Topics in Distributional Macroeconomics

Lecture 1

A. History

B. Introduction

1. Recap finite state Markov Chains
2. The RBC model is a Markov Chain
3. The Aiyagari model is almost the same (but more sophisticated)
4. Demographics : Infinite horizon vs perpetual youth vs life cycle model

C. Debt and Default

1. Recap Finite state Markov chains

state values

state space $S = \{x_1, x_2, \dots, x_n\}$

a Markov chain X_t on S is a sequence of R.V. that satisfy the Markov property

$$\forall t \quad \forall y' \in S \quad Pr(X_{t+1} = y' \mid X_t, X_{t-1}, \dots) = Pr(X_{t+1} = y' \mid X_t)$$

⇒ knowing current state is enough to know all probabilities of future states

⇒ Dynamics are fully determined by $P(x,y) = \Pr(X_{t+1}=y \mid X_t = x)$

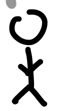
⇒ can view P as a stochastic matrix $\in [0,1]^{n\times n}$ where $P_{ij} = P(x_i, x_j)$ (cf $\mathbb{R}^{n\times n}$)

Example : Productivity can be modeled as a MC

2. (Finite state) Real Business Cycle Model is a MC

(1st Gen Macro)

representative agent (RA)



(z_t)

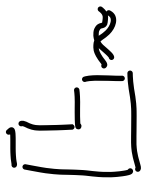
$$\max_{(c_t, \ell_t)} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t)$$

s.t. $k_{t+1} + c_t = k_t(1-\delta+r_t) + \underbrace{z_t \ell_t}_{\text{eff. labor}} w_t$

productivity

$z_t \sim$ finite Markov Chain P_z

rep. firm



$$\max_{K_t, L_t} F(K_t, \tilde{L}_t) - r_t K_t - w_t \tilde{L}_t$$

$$w_t = F_{\tilde{L}}(K_t, \tilde{L}_t)$$

$$r_t = F_K(K_t, \tilde{L}_t)$$

Market clearing: $\begin{cases} K_t = k_t \\ \tilde{L}_t = z_t \ell_t \\ \ell_t = 1 \end{cases}$

Solution : price schedules $r(k_t, z_t)$ $w(k_t, z_t)$
policy functions $c^*(k_t, z_t)$ $k^*(k_t, z_t)$

What's the state of the economy?

(k_t, z_t) ... everything is a function of just that.

Assume : $k_t \in \{k^1, k^2, \dots, k^n\}$

How does the economy evolve?

need to track the evolution of the state

if today $(\tilde{k}_t, \tilde{z}_t)$

then tomorrow $\tilde{k}_{t+1} = k^*(\tilde{k}_t, \tilde{z}_t)$

$$(\tilde{k}_t, \tilde{z}_t) \to \begin{cases} (\tilde{k}_{t+1}, z_1) & \text{with Pr } P_z(\tilde{z}_t, z_1) \\ \vdots & \\ (\tilde{k}_{t+1}, z_m) & \text{with Pr } P_z(\tilde{z}_t, z_m) \end{cases}$$

We only need to know today's state for the probability of tomorrow's state

$$\boxed{(k_t, z_t) \text{ follows a MC on } S = \{k_1, \dots, k_n\} \times \{z_1, \dots, z_n\}}$$

$$\Pr(k_{t+1}, z_t \mid k_t, z_t) = \begin{cases} \Pr(z_{t+1} \mid z_t) & \text{if } k_{t+1} = k^*(k_t, z_t) \\ 0 & \text{o.w.} \end{cases}$$

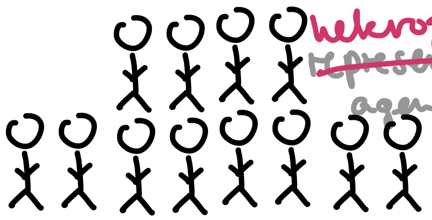
Exercise : simulate the economy (= one path)

Simplified statement :

RBC : simulate one path

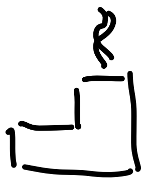
Aiyagari : simulate the whole distribution ($\approx$ many path)

3 (Finite state) Aiyagari model 2nd Gen



~~representative~~ heterogeneous agent (~~R~~HA)

$z_1 \; z_2 \; \dots \; z_m$



rep. firm

$$\max_{(c_t^i, \ell_t^i)} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i, \ell_t^i)$$

$$\text{s.t. } k_{t+1}^i + c_t^i = k_t^i (1 - \delta + r_t) + \underbrace{z_t^i \ell_t^i}_{\text{eff. labor}} w_t$$

productivity

$$\max_{k_t, L_t} F(K_t, \tilde{L}_t) - r_t K_t - w_t \tilde{L}_t$$

$$w_t = F_{\tilde{L}}(K_t, \tilde{L}_t)$$

$$r_t = F_K(K_t, \tilde{L}_t)$$

$z_t^i \sim$ finite Markov Chain P_z

Market clearing: $\begin{cases} K_t = \frac{1}{N} \sum_{i=1}^{N} k_t^i \\ \tilde{L}_t = \frac{1}{N} \sum_{i=1}^{N} z_t^i \ell_t^i \\ \ell_t^i = 1 \end{cases}$ per capita

Solution: price schedules $\quad r(K_t, \tilde{L}_t) \quad w(K_t, \tilde{L}_t)$

policy functions $\quad c^*(k_t, z_t, K_t, \tilde{L}_t) \quad k^*(k_t, z_t, K_t, \tilde{L}_t)$

What's the state of the economy?

$$(k_t^i, z_t^i)_{i=1}^{N}$$

$$k_{t+1}^i = k^*(k_t^i, z_t^i, K_t, \tilde{L}_t)$$

The stationary equilibrium $\quad r, w$ are time-constant

as long as there is no aggregate risk (cf Krusell-Smith)

this economy will have a stationary equilibrium.

How to get there?

either for many i $(k_t^i, z_t^i) \to (k_{t+1}, z_1), \ldots, (k_{t+1}, z_n)$

or (the cleverer way)

recall that an MC is a $Q^* \in [0,1]^{n\cdot m \times n\cdot m}$

given a distribution today π_t,

tomorrows distribution is $\pi_{t+1}' = \pi_t' Q^*$

$\pi_{t+k}' = \pi_t' (Q^*)^k \xrightarrow[\text{under some conditions}]{} \pi_\infty'$ stationary distrib.

$(\pi_\infty' = \pi_\infty' Q^*)$

4. Demographics : Infinitely-lived
Perpetual youth $\rightarrow$ $\pi_1^{alive} = (1-m)$, $\pi_2^{alive} = (1-m)^2$, $\pi_t^{alive} = (1-m)^t$
Lifecycle

Infinitely lived

$$\max_{(c_t)_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\Downarrow$$

$$Q_\beta^*$$

summarize optimal choices and evolution of states

$$\pi_{t+1}' = Q_\beta^* \pi_t'$$

$$\pi_\infty' = Q \pi_\infty'$$

eigenvalue problem

Perpetual youth

$$\max_{(c_t)_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \underbrace{\beta^t \pi_t^{alive}}_{\tilde{\beta}^t = (\beta(1-m))^t} u(c_t)$$

$$\Downarrow$$

$$Q_{\beta(1-m)}^*$$

summarize optimal choices and evolution of states of survivors !

$$\pi_{t+1}' = (1-m) Q_{\beta(1-m)}^* \pi_t' + m \pi_0'$$

$$\pi_\infty' = (1-m) Q \pi_\infty' + m \pi_0'$$

$$\Leftrightarrow (I - (1-m)Q)^{-1} m \pi_0' = \pi_\infty'$$

Lifecycle

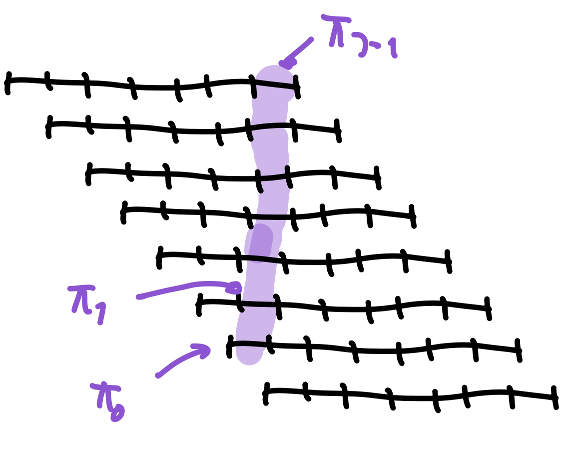
$$\max_{c_t} \mathbb{E}_0 \sum_{t=0}^{J-1} \beta^t u(c_t)$$

$$\Downarrow$$

$$(Q_1^*, Q_2^*, \dots Q_{J-1}^*)$$

$$\pi_{t+1}' = Q_{t+1}^* \pi_t'$$

$$\bar{\pi}_t = \frac{1}{J} \sum_{j=0}^{J-1} \pi_j$$



C Consumer Credit & Default

1. A model of unsecured debt and default

- lifecycle model — sick, divorced, pregnant
- uncertainty: income, expense shocks
- agents can choose to default (= file bankruptcy)
  $\Rightarrow$ debt is relieved, but <u>this comes at a cost</u>

debt income expense shock

$$V_j(d, y, \kappa) = \max_{c, d'} u(c) + \beta \mathbb{E}\Big[\boxed{\max}\{\underbrace{V_{j+1}(d', y', \kappa')}_{\text{repay}}, \underbrace{B_{j+1}(y', \kappa')}_{\text{default}}\} \,\big|\, y, \kappa\Big]$$

$$c + \quad = y - d - \kappa + d' \underbrace{q_j(d', y)}_{\text{price of borrow.}}$$

stigma

$$B_j(y, z) = u(c) - x + \beta \mathbb{E}\left[V_{j+1}(0, y', \kappa') \mid y, \kappa\right]$$

$$\text{s.t.} \quad c(1+\lambda) = (1-\gamma)(y - \varphi)$$

additional costs (phone contracts) — wage garnishment — fixed cost of bankr.

$(x, \lambda, \gamma, \varphi)$ implicit and explicit punishment

3. The cost of borrowing: The bank's arbitrage condition

risk neutral

safe investment vs lending to (j, d', y)

$$\frac{1}{q^s} 1 \stackrel{!}{=} \frac{1}{q_j(d', y)} \big((1 - \theta_j(d', y)) \cdot 1 + \theta_j(d', y)\, \rho(d', y)\big)$$

$\frac{1}{\text{price}} \cdot$ return — $\frac{1}{\text{price}}$ — $\mathbb{E}(\text{return})$ — recovery

$$\rho(d', y) = \mathbb{E}\left(\frac{\gamma y'}{d' + \kappa'} \,\Big|\, y\right)$$

… how much the bank gets in case of bankruptcy

$\theta_j(d', y)$ ... Pr of default next period of somebody with state/choice (j, d', y)

??? solved by numerical iteration

3. Solve using DiscreteDP $\rightarrow Q_1^*, \ldots, Q_{J-1}^*$

4. Look at Exer & Terhilb 2020