

Firm-Borne Financial Contagion: When Rollover Risk Ripples

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Abstract

A financial network becomes more *resilient to large shocks* when it is split into two weakly connected components. A shock will be contained in the component of the network that it hits. This paper shows that shocks can travel across unconnected components if firms have multiple lenders and both short-term and long-term debt. Because of their short-term debt, firms are subject to rollover risk. When Bank *S* does not rollover (withdraw) their short-term loan the firm might be forced to suspend its long-run debt service with its *other lender* Bank *L*. *The roll-over risk ripples:* a financial shock to one bank (*S*) travels to another bank (*L*), that need not be connected on the interbank market. I show that the shock will reach Bank *L* if three conditions are satisfied. First, the firm's *reliance on short-term debt* must be in an intermediate range. Second, Bank *S* provides a big enough share of the firm's short-term debt. And third, Bank *L* provides a big enough share of the firm's long-term debt. (*JEL D85, E44, G21, G28, L14*)

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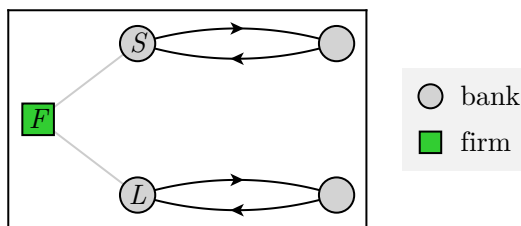


Figure 1: The research question: Can a shock travel from Bank S to Bank L if they have a common borrower Firm F ?

1 Introduction

In financial networks, a shock can travel from one institution to another if these institutions are linked. For studying financial stability it is crucial to understand what the relevant links between financial institutions are. It is well established that shocks can travel if banks are connected via interbank debt, or via correlated asset holdings.¹ In this paper I show that shocks can also travel between banks if they lend to the same firm—a *common corporate borrower*.

The mechanism depends on two key assumptions. First, the firm requires both short-term and long-term funding. (Short-term funding is used to pay workers in advance and long-term funding to finance its capital.) Second, the firm has multiple lenders. Say, Bank S provides some short-term funding and Bank L provides some long-term funding.²

Because of its reliance on short-term funding, the firm is subject to *rollover risk* (as in Acharya, Gale, and Yorulmazer, 2011): When Bank S is hit by a shock, it will refuse to roll-over their *short-term* loan. As a consequence, the firm is short in liquidity. To keep operations running it has to suspend its *long-term* debt service with bank L . The firm's rollover risk ripples: A shock to Bank S travels to the other Bank L via their common corporate borrower.

Propagation depends on two parameters. First, the firm's *reliance of short-term funding* (which is the firm's labor share $1 - \alpha$) needs to be in an intermediate range. Consider the two extreme cases. If $\alpha = 1$, there is no short-term debt and thus no rollover risk. If $\alpha = 0$ there is no long-run debt, so the rollover risk cannot ripple.

¹Interbank debt: e.g. Allen and Gale (2000), Eisenberg and Noe (2001), Elliott, Golub, and Jackson (2014) and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015); correlated (portfolios of) assets: e.g. Caballero and Simsek (2013), Cabrales, Gottardi, and Vega-Redondo (2017). See also the survey by Jackson and Pernoud (2021)

²These assumptions can easily be microfounded. Brunnermeier and Oehmke (2013) who show that banks have an incentive to provide funding of ever shorter maturities (*maturity rat race*) and Kolm, Laux, and Lóránth (2018) who show that it may be optimal for firms to have multiple lenders and multiple maturities (*staggered debt*).

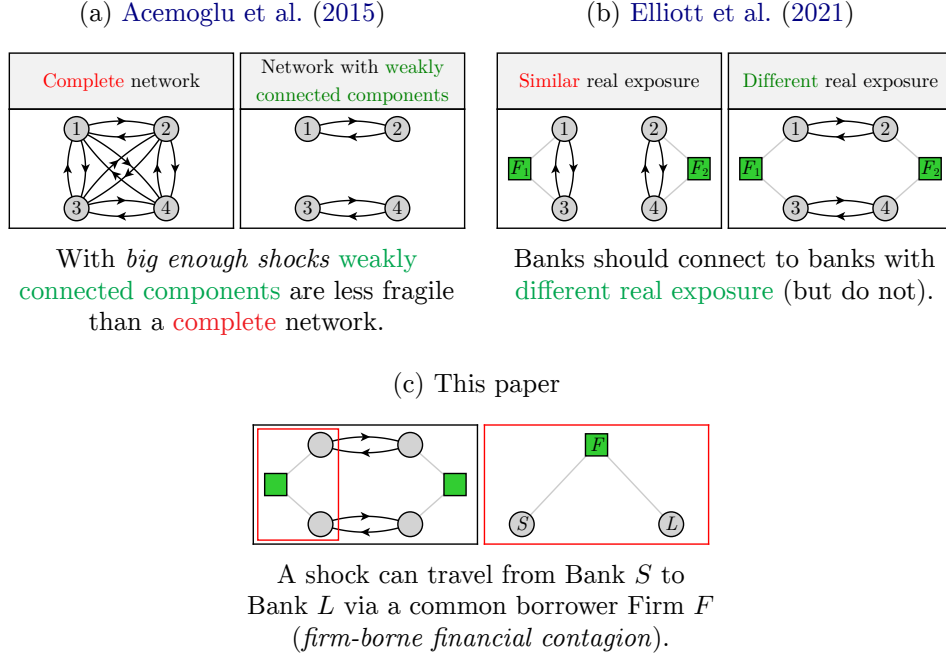


Figure 2: Relationship to the literature on socially optimal network topologies

Second, propagation depends on the *asymmetry of their lenders*. The firms needs to have one Bank S that provides *most of the short-term funding* (to get strong propagation from Bank S to the firm) and another Bank L that provides *most of the long-term funding* (to get strong propagation from the firm to Bank L). In the symmetric case where two banks provide both types of debt in equal shares, there will be *no contagion*.

(Third, it will also depend on the size of the firms—or the importance of the firm as a borrower. But that’s still to be developed.)

CONTRIBUTIONS This paper makes two main contributions. First, it provides a mechanism that links the real economy and the financial sector in two ways. Shocks can travel from the financial sector to the real economy and the other way around. The firm’s suspension of debt service is a real effect of the financial shock.

Second, the paper sheds new light on two important results on socially optimal network topologies. Acemoglu et al. (2015) show that in a *large shock regime* financial stability increases whenever the *interbank* network is split into multiple components. This paper highlights that these partitions should take into account lending relationships with firms. Under some circumstances such a bank-firm-bank links can act like an interbank link. This is illustrated in Panels a and c of Figure 2.

Moreover, [Elliott et al. \(2021\)](#) show that banks want (and empirically have) *similar real exposure*—connected banks tend to have correlated portfolios of loans to firms—even though that is a source of systemic risk. I provide an argument why similar real exposure can in fact improve welfare because it prevents huge shocks from traveling across otherwise unconnected components in the financial system. This is illustrated in Panels [b](#) and [c](#) of Figure [2](#).

RELATIONSHIP TO THE LITERATURE First and foremost, this paper relates to the literature on *systemic risk in financial networks* pioneered by [Allen and Gale \(2000\)](#), [Eisenberg and Noe \(2001\)](#), [Acemoglu et al. \(2015\)](#), [Elliott et al. \(2014\)](#) and [Cabrales et al. \(2017\)](#), amongst others.³ We add to this literature in multiple ways.

First, while most of the literature keeps the bank-firm links very simple—each bank lends to exactly one firm—we study how the nature of the bank-firm network impacts financial stability. One notable exception is [Elliott et al. \(2021\)](#). We contrast their findings by show-casing an alternative mechanism that suggests that such a network topology can have negative side-effects as well. (This is the second contribution.)

Third, we add to the strand of the literature that studies *transmission of systemic risk via the asset side*. While most of the literature has studied shocks to widely held assets (the *fire sales* externality, see for example [Caballero and Simsek, 2013](#)), we study the case where a small number of banks hold the same asset (a firm with multiple lenders, a *common corporate lender*).

Moreover, we complement [Donaldson, Piacentino, and Yu \(2022\)](#) who show that financial network models can yield very different predictions if interbank debt contracts are long-term instead of short-term. Instead of varying the maturity of interbank debt, I vary the maturity of corporate debt. Finally, like [Acemoglu et al. \(2015\)](#) and [Cabrales et al. \(2017\)](#), I am looking for socially optimal network topologies.

This paper builds on insights from the literature on rollover risk. [Acharya et al. \(2011\)](#) and [He and Xiong \(2012\)](#) show how short-term debt creates roll-over risk, which raises default thresholds and then lead to sudden market freezes. [Eisenbach \(2017\)](#) finds that rollover risk leads to inefficient market discipline of banks in both booms and crises. This paper shows that with rollover risk shocks can propagate between financial institutions that are unconnected on the interbank market.

In our model, financial contagion can affect the real economy. This relates to an ever growing *empirical literature* studying the real effects of financial shocks. [Huber \(2018\)](#) shows that lending cuts by a big German

³The literature on systemic risk in financial networks has recently been surveyed by [Jackson and Pernoud \(2021\)](#).

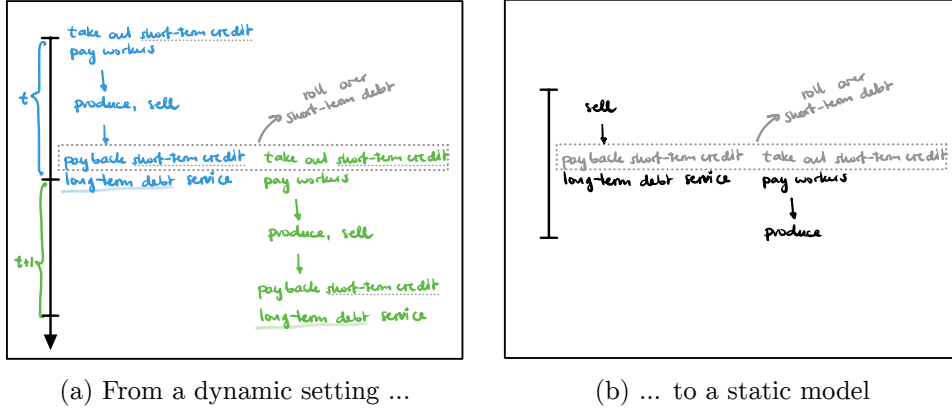


Figure 3: How the firm operates

bank (Commerzbank) lead to lower growth in counties that are more exposed to this bank. [Chodorow-Reich \(2014\)](#) shows that US firms with less healthy lenders had a lower likelihood of receiving a new loan after Lehman's failure. [Chodorow-Reich and Falato \(2022\)](#) show that banks in worse health are more likely to renegotiate loan terms or require faster repayment of longer term loans if borrowers violate *loan covenants* (that is, contractual obligations of the borrower to be transparent and reduce risk). [Cingano, Manaresi, and Sette \(2016\)](#) show that bank lending collapsed following the collapse of the Italian interbank market following the collapse of Lehman. [Martín, Moral-Benito, and Schmitz \(2021\)](#) show that the mortgage boom in Spain had effects on credit supply for the non-housing sector.

2 Model

Our model consists of a finite number of firms and a finite number banks. Banks provide two types of funding (short-term and long-term) to firms. Firms have multiple lenders. The bank-firm network is taken as exogenous. Besides lending to firm, banks lend to each other on the interbank market.

The model extends that of [Acemoglu et al. \(2015\)](#) in three ways. First, firms are modeled as independent entities, not as bank-owned projects. Second, firms require both short-term and long-term funding. Third, the firm can have multiple lenders. The latter two assumptions are microfounded in [Kolm et al. \(2018\)](#) who show that it may be optimal for firms to have multiple lenders and multiple maturities (*staggered debt*) and [Brunnermeier and Oehmke \(2013\)](#) who show that banks have an incentive to provide firms with funding of ever shorter maturities (*maturity rat race*).

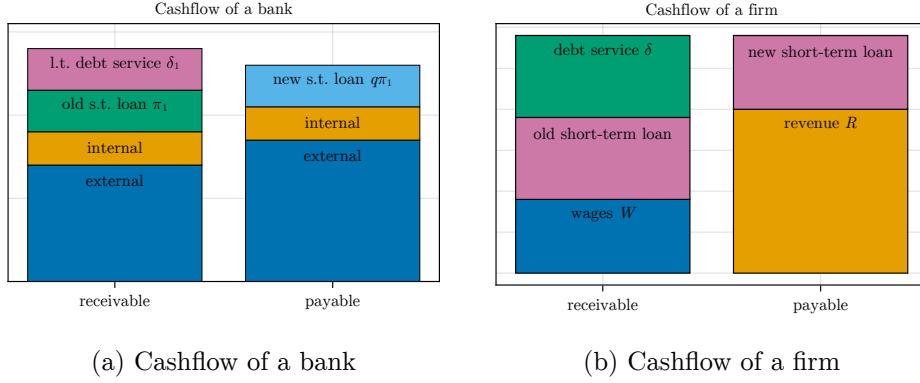


Figure 4: Cashflows of banks and firms

2.1 Firms

Production and sales happens during the day while financial transactions happen at night. The firm's timeline described below is depicted in Figure 3.

During the day a firm produces output Y using capital K and labor L with Cobb-Douglas technology

$$Y = K^\alpha L^{1-\alpha}.$$

The workers receive advance payment of wages w in the morning, before production. In order to finance the workers' wages the firm takes out short-term (one-day) debt $q\pi = wL$ the night before. Total labor costs include wages and financing costs

$$wL + (1 - q)\pi = \pi.$$

Capital K is financed using long-term debt with debt service $\delta_0 = rK$. Long-term debt service is paid at night, once the output is sold.

Each evening, after selling the goods on the market, the firm repays the one-day debt π . At the same time it asks for a new one-day debt to pay her workers the next morning. Effectively, the firm asks to *roll over the short-term debt*. Only after the firm has ensured its working capital for the next day, it services the long-term debt δ_0 .

The firm's optimal profit is zero, with a fraction α used for financing capital and $(1 - \alpha)$ on financing labor.

$$\begin{aligned} \alpha pY &= rK = \delta_0 \\ (1 - \alpha)pY &= wL + (1 - q)\pi = \pi \end{aligned}$$

Short-term debt is $\pi = (1 - \alpha)pY$, long-term debt is $D = K = \frac{\alpha pY}{r}$.

2.2 Banks

Banks are modeled after [Acemoglu et al. \(2015\)](#), with two exceptions. First, I interpret long-term funding as debt (as opposed to equity investment). Troubled firms can temporarily suspend long-term debt service payments. Banks will accept that to avoid default of the firm. Second, the bank has a second type of asset, short-term loans, that it can choose to withdraw (that is, not roll-over) without cost.

Bank i has external assets c (think cash) and liabilities ν (think deposits) that mature in the intermediate period. Banks have interbank assets x_i and interbank liabilities y_i . They provide two types of funding to a firm. The long-term loan pays interest a and has a continuation value of A . It can be partially liquidated at scrap value ζA where $\zeta \geq 0$ (but $\zeta \approx 0$).

The short-term loan pays face value π . The bank can (and usually will) roll-over the short-term loan, paying out $q\pi$ ($0 < q \leq 1$).

Banks' actions The state of the bank is described by the tuple $(c, \nu, a, x, y, \pi, \zeta A)$. Define the funds available

$$h_i = c + x_i + a + \pi.$$

If funds available h_i are sufficient to cover liabilities and the short-term loan, the bank will roll over (corresponds to Condition (0) in Table 1). If not, the bank will take following sequence of steps until its receivables are greater than its payables. First, it will refuse to roll-over the short-term loan ($\chi_i < 1$; Condition (1)). Second, it will liquidate the long-term loan ($\tilde{\ell}_i > 0$; Condition (2)). Third, it will default on its interbank debt (Condition (3)). Fourth, it will default on deposits (Condition (4)).

Table 1: Banks' actions

| | condition | roll over short-term loan | liquidate long-term loan | repay interbank debt |
|-----|----------------------------|------------------------------|-----------------------------|-------------------------|
| (0) | $h_i \geq v + y + q\pi$ | fully | no | fully |
| (1) | $h_i \geq v + y$ | partly | no | fully |
| (2) | $h_i + \zeta A \geq v + y$ | no | partly | fully |
| (3) | $h_i + \zeta A \geq v$ | no | fully | partly |
| (4) | $h_i + \zeta A < v$ | no | fully | no |

It will be helpful to define $[x]_a^b = \begin{cases} a & \text{if } x < a \\ x & \text{if } x \in [a, b] \\ b & \text{if } x > b \end{cases}$.

The if the bank cannot fully roll over its short term debt, it will roll-over as much as possible,

$$\chi_i = \left[\frac{h_i - (v + y)}{q\pi} \right]_0^1.$$

Similarly, if the funds available are not enough to cover its liabilities it will liquidate just as much as needed.

$$\tilde{\ell}_i = \left[\frac{(v + y) - h_i}{\zeta A} \right]_0^1$$

And if even full liquidation is not sufficient to cover its liabilities it will (partially) default on interbank debt, repaying a fraction

$$\sigma_i = \left[\frac{\zeta A + h_i - v}{y} \right]_0^1.$$

HOW DOES A BANK REACT TO A REAL SHOCK?

Definition 1 (A bank's buffer). Let $\Delta = \nu - c$, δ be the contractual debt service on the long-term loan and π be the short-term loan. A bank's buffer β is defined as

$$\beta = \delta + \pi - \Delta.$$

Proposition 1. *The firm does not repay its long-term debt δ_1 with a bank. This bank will not default on its interbank debt if the buffer is bigger then the long-term loan.*

$$\delta < \beta \iff \Delta > \pi.$$

2.3 Interbank market

Banks lend to each other in an interbank market. The matrix (y_{ij}) of liabilities is taken as exogenous. $y_{ij} > 0$ means that bank i has debt with (a promise to pay to) bank j . We assume that the interbank market is *regular* as in [Acemoglu et al. \(2015\)](#),

$$\sum_j y_{ij} = \sum_j y_{ji} = y \quad \text{for all } i.$$

That is, total interbank lending equals total interbank borrowing for each bank. And it is constant across banks.

2.4 Payment equilibrium

For a given $((y_{ij})_{ij}, (c_i, \nu_i)_i)$ the collection $(\sigma_i, \chi_i, \tilde{\ell}_i)_i$ is a payment equilibrium if

$$\begin{aligned} x_i &= \sum_j \sigma_j y_{ji} \\ h_i &= c + x_i + a + \pi \\ \chi_i &= \left[\frac{h_i - (v + y)}{q\pi} \right]_0^1, \\ \tilde{\ell}_i &= \left[\frac{(v + y) - h_i}{\zeta A} \right]_0^1, \\ \sigma_i &= \left[\frac{\zeta A + h_i - v}{y} \right]_0^1. \end{aligned}$$

This definition follows [Acemoglu et al. \(2015\)](#), with the only addition being the fraction rolled-over $(\chi_i)_i$.

3 Financial contagion through rollover risk

Ordinarily, the short-term debt is rolled over in each period because the firm needs to pay its workers before it earns the revenue from production. However, the bank can decide not to roll over their short-term debt at any time. So, what happens if some bank refuses to roll over (a fraction $\tilde{\chi}$ of) the short-term loan? The firm knows that it needs liquidity to pay its workers the following morning. So it retains some of its earnings from the day, suspending part of the debt service payment.

This vulnerability of the firm is called *rollover risk*. If the bank decides to not rollover their short-term debt the firm has a liquidity shortage. In order to pay its workers (and, thus, keep producing) it will suspend its long-run debt service. The *rollover risk ripples*. The firm's long-term lenders will accept this temporary suspension to avoid even higher costs from default of the firm.⁴

3.1 One firm and n banks

Consider the case of n lenders, that might provide short-term funding or long-term funding, or both, to a single firm. The shares in short-term lending are denoted by σ_i and the shares in long-term lending by λ_i , with

$$\sum_{i=1}^n \sigma_i = \sum_{i=1}^n \lambda_i = 1.$$

⁴The long-term lenders know that debt service will be paid after production.

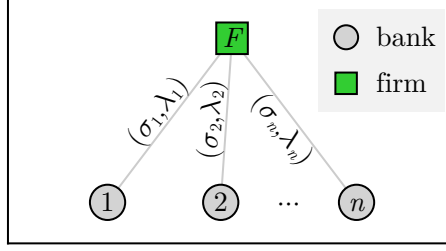


Figure 5: One firm (F) and n banks with short-term lending σ_i and long-term lending λ_i

Suspension of debt service Consider the situation that Bank S is hit by a shock so that it is forced to withdraw its short-term loan $\sigma_S \cdot q\pi$ completely. The firm is short of liquidity, so it will reduce its debt service by

$$\begin{aligned}\Delta\delta &= \min\{\sigma_S \cdot q\pi, \delta\} \\ &= \min\{\sigma_S \cdot q(1 - \alpha), \alpha\}pY.\end{aligned}$$

(The firm can at most suspend its full debt service δ .) Bank L bears fraction λ_L of that loss (their share of long-term lending λ_L),

$$\Delta\delta_L = \lambda_L \min\{\sigma_S \cdot q(1 - \alpha), \alpha\}pY.$$

We see that $\Delta\delta_L$ crucially depends on the importance of Bank S as a short-term lender (σ_S), the importance of Bank L as a long-term lender (λ_L) and the firm's dependence on long-term debt α . σ_S determines the transmission of the shock from Bank S to the firm, and λ_L determines the transmission of the shock from the firm Bank L . It is easy to see that $\Delta\delta_L$ is maximized when $\sigma_S = \lambda_L = 1$ and $\alpha = \frac{q}{1+q}$.

Proposition 2 (Maximal suspension of L 's long-term debt service). *Assume that Bank S withdraws all of their short-term debt. The parameters that maximize the suspension of Bank L 's debt service $\Delta\delta_L$ are $\sigma_S = \lambda_L = 1$ and $\alpha = \frac{q}{1+q}$.*

This result is illustrated in Figure 6. We see that Bank L 's loss is high for intermediate values of α . Consider the extreme cases. When $\alpha = 1$ there is no short-term debt, so there is no rollover risk to begin with. If $\alpha = 0$, there is rollover risk, but there is no long-term debt service that can be suspended.

Pass-through We want to measure how strongly Bank L is affected by the suspension of debt service. In particular, we want to measure the impact on the likelihood of default.

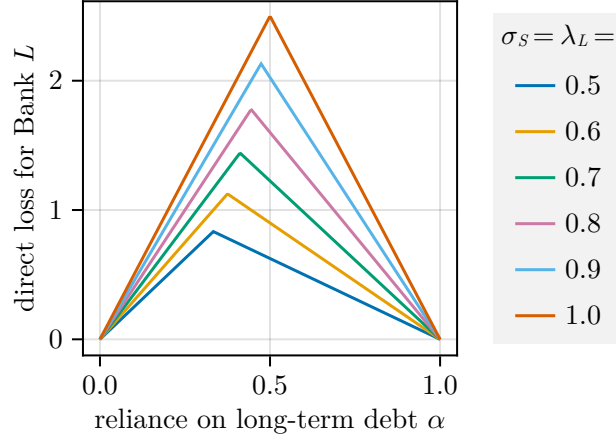


Figure 6: Direct reduction in debt service $\Delta\delta_L$

Note that $\Delta\delta_L$ is a first-round effect. The total effect on Bank L can be higher (if more banks refuse to rollover their short-term loans to the firm in second round effects). But the effect can also be lower because one needs to take into account that Bank L can withdraw their own short-term loan $\sigma_L \cdot q\pi$ to avoid a liquidity shortfall. (This can, in turn, lead to more suspension in debt service.)

In case of liquidity shortage the bank will first withdraw (not rollover) its short-term debt. That is why we define *direct pass-through* to be Bank L 's loss in long-run debt service net of its short-run debt

$$\begin{aligned} & \max\{\Delta\delta_L - \sigma_L q\pi, 0\} \\ &= \max\left\{\lambda_L \min\{\sigma_S q(1 - \alpha), \alpha\} - \sigma_L q(1 - \alpha), 0\right\} pY. \end{aligned}$$

The direct pass-through additionally depends on Bank L 's short-term lending $\sigma_L \in [0, 1 - \sigma_S]$, which determines its buffer from not rolling over. The right panel of Figure 7 shows the direct pass-through. The solid line holds for the case of two banks, where $\sigma_L = \sigma_S$.

The picture suggests that if both Banks are too symmetric (they both provide about half of both short-term and long-term debt) then there is no pass-through. This result holds more generally.

Proposition 3 (Contagion in the asymmetric case). *If there is one “important enough” lender of short-term debt and one “important enough” lender of long-term debt, then there will be financial contagion if α is in an intermediate range.*

Let us now consider the symmetric case, where all banks provide both long-term and short-term funding in equal shares,

$$\sigma_i = \lambda_i = \frac{1}{n} \text{ for all banks } i \in \{1, \dots, n\}.$$

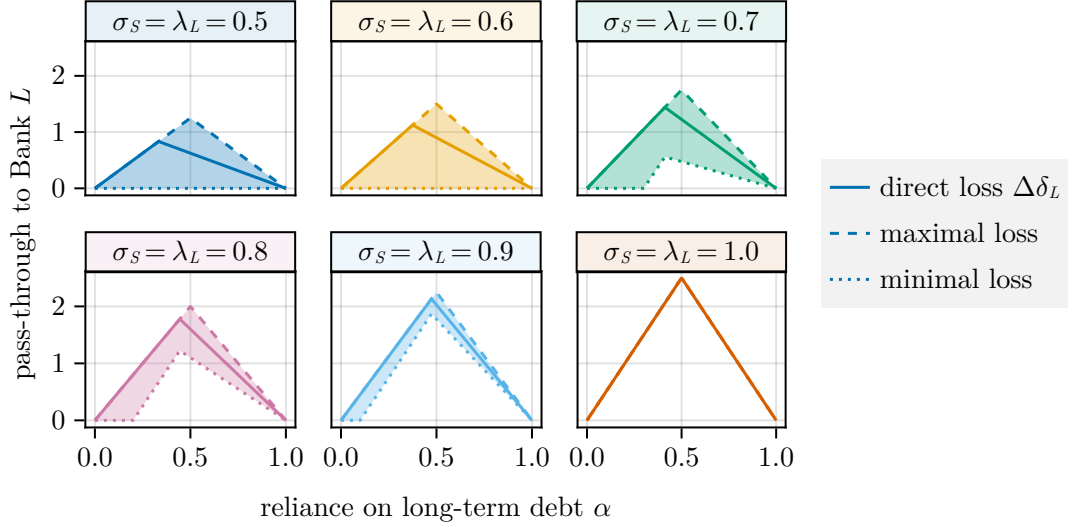


Figure 7: Bounds on the pass-through from Bank S to Bank L

Proposition 4. *In the symmetric, there will be no contagion between banks.*

Proof. The pass-through is positive whenever the loss from suspended long-term debt service exceeds the buffer of short-term loans,

$$\begin{aligned}
 & (\lambda_L \min\{\bar{\sigma}q(1-\alpha), \alpha\} - \sigma_L q(1-\alpha))pY > 0 \\
 & \iff \lambda_L \min\{\bar{\sigma}q(1-\alpha), \alpha\} > \sigma_L q(1-\alpha) \\
 & \iff \frac{1}{n} \min\{\bar{\sigma}q(1-\alpha), \alpha\} > \frac{1}{n} q(1-\alpha).
 \end{aligned}$$

This inequality can never be satisfied because total suspension $\bar{\sigma}$ is bounded above by $\sum \sigma_i = 1$. \square

4 Conclusion

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