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**Borroni** Alessandro  
**D'Intinosante** Fabrizio  
**Perletti** Massimiliano



A circular graphic in the center of the slide features the text "FORECASTING in the DARK" in a stylized font. The word "FORECASTING" is written vertically along the top inner edge of the circle, while "in the" and "DARK" are written horizontally across the bottom. The circle is outlined in yellow and set against a dark background with several yellow lightning bolt icons.

FORECASTING  
in  
the  
DARK



# OVERVIEW

# CONTEXT



Operates in **power, gas** and **environmental** markets.  
It is the exchange place for electricity and natural gas spot trading in Italy  
In the power market platform, *producers* and *purchasers* sell and buy wholesale electricity



An auction for every hour of the day

GME releases data in files .xml with a delay of one week



Forecasting this Supply Function could be interesting for every energy producer



# OUR GOAL

Predict the «**supply function**» for auctions in the Italian electricity market exploiting **functional time series forecasting**, through both statistical methods and machine learning

1. The supply function is a "curve" that relates the amount of electricity (in MWh) with the price (in €)

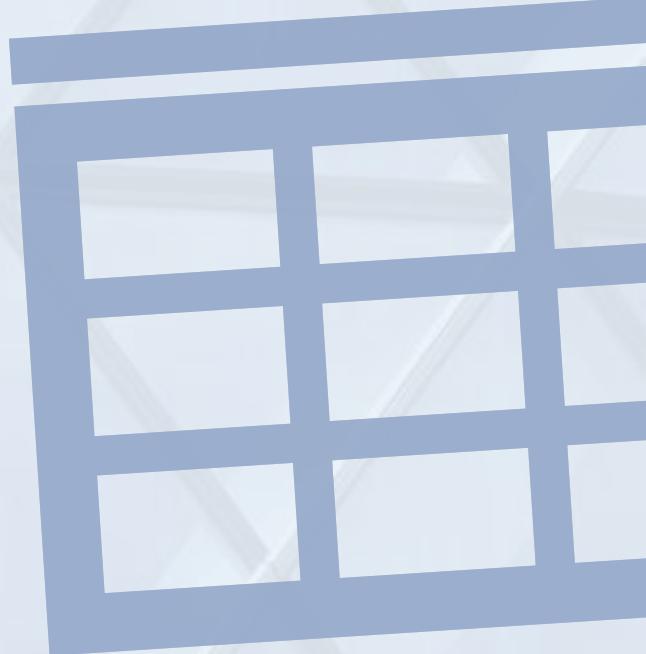


- The results might provide the electricity producers with an improvement for their bidding strategy
- 2.

# RAW DATASET

- PURPOSE\_CD
- STATUS\_CD
- UNIT\_REFERENCE\_NO
- INTERVAL\_NO
- BID\_OFFER\_DATE\_DT
- QUANTITY\_NO
- AWARDED\_QUANTITY\_NO
- ENERGY\_PRICE\_NO
- MERIT\_ORDER\_NO
- PARTIAL\_QTY\_ACCEPTED\_IN
- ADJ\_QUANTITY\_NO
- ZONE\_CD
- AWARDED\_PRICE\_NO
- OPERATORE

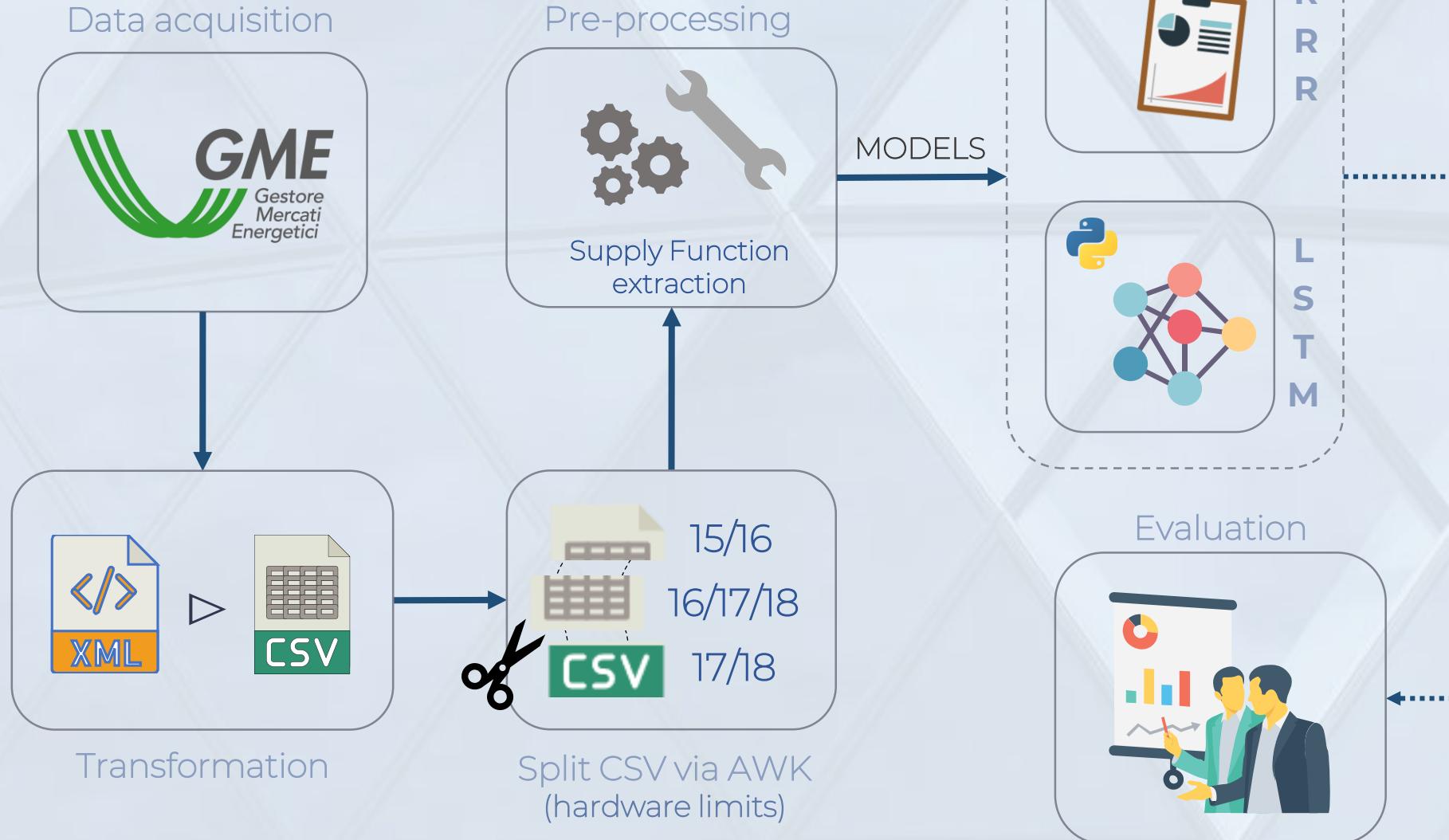
**54 M rows**  
**(4 GB)**



OFF = selling offer  
BID = purchase offer



# PIPELINE



# PRE-PROCESSING



# FOCUS

Target on 2015-2016 data:

- Dimensionality of the dataset
- Data density



Due to hours change during the year some days have **23** or **25** INTERVALS

- ◆ 25 » Delete the 25th hour
- ◆ 23 » Replicate the 23rd to obtain the 24th

INTERVAL\_NO  
and

BID\_OFFER\_DATE\_DT  
are concatenated into  
*datetime format*

# GRID

The **grid** is realized using 50-iles of an interval going from 0 to 260 (cap. price in the Italian market, based on this paper\*)



# DATA MANIPULATION

# FOCUS

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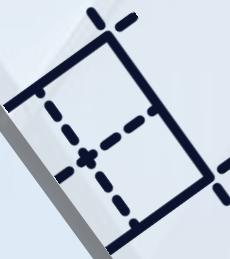
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# DATA MANIPULATION

# LOG-TRANSFORMATION

Apply groupby on prices  
in order to take only  
unique values

The offers are ordered with  
respect to the price cumulating  
the quantities

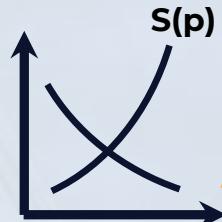
From each supply function 51 prices  
based on the grid values are sampled,  
obtaining 51 time series

Transform the original ordinate points preserving  
positivity and non-decreasing monotonicity

$$q_{i,t} := \begin{cases} \log S_t(p_i), & \text{for } i = 0; \\ \log (S_t(p_i) - S_t(p_{i-1}) + c), & \text{for } i = 1, \dots, 50, \end{cases}$$

After the prediction phase, the inverse  
transformation is applied to the results.

$$\hat{S}_t(p_i) = \begin{cases} \exp(\hat{q}_{i,t} + s_{i,t}^2/2), & \text{for } i = 0; \\ \exp(\hat{q}_{i,t} + s_{i,t}^2/2) + \hat{S}_t(p_{i-1}) - c, & \text{for } i = 1, \dots, 50. \end{cases}$$



## SUPPLY FUNCTION

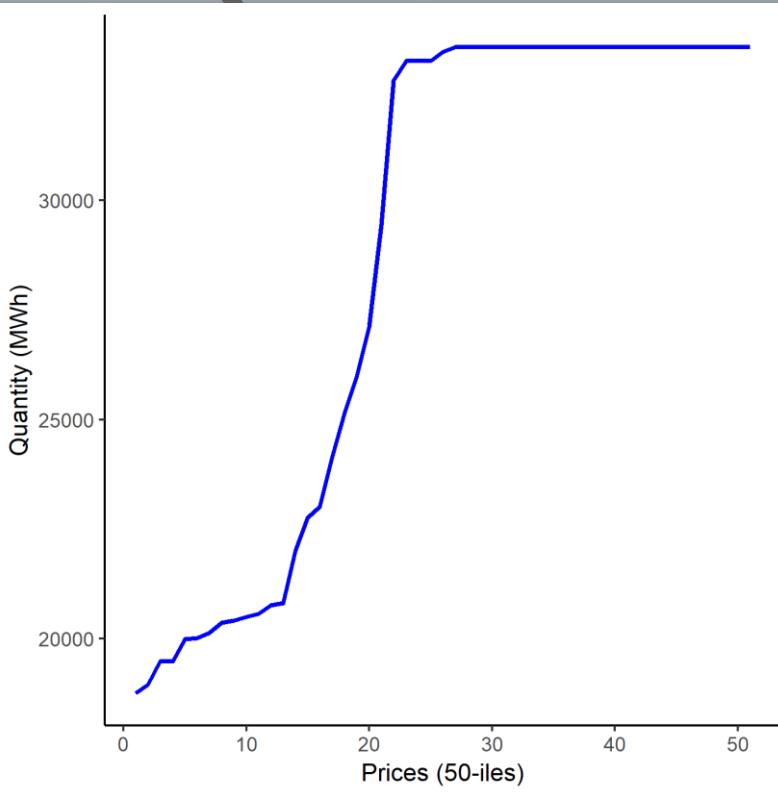


## SUPPLY FUNCTION

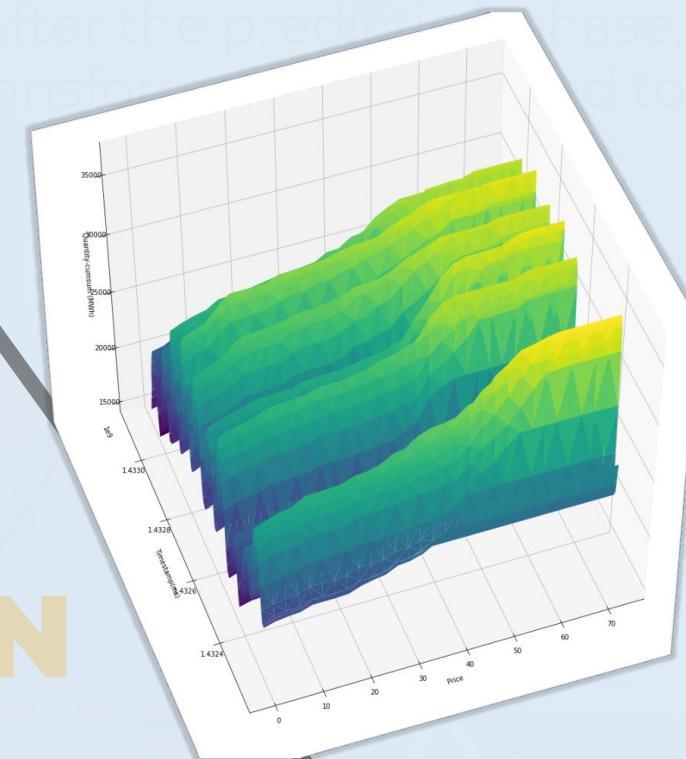
Apply groupby  
in order to take  
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The offers are  
respect to the  
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From each supply function 51 prices  
based on the grid values are sampled,  
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The cumulative quantity  
based on the ordered  
prices is taken in order to  
obtain a Supply Function

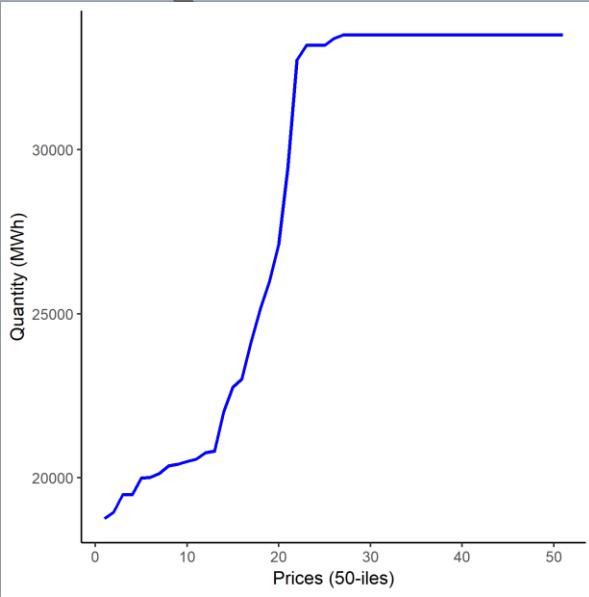


Apply groupby on  
in order to take  
singular values

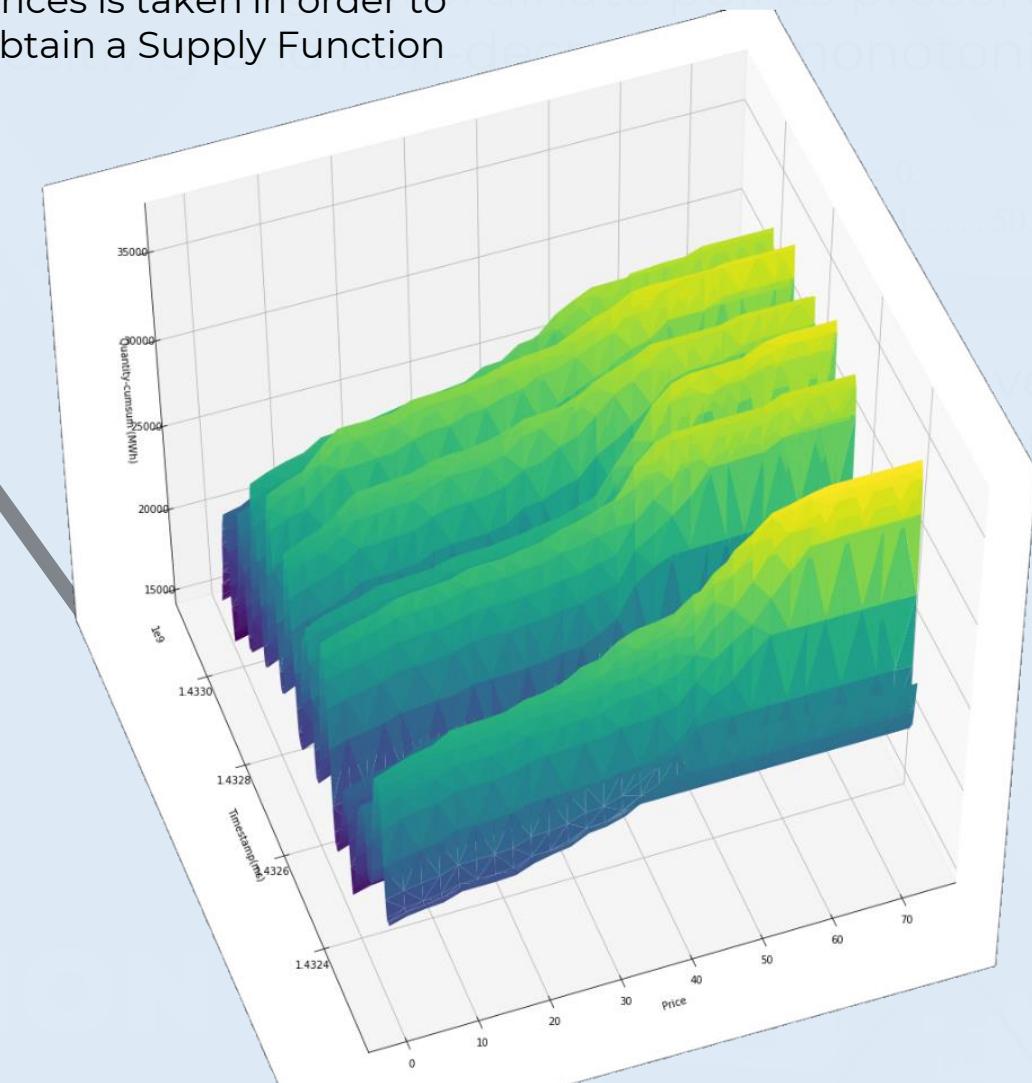
The offers are ordered  
respect to the price  
the quantities

From each supply function 5 prices are  
sampled based on the grid values  
obtained

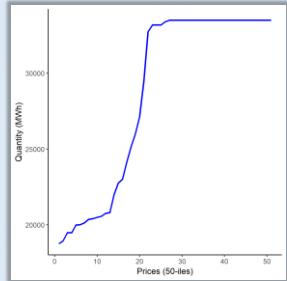
There will be a curve for each  
time interval in the dataset



The cumulative quantity  
based on the ordered  
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# LOG-TRANSFORMATION



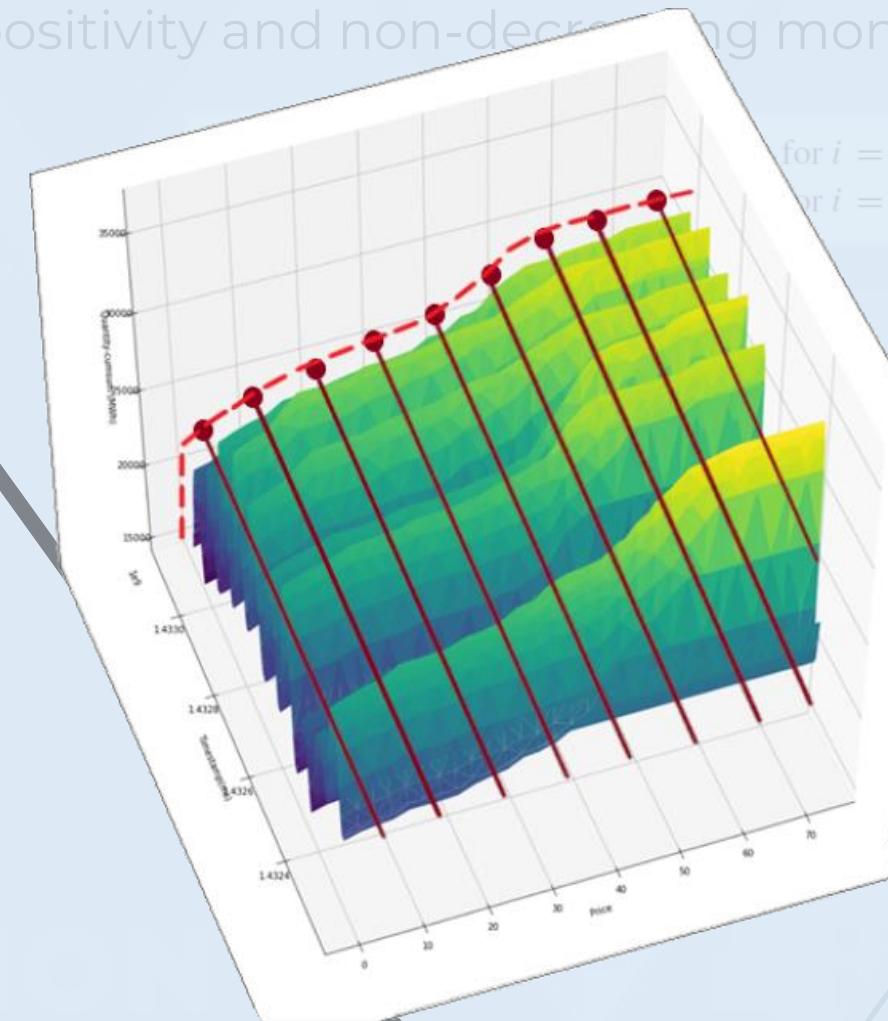
The cumulative quantity based on the ordered prices is taken in order to obtain a Supply Function

There will be a curve for each time interval in the dataset

**Objective:** to predict this curve in a future time

The **estimated Supply Function** is obtained by linking together the predictions of each time series

Transform the original ordinate points preserving positivity and non-decreasing monotonicity



# LOG-TRANSFORMATION

Apply groupby on prices  
in order to take only  
unique values

The offers are ordered with  
respect to the price accumulating  
the quantities

From each supply function 51 prices  
based on the grid values are sampled,  
obtaining 51 time series of ordinate points

Transform the original ordinate points preserving  
positivity and non-decreasing monotonicity

$$q_{i,t} := \begin{cases} \log S_t(p_i), & \text{for } i = 0; \\ \log (S_t(p_i) - S_t(p_{i-1}) + c), & \text{for } i = 1, \dots, 50, \end{cases}$$

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## SUPPLY FUNCTION

# MODELS



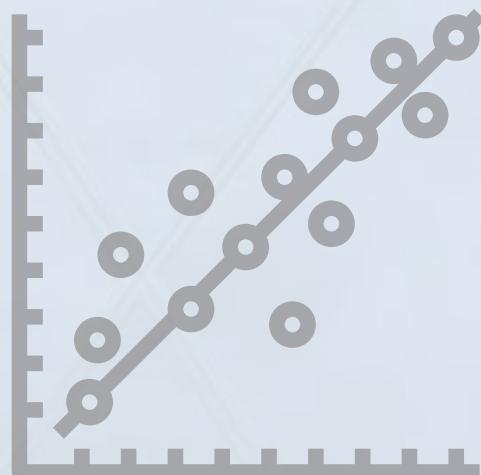
# RRR



The **Reduced Rank Regression** (RRR) model is a multivariate regression model with a coefficient matrix of reduced rank

Reducing the rank of the coefficient matrix means that few linear combinations of the regressors are enough to take into account all the variability of dependent variable

The optimal rank is chosen evaluating iteratively minimizing the out-of-sample MSE or RMSE using Cross Validation





# RRR

The applied models include **1**-hour, **24**-hours and **168**-hours (1 week) forecasting



Every forecasting model make use of the corresponding **Lag** in time and the subsequent ones  
(e.g.  $y_t$  on  $y_{t-1}, y_{t-24}, y_{t-168}$ )

Each model is performed also on 3 different matrices of **deterministic variables**, called Reg. 1, Reg. 2 and Reg. 3

[:::]



# RRR

## Reg. 1

- ◆ Sinusoids for annual seasonality



## Reg. 2

- ◆ Sinusoids for annual seasonality
- ◆ Dummies indicating Saturday, Sunday and Monday

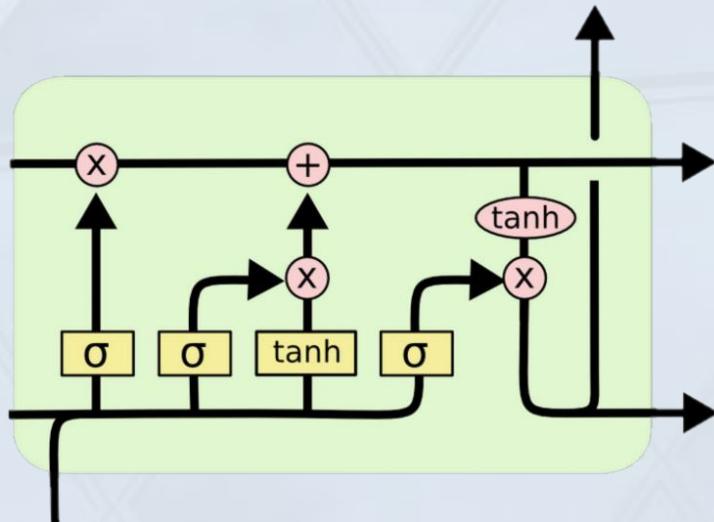
## Reg. 3

- ◆ Sinusoids for annual seasonality
- ◆ Dummies indicating Saturday, Sunday and Monday
- ◆ Sinusoids for daily seasonality



# LSTM

The Long Short-Term Memory network (LSTM) is a **recurrent neural network** trained using backpropagation  
This family of NN is particularly suitable for sequence modelling



The core concept of LSTM is the cell state, and it's various gates

The cell state acts as a transport channel that propagates information all the way down the sequence chain

In order to emulate a reduced rank effect (like RRR models), the network flow is **compressed** using a hidden layer with a reduced number of neurons in order to re-create a kind of information bottleneck

# LSTM - The Model



Loss function = MSE

```
from keras.models import Sequential
from keras.layers.core import Dense, Activation, Dropout, Flatten
from keras.layers import CuDNNLSTM
from keras.callbacks import EarlyStopping, TensorBoard, ModelCheckpoint
from keras.layers.normalization import BatchNormalization
from keras.optimizers import Adam, SGD, Nadam
from time import time
#!pip install livelossplot
#from livelossplot import PlotLossesKeras
from keras.layers.advanced_activations import LeakyReLU, PReLU
import tensorflow as tf
from tensorflow.python.client import device_lib
from sklearn.preprocessing import StandardScaler

def create_model(look_back, bott, reg=True):
    model = Sequential()
    #1 layer
    model.add(CuDNNLSTM(500, input_shape=(look_back, n_features), return_sequences=True, kernel_initializer='TruncatedNormal'))
    if reg:
        model.add(BatchNormalization())
    model.add(LeakyReLU())
    if reg:
        model.add(Dropout(rate=0.1))
    #2
    for _ in range(1):
        model.add(CuDNNLSTM(128, kernel_initializer='TruncatedNormal', return_sequences=True))
        if reg:
            model.add(BatchNormalization())
        model.add(LeakyReLU())
        if reg:
            model.add(Dropout(rate=0.1))
    #3
    for _ in range(1):
        model.add(CuDNNLSTM(51, kernel_initializer='TruncatedNormal', return_sequences=False))
        if reg:
```

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    #4
    for _ in range(1):
        model.add(Dense(rank))
        if reg:
            model.add(BatchNormalization())
        model.add(LeakyReLU())
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    #5
    model.add(Dense(n_features))

    return model
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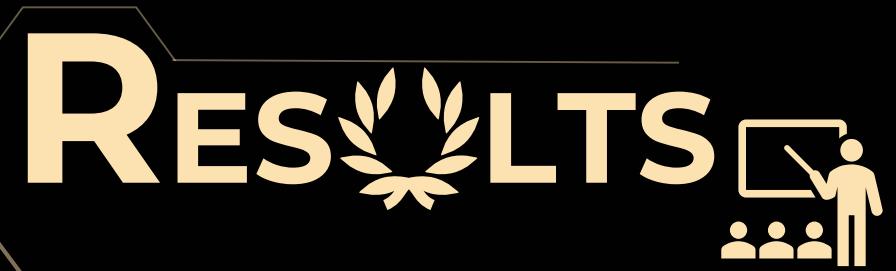
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# RESULTS

Each model is applied to a **training set** of 12'992 records and evaluated on a **test set** of 1'000 records

The out-of-sample evaluations are performed using **RMSE** (Root Mean Square Error) and **MAPE** (Mean Absolute Percentage Error) as metrics



# EVALUATION - RRR

Table 1: Out-of-sample root mean square error for the three forecast and 9 models

	Reg. 1		Reg. 2		Reg. 3	
	Rank	RMSE	Rank	RMSE	Rank	RMSE
1-step	51	3494.8	51	3733.5	51	4555.2
24-step	50	4678.9	50	4415.4	50	5179.5
168-step	51	5567.5	51	5440.9	50	4775.4

Table 2: Out-of-sample mean absolute percentage error computed for each day of the week

Pred.	Reg.	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Ave.
1-step	Reg. 1	10.6	11.9	<b>11.1</b>	11.6	11.4	9.94	10.1	11.0
	Reg. 2	12.5	12.1	13.1	12.6	12.7	<b>7.7</b>	<b>6.4</b>	11.0
	Reg. 3	<b>9.8</b>	<b>10.0</b>	11.5	<b>9.4</b>	<b>10.0</b>	10.0	9.5	<b>10.0</b>
24-step	Reg. 1	14.5	14.4	<b>11.6</b>	14.2	13.3	11.7	10.3	12.9
	Reg. 2	15.4	<b>12.1</b>	12.0	13.0	12.5	<b>11.1</b>	<b>9.4</b>	12.2
	Reg. 3	<b>14.4</b>	12.4	12.8	<b>10.7</b>	<b>10.7</b>	11.9	11.8	<b>12.1</b>
168-step	Reg. 1	15.1	13.5	13.4	15.9	13.6	14.8	<b>12.0</b>	14.0
	Reg. 2	14.9	12.9	12.4	15.0	12.8	14.9	13.5	13.8
	Reg. 3	<b>13.6</b>	<b>12.3</b>	<b>11.8</b>	<b>11.4</b>	<b>10.7</b>	<b>13.4</b>	<b>12.0</b>	<b>12.2</b>

# EVALUATION - LSTM

Table 3: Out-of-sample root mean square error

	Rank	RMSE
1-step	35	<b>1074.8</b>
24-step	30	2245.1
168-step	35	3538.9

Table 4: Out-of-sample mean absolute percentage error computed for each day of the week

Pred.	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Ave.
1-step	<b>4.0</b>	<b>3.3</b>	<b>3.2</b>	<b>3.2</b>	<b>3.0</b>	<b>2.9</b>	<b>4.7</b>	<b>3.5</b>
24-step	6.1	5.2	5.8	5.5	5.1	4.5	6.9	5.6
168-step	14.4	9.3	6.9	11.2	8.6	7.0	10.4	9.7

# CONCLUSIONS



# CONSIDERATIONS



RRR is faster and simpler



LSTM has better performances

RRR potentially interpretable



LSTM is more complex to implement

RRR allows to use cross validation in order to find the best rank



LSTM as NN is not interpretable

RRR achieves poor performances



LSTM tends to overfit the train set, ideally needs more data

# IMPROVEMENTS



**More** continuous data

LEVEL  
UP!



More time



Specific models  
for time series  
forecasting



More powerful  
hardware



THANK YOU