.044	0.274
-0.903	0.015	0.017	-0.122	0.237	-0.063	0.106	0.106	0.144	-0.005	0.062	0.050	0.243

Table 5.4: Eigenvectors of the Covariance Matrix of the Forward Prices

-0.108	0.020	-0.003	0.077	-0.201	0.117	0.328	0.082	-0.785	-0.241	-0.231	0.294
-0.042	0.083	-0.047	-0.231	-0.271	-0.491	-0.279	0.547	0.177	-0.153	-0.263	0.348
-0.043	0.003	-0.001	0.077	0.015	0.201	0.180	-0.380	0.475	-0.560	-0.367	0.317
-0.035	-0.555	0.624	0.289	0.150	0.159	-0.071	0.263	0.070	0.144	-0.062	0.257
-0.057	0.218	-0.510	0.540	0.345	0.252	-0.105	0.327	0.068	0.164	-0.062	0.248
-0.010	-0.080	-0.156	-0.586	-0.188	0.625	0.039	0.120	0.114	0.319	-0.003	0.264
-0.014	-0.084	-0.102	0.007	0.049	-0.413	0.712	-0.080	0.180	0.435	0.015	0.272
-0.039	0.738	0.508	0.084	-0.088	0.059	-0.086	-0.156	0.020	0.249	0.063	0.282
-0.021	-0.268	-0.229	0.289	-0.527	-0.099	-0.381	-0.456	-0.053	0.231	0.102	0.291
-0.018	-0.020	-0.020	-0.026	0.023	-0.001	0.085	0.114	0.036	-0.385	0.845	0.337
-0.403	-0.081	-0.044	-0.339	0.601	-0.195	-0.295	-0.311	-0.215	0.054	-0.046	0.283
0.903	-0.014	-0.017	-0.100	0.244	-0.063	-0.106	-0.106	-0.148	-0.004	-0.062	0.251

Table 5.5: Eigenvalues of the Covariance Matrix of the Spot and Forward Prices

1	2	3	4	5	6	7	8	9	10	11	12	13	SUM
1.577	0.442	0.183	0.124	0.086	0.072	0.068	0.057	0.051	0.048	0.030	0.028	0.016	2.783

Table 5.6 Eigenvalues of the Covariance Matrix of the Forward Prices

1	2	3	4	5	6	7	8	9	10	11	12	SUM
1.487	0.184	0.124	0.087	0.072	0.068	0.057	0.051	0.049	0.030	0.028	0.016	2.253

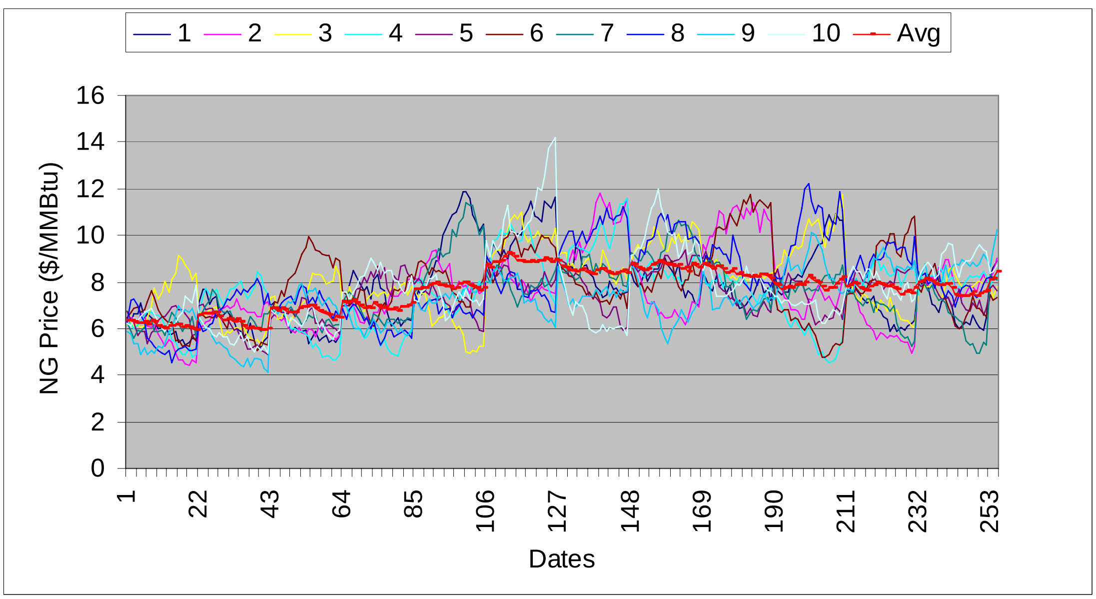

Figure 5.3: Simulated Spot Prices from July 07 to June 08

5.2 Results

5.2.1 The Storage Contract

The results are from the valuation of a natural gas storage contract:

Term: from July 1, 07 to June 30, 08

Location: Henry Hub

Working gas capacity: 1,000,000 MMBtu

Start volume: 0

End volume: 0

Maximum daily injection rate: 35,000 MMBtu

Maximum daily withdrawal rate: 75,000 MMBtu

Withdrawal cost: 0

Injection cost: 1.5% of the fuel price

We also assume that the ask price equals mid price plus one cent and the bid price equals mid price minus one cent.

The valuation is implemented with MATLAB on a desktop personal computer with 2.8 GHz CPU and 2.5 GB RAM.

5.2.2 The Results by the Method of “Intrinsic Rolling with Spot and Forward”

Table 5.7 shows the value of the storage contract estimated by the method of “intrinsic rolling with spot and forward” based on the market information on June 28, 2007. The simulated value is about $1.7 million, which is close to the value estimated by FEA (Financial Engineering Association) provided model MCSTORAGEOPT. As the number of simulations increases, the CPU time linearly increases. For instance, one run with 1000 simulations takes about 38 minutes. Note the term of the contract is one year. If it increases, then the CPU time also will increase. Fortunately, the value of the storage converges relatively well. So for valuation purpose 100 simulations seem enough.

MCSTORAGEOPT uses Monte Carlo simulation and uses one risk factor model for prices. Its inputs include spot and forward price on valuation date, volatility of forward months, etc. However, neither the covariance matrix nor the correlation matrix is required, so it’s unclear whether the model takes correlation into account. In addition, other key issues such as the method for computing conditional expectation are not disclosed.

Table 5.7: The Value of the Storage Contract Estimated by the Method of "Intrinsic Rolling with Spot and Forward" on June 28, 2007

Number of simulations	Value of the 1st Run ($ 1000)	Value of the 2nd Run ($ 1000)	CPU Time of the 1st Run (Sec)	CPU time of the 2nd Run (Sec)	FEA ($ 1000)
10	1628	1680	25	24	2370
20	1678	1611	47	46	2065
40	1703	1745	93	93	1938
100	1661	1643	232	234	1677
160	1676	1698	370	369	1773
320	1679	1674	739	743	1717
500	1686	1678	1155	1164	1699
640	1673	1701	1480	1478	1691
1000	1675	1681	2305	2319	1708
2000	1680	1676	4604	4625	1836
5000	1678	1680	11553	11550	1862
10000	1677	1676	23092	23106	1859

Figure 5.4 shows a sample optimization result on day 1 and day 20, respectively. Based on the simulated prices on day 1, the optimization model suggests one turnover during the contract term: inject 735000 MMBtu in July 07 and 265000 MMBtu in September 07 and them withdrawal the full amount in January 08. On day 20, the optimization model suggestions two turnovers.

Figure 5.5 shows the histogram of storage value from one run with 1000 simulations. It shows the simulated storage values are normally distributed.

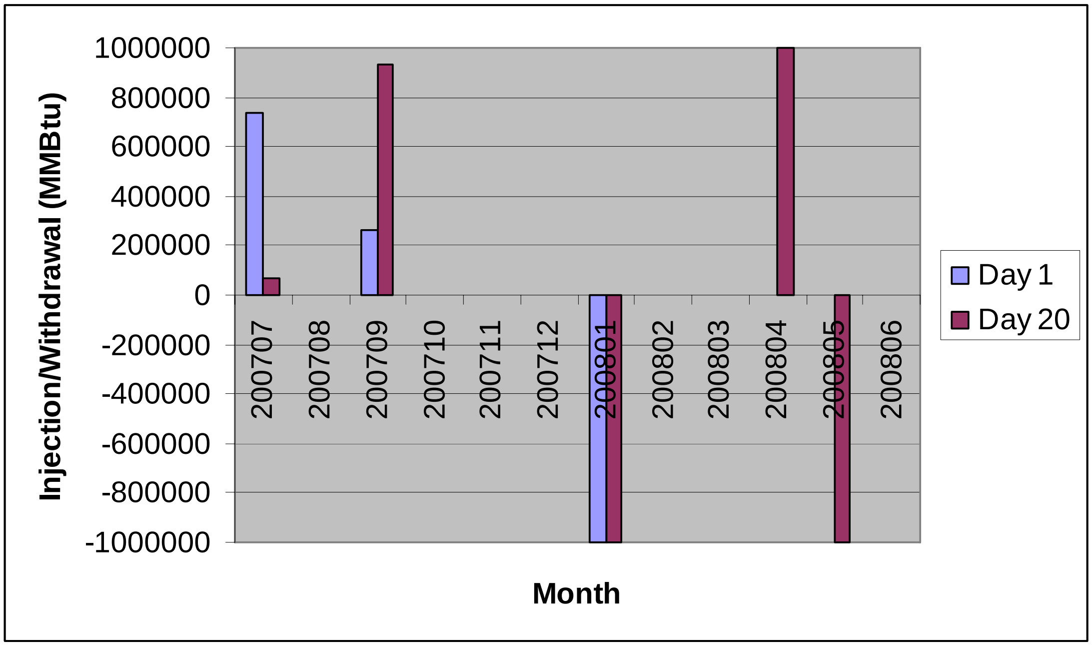

Figure 5.4: Sample Optimization Result on Day 1 and Day 20

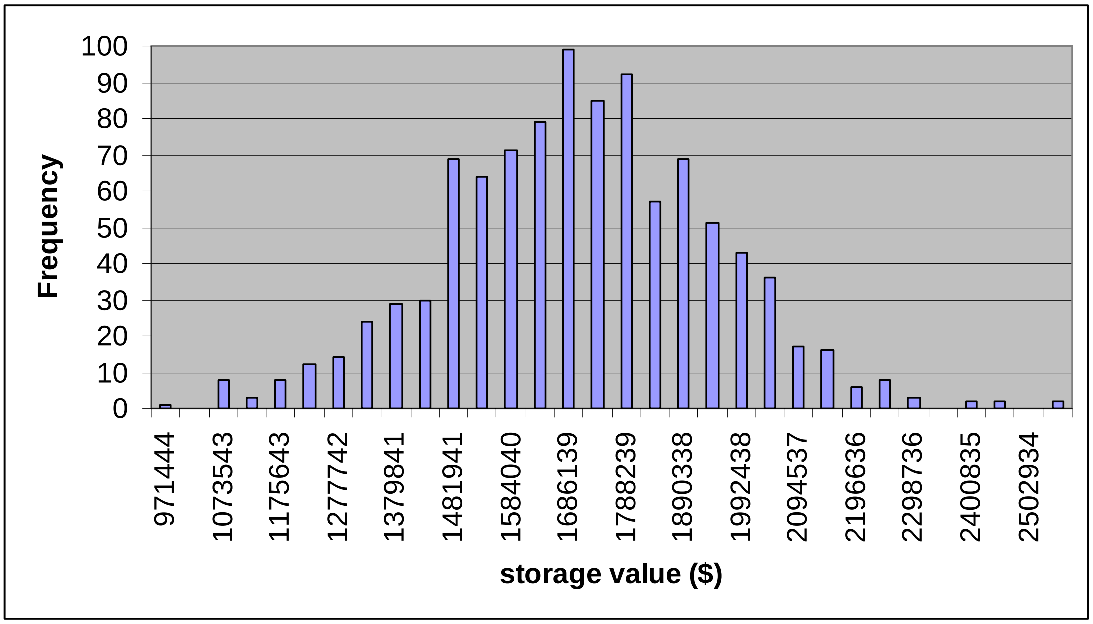

Figure 5.5: The Histogram of Storage Value from One Run of 1000 Simulations

To study the impact of market information, the value of the storage based on the information of June 29, 2007 is also estimated, which is listed in table 5.8. The impact of market information is significant, the value from June 29 price is much higher than the value from one day before. This is due to a very low spot price on June 29 compared to the forward prices (see figure 5.2), which creates bigger spreads between the spot and the forwards.

Table 5.8: The Value of the Storage Contract Estimated by the Method of "Intrinsic Rolling with Spot and Forward" on June 29, 2007

Number of simulations	Value of the 1st Run ($ 1000)	Value of the 2nd Run ($ 1000)	CPU Time of the 1st Run (Sec)	CPU time of the 2nd Run (Sec)	FEA ($ 1000)
10	2386	2423	25	24	2640
20	2466	2493	47	45	2244
40	2430	2435	92	90	2145
100	2416	2433	227	228	1783
160	2420	2417	366	366	1925
320	2445	2428	735	729	1837
500	2413	2420	1146	1147	1806
640	2421	2424	1469	1461	1800
1000	2418	2420	2299	2288	1806
2000	2430	2401	4585	4567	1945
5000	2421	2422	11452	11394	1982
10000	2428	2426	22967	22959	1977

Table 5.9 shows the value of the storage estimated by forwards only. The results show that the estimated storage value decreases significantly when spot is not taken into account. Since the spot is moved away, the selected eigenvalues and eigenvectors will be different, but the same regression model is applied.

Table 5.9: The Value of the Storage Contract Estimated by the Method of "Intrinsic Rolling with Forward Only"

Number of simulations	Value from inputs of 6/28/07 ($ 1000)	Value from inputs of 6/29/07 ($ 1000)	CPU Time (6/28/07) (Sec)	CPU time (6/29/07) (Sec)
10	1040	1010	21	19
20	987	967	39	36
40	940	981	73	74
100	946	974	183	181
160	949	978	293	290
320	948	972	585	584
500	957	964	912	912
640	956	969	1179	1163
1000	963	969	1834	1817
2000	958	969	3661	3634
5000	958	968	9154	9093

Principal component analysis is helpful for obtain a practicable regression model, but it may reduce the value due to the reduced volatility, especially in this exercise, where the ratio of the three biggest eigenvalues to the total eigenvalues is only about 80%.

5.2.3 The Results by Spot Only Methods

Tables 5.10 and 5.11 list the valuation results by the method of Monte Carlo with ordinary least square regression on June 28 and June 29, 2007 respectively. This method shows that the value of the storage contract is about $7 million, which is much higher than the value estimated by the previous methodology. This is mostly due to the higher volatility of the spot price, which makes frequent position adjustment profitable. However, storage traders don't adjust their position very frequently. A typical frequency of position adjustment is around once a week. Note that the spot curve is also taken into account in the decision process in the method of "intrinsic rolling with spot and forward", but it is simply treated as a forward curve, so the value from the spot volatility can not be fully captured in the valuation. The volatilities are calibrated from historic data, but it can be replaced by implied volatilities.

Tables 5.12 and 5.13 list the valuation results by the method of Monte Carlo with stochastic dual dynamic programming on June 28 and June 29, 2007 respectively. This method also gives a significantly higher storage value of about $11 million.

The empirical valuation shows that the value of the storage converges very well by the stochastic dual methodology. 100 simulations are enough for an accurate valuation, which takes only about 4 seconds. It requires more simulations (about

2000) to obtain an accurate value for the least square method. Boogert and Jong (2006) observed that as few as 50 simulations are required to obtain a precise value of storage and explained it is possibly due to the mean-reversion model.

CPU time does not increase significantly for the stochastic dual dynamic method as the number of simulation increases. This is due to the fact that there are only 10 price scenarios in all of the runs. When the number of simulation is 10, then the first scenario uses the lowest price and the second scenario uses the second lowest price, etc. When the number of simulation is 20, the first scenario uses the average of lowest two prices and the second scenario uses the average of the second lowest two prices, etc.

Table 5.10: The Value of the Storage Contract Estimated by the Method of Monte Carlo with Ordinary Least Square Regression on June 28, 2007

Number of simulations	Value of the 1st Run ($ 1000)	Value of the 2nd Run ($ 1000)	CPU Time of the 1st Run (Sec)	CPU time of the 2nd Run (Sec)
10	8440	7939	4	4
20	7499	6984	5	5
40	7104	7188	9	9
100	7293	7345	20	20
160	7252	7201	31	31
320	6918	7110	60	60
500	6912	7002	95	94
640	7047	6979	124	120
1000	6924	6971	194	187
2000	6980	7028	386	374
5000	6986	6930	972	952
10000	6971	6974	1918	1926

Table 5.11: The Value of the Storage Contract Estimated by the Method of Monte Carlo with Ordinary Least Square Regression on June 29, 2007

Number of simulations	Value of the 1st Run ($ 1000)	Value of the 2nd Run ($ 1000)	CPU Time of the 1st Run (Sec)	CPU time of the 2nd Run (Sec)
10	7300	8690	4	4
20	7390	7301	5	5
40	6833	7205	9	9
100	7218	6850	20	20
160	6878	6779	31	31
320	6810	6839	62	62
500	6906	6695	96	96
640	6761	6711	126	122
1000	6859	6839	194	193
2000	6746	6753	386	384
5000	6752	6695	978	988
10000	6741	6729	1977	1937

Table 5.12: The Value of the Storage Contract Estimated by the Method of Monte Carlo with Stochastic Dual Dynamic Programming on June 28, 2007

Number of simulations	Value of the 1st Run ($ 1000)	Value of the 2nd Run ($ 1000)	CPU Time of the 1st Run (Sec)	CPU time of the 2nd Run (Sec)
10	12086	12003	4	4
20	12002	12763	4	4
40	12090	11533	4	4
100	11629	11880	4	4
160	11831	11827	4	4
320	11941	11900	4	4
500	11958	11869	4	4
640	12128	11929	4	4
1000	11895	12029	4	4
2000	11996	11923	5	5
5000	12006	11925	6	6
10000	11937	11980	8	8

Table 5.13: The Value of the Storage Contract Estimated by the Method of Monte Carlo with Stochastic Dual Dynamic Programming on June 29, 2007

Number of simulations	Value of the 1st Run ($ 1000)	Value of the 2nd Run ($ 1000)	CPU Time of the 1st Run (Sec)	CPU time of the 2nd Run (Sec)
10	11942	10281	4	4
20	10890	11243	4	4
40	11269	11462	4	4
100	11840	11541	4	4
160	11857	11585	4	4
320	11789	11717	4	4
500	11626	11781	4	4
640	11621	11765	4	4
1000	11604	11819	4	4
2000	11740	11669	5	5
5000	11763	11764	6	6
10000	11781	11778	8	8

The value of the storage is also impacted by the operational flexibilities. Storage increases in value with the level of flexibility. Increased flexibilities give more value to the storage when spot only methods are used for valuation.

5.3 Valuation with a Changing Bid-ask Spread

In above simulation, bid-ask spread is set as constant, but it changes over time in reality. The spot and prompt forward are very liquid, thus the associated spread can be less than 1 cent. As the maturity of the forwards increases, the spread increases as well. Also, the spread is determined by the location. Henry Hub market is very liquid and the other markets are less liquid. Considering the specific storage contract, the following bid-ask spread is applied:

Bid-ask spread = $(0.1+0.06t)\%$ of fuel price ($t=0, 1, \ldots, 11$),

If the price of natural gas is \$8/MMBtu, then the above equation implies a spread of 0.8 cent for the spot and a spread of 6.08 cents for the contract expires in one year. Table 5.14 shows the results with changing bid-ask spread. Those values are close to the value estimated by a constant bid-ask spread of 2 cents.

Table 5.14: The Valuation of the Storage Contract with Changing Bid-ask Spread

Number of simulations	Intrinsic rolling with spot and forwards ($1000)	Least-square MC, spot only ($1000)	SDDP, spot only ($1000)
10	2102	7976	11972
20	2019	7711	12005
40	2089	7140	11889
100	2050	7071	11897
160	2073	7241	11786
320	2019	6849	11895
500	2040	7018	11777
640	2025	6980	11752
1000	2035	6921	11768
2000	2032	6917	11741
5000	2037	6904	11759

5.4 Further Discussions on the Methodologies

The "intrinsic rolling" may only capture very limited time value compared with the intrinsic value obtained on the first day. For instance, we have a contract that has a term from September 1, 2007 to December 31, 2007. When we are in September, we can adjust the positions and capture the time value, but on and after September 26, the

October contract expires, so the choices for injection are limited to the spot and November contract, which is usually more expensive than October contract. Thus, after September 26, the storage value tends to decrease. That means the method may not capture the value in the price volatilities after moving into the next month. The problem is that when we compare the storage values in two periods, they always have a new start (empty storage) but have different choices (more or less forward contracts are available).

This method can be improved by excluding the realized activities when comparing the values in two periods. For example, if the storage value on September 25 is 100 and the storage value on September 26 is 90, then follow the originally method we set the value on September 25 as 100. But what we should do is as follows:

> First, check the results of the optimization on September 25, if there are not activities associated with the spot of September 25 and October forward, then there is no need for further analysis and we still set the value on September 25 as 100.
>
> Second, if there are activities on the spot of September 25 and October forward, then we separate the value of the storage into two parts. One is the injection on the spot of September 25 and October forward, saying -1000 (since it is the cost), and the other is withdrawal on December forward, saying 1100.

Third, redoing the optimization on September 26 given the market information on September 26 and additional constrains that are the results from the optimization on September 25 associated with the spot on September 25 and October contract.

Finally, if the new value of the storage on September 26 is less than 1100, then set the value on September 25 as 100. If the new value of the storage on September 26 is greater than 1100, saying 1105, then set the value on September 25 as 105.

By this way, the valuation can capture more time value. However, this method may take more time.

In this exercise, we use very simple price models. Since the simulation of the spot and forward prices are separated from the optimization models, alternative price models can be used in the valuation.

CHAPTER 6

CONCLUSIONS

Pricing natural gas storage is a very challenging topic. It is necessary to develop both appropriate price models and optimization models.

For the price models, we assume that each forward contract follows a geometric Brownian motion with zero drift. The volatility of each forward curve is calibrated from the historic data. The spot price curve also follows a geometric Brownian motion, but its expectation changes from month to month. Specifically, its expectation in a certain month equals the corresponding forward price on the valuation date. Since price models don't impact the optimization models and their implementation, alternative models can be selected in the valuation. The volatilities of the spot and forward curves are calibrated based on the historic data, but they can be replaced by implied volatilities that can be derived from the traded options.

We developed three methodologies for storage valuation. The first methodology is called "intrinsic rolling with spot and forward". It takes both the spot and forward prices into account. This method applies the so-called "Intrinsic rolling" trading strategy where a trader locks his/her positions based on the market information on the first day and the trader adjust his/her positions in the following days based on new market information to obtain extra value.

The second methodology is called Monte Carlo with ordinary least square regression and the third is called Monte Carlo with stochastic dual dynamic programming. Both the second and the third methodologies consider the spot price only in the decision making. These two methods are based on the work of Boogert and Jong (2006) and Bringedal (2003) respectively, but they are improved so the bid and ask spread can be simulated in the valuation. Both the second and third methods apply a spot trading strategy: inject at current period if current price is lower than the marginal value of next period and withdraw at current period if current price is higher than the marginal value of next period.

All of the methodologies are implemented by backward induction. A key issue is how to compute the expected value in the next period conditioned on current information. The third methodology assumes that the conditional expectation value in the next period is a piece-wise linear function of the storage level. The second methodology uses a regression model to compute the conditional expectation value. The first methodology also uses a regression model. However, there are too many terms if all of the spot and forward prices are included in the regression. Therefore only three risk factors are selected based on principal component analysis, thus greatly reducing the dimension of the regression model.

The value of the storage estimated by the first methodology is close to the market value[1] and the value estimated by FEA provided functions. Both the second and the third method give much higher value for the same storage contract. This may be partially due to the spot curve used in this valuation, but methods that only use the spot price may overstate the market value of the storage contract, since frequently position adjustment is profitable due to the high volatility of the spot price, but in the reality, traders don't adjust their position very often. Empirical results indicate that storage values estimated by all of the three methodologies converge in terms of the purpose of valuation.

Many other factors impact the storage value including market information, operational flexibility of the storage. The fundamentals of the natural gas market are also very important to the value of the storage. An important component of the storage value comes from the spreads among forward contracts, which is the result of seasonality of natural gas demand. The increased demand of filling season will reduce the value of storage. The increased supply of storage will also reduce the value of storage. So in the long-run, there is an equilibrium when the marginal cost of building storage equals the value of storage.

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[1] This is only based on one observation. Due to confidential policies at FPL Energy, the market value of the storage contract can't be disclosed in the thesis.

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