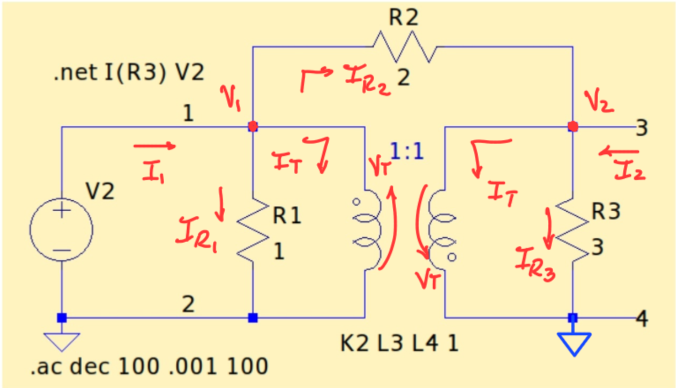
#Ejercicio 1:

Para el siguiente cuadripolo se pide calcular los parámetros Z.



$$T_{TFI} = \begin{bmatrix} 1/a & 0 \\ 0 & -1/a \end{bmatrix} \underset{a=1}{=} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$Z_{11} = \frac{V_1}{I_1}\Big]_{I_2=0} \qquad Z_{12} = \frac{V_1}{I_2}\Big]_{I_1=0}$$

$$Z_{21} = \frac{V_2}{I_1}\Big]_{I_2=0} \qquad Z_{22} = \frac{V_2}{I_2}\Big]_{I_1=0}$$

$$V_T = V_1 = -V_2$$

(1) $I_1 = I_{R_1} + I_T + I_{R_2} = \frac{V_1}{R_1} + I_T + \frac{V_1 - V_2}{R_2} = \frac{V_1}{R_1} + I_T + \frac{V_1 - (-V_1)}{R_2} = \frac{V_1}{R_1} + \frac{2V_1}{R_2} + I_T$

(2) $I_{R_2} = I_T + I_{R_3} \longrightarrow I_T = \frac{2V_1}{R_2} - I_{R_3} = \frac{2V_1}{R_2} - \frac{V_2}{R_3} = \frac{2V_1}{R_2} + \frac{V_1}{R_3}$

(2) $\rightarrow$ (1) : $I_1 = \frac{V_1}{R_1} + \frac{2V_1}{R_2} + \frac{2V_1}{R_2} + \frac{V_1}{R_3} = V_1\left(\frac{1}{R_1} + \frac{4}{R_2} + \frac{1}{R_3}\right)$

$$Z_{11} = \frac{V_1}{I_1}\Big]_{I_2=0} = \left(\frac{1}{R_1} + \frac{4}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(1 + \frac{4}{2} + \frac{1}{3}\right)^{-1} = 0,3\ \Omega$$

por pasividad: $Z_{12} = Z_{21}$

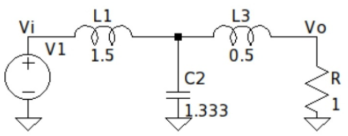
$$\frac{V_1}{I_2}\Big]_{I_1=0} = \frac{V_2}{I_1}\Big]_{I_2=0} = -\frac{V_1}{I_1}\Big]_{I_2=0} = -Z_{11} = -0,3\,\Omega$$

$$Z_{22} = \frac{V_2}{I_2}\Big]_{I_1=0} = -\frac{V_1}{I_2}\Big]_{I_1=0} = Z_{11} = 0,3\ \Omega$$

$$Z = \begin{bmatrix} 0,3 & -0,3 \\ -0,3 & 0,3 \end{bmatrix} = \frac{3}{10}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

#Ejercicio 2:

Dado el siguiente circuito:



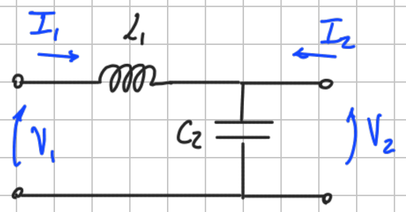
👉 Obtener la transferencia de tensión $\frac{V_o}{V_i}$ por método de cuadripolos (se sugiere referirse a alguno de los métodos de interconexión ya vistos). Ayuda: si $C_2 = \frac{4}{3}$ (se utilizó 1.333 para la simulación), los polos de la transferencia están ubicados sobre una circunferencia de radio unitario

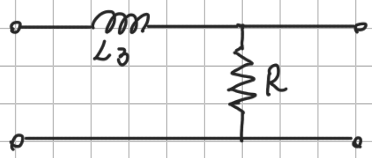
👉 Construya la matriz de admitancia indefinida (MAI) del circuito.

👉 Compute la transferencia de tensión con la MAI.

a) Lo puedo pensar como:

- Una T en cascada con una R
- Dos L en cascada ✓
- El inductor L_1 en cascada con una π

(1) 

(2) 

(1) $\begin{cases} V_1 = Z_{11} I_1 + Z_{12} I_2 \\ V_2 = Z_{21} I_1 + Z_{22} I_2 \end{cases}$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Parámetros T:

$$A = \left.\frac{V_1}{V_2}\right|_{(-I_2)=0} \qquad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{(-I_2)=0} \qquad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

$$Z_1 = \begin{bmatrix} sL_1 + 1/sC_2 & 1/sC_2 \\ 1/sC_2 & 1/sC_2 \end{bmatrix} \longrightarrow T \text{ ¿?}$$

Relaciones entre los parámetros:

$$A = \frac{Z_{11}}{Z_{21}} \qquad B = \frac{\Delta Z_1}{Z_{21}}$$

$$C = \frac{1}{Z_{21}} \qquad D = \frac{Z_{22}}{Z_{21}}$$

$$\Delta Z_1 = L_1/C_2 + 1/s^2C_2^2 - 1/s^2C_2^2 = L_1/C_2$$

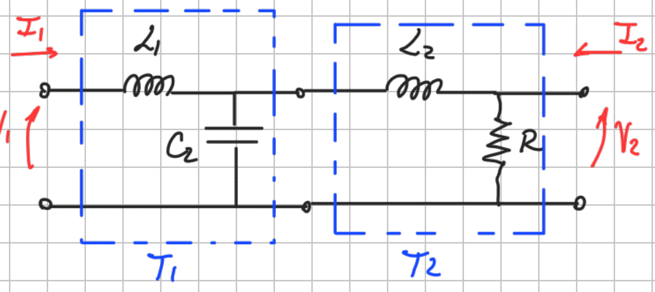
$$T_1 = \begin{bmatrix} s^2L_1C_2 + 1 & sL_1 \\ sC_2 & 1 \end{bmatrix}$$

(2)

$$Z_2 = \begin{bmatrix} sL_3 + R & R \\ R & R \end{bmatrix} \quad ; \quad \Delta Z_2 = sL_3 R + R^2 - R^2 = sL_3 R$$

$$T_2 = \begin{bmatrix} sL_3/R + 1 & sL_3 \\ 1/R & 1 \end{bmatrix}$$

$$T = T_1 \cdot T_2$$



$$T = \begin{bmatrix} s^2 L_1 C_2 + 1 & sL_1 \\ sC_2 & 1 \end{bmatrix} \cdot \begin{bmatrix} sL_3/R + 1 & sL_3 \\ 1/R & 1 \end{bmatrix}$$; me ayudo con la compu

$$T = \begin{bmatrix} \dfrac{(sL_3 + R)(s^2 L_1 C_2 + 1) + sL_1}{R} & sL_3 (s^2 L_1 C_2 + 1) + sL_1 \\ \dfrac{sC_2 (sL_3 + R) + 1}{R} & s^2 L_3 C_2 + 1 \end{bmatrix}$$

$$A = \frac{V_1}{V_2} = \frac{s^3 L_1 L_3 C_2 + s^2 R L_1 C_2 + s(L_1 + L_3) + R}{R}$$

$$\frac{V_0}{V_i} = \frac{V_2}{V_1} = \frac{R}{L_1 L_3 C_2 \left(s^3 + s^2 R/L_3 + s \dfrac{(L_1 + L_3)}{L_1 L_3 C_2} + \dfrac{R}{L_1 L_3 C_2} \right)}$$

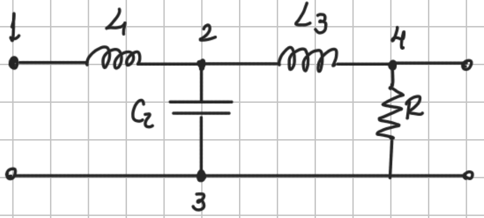
$$T(s) = \frac{V_0}{V_i} = \frac{\dfrac{R}{L_1 L_3 C_2}}{s^3 + s^2 R/L_3 + s \dfrac{(L_1 + L_3)}{L_1 L_3 C_2} + \dfrac{R}{L_1 L_3 C_2}}$$

$$\begin{cases} R = 1 \\ L_1 = 1{,}5 \\ L_3 = 0{,}5 \\ C_2 = 4/3 \end{cases}$$

$$T(s) = \frac{1}{s^3 + 2s^2 + 2s + 1} = \frac{1}{(s+1)(s^2 + s + 1)}$$

Es un LP Butterworth de 3er orden

b)



$$Y = \begin{bmatrix} Y_{12}+Y_{13}+Y_{14} & -Y_{12} & -Y_{13} & -Y_{14} \\ -Y_{12} & Y_{12}+Y_{23}+Y_{24} & -Y_{23} & -Y_{24} \\ -Y_{13} & -Y_{23} & Y_{13}+Y_{23}+Y_{34} & -Y_{34} \\ -Y_{14} & -Y_{24} & -Y_{34} & Y_{14}+Y_{24}+Y_{34} \end{bmatrix}$$

$$Y = \begin{bmatrix} 1/sL_1 & -1/sL_1 & 0 & 0 \\ -1/sL_1 & 1/sL_1+sC_2+1/sL_3 & -sC_2 & -1/sL_3 \\ 0 & -sC_2 & sC_2+1/R & -1/R \\ 0 & -1/sL_3 & -1/R & 1/sL_3+1/R \end{bmatrix}$$

c) $A_{mn}^{ij} = \operatorname{sgn}(i-j)\operatorname{sgn}(m-n)\dfrac{Y_{ij}^{mn}}{Y_{mn}^{mn}} = \dfrac{V_{ij}}{V_{mn}}$

filas

columnas

$i, j = 4, 3$ (puerto de salida $V_{43} = V_o$)

$m, n = 1, 3$ (puerto de entrada $V_{13} = V_i$)

$$Y_{43}^{13} = (-1)^{11}\left[(-1/sL_1)(-1/sL_3) - 0\right] = -1/s^2L_1L_3$$

$$Y_{13}^{13} = (-1)^{8}\left[1/s^2L_1L_3 + 1/sL_1R + C_2/L_3 + sC_2/R + \cancel{1/s^2L_3^2} + 1/sL_3R - \cancel{1/s^2L_3^2}\right]$$

$$Y_{13}^{13} = \frac{s^3L_1L_3C_2 + s^2L_1C_2R + s(L_1+L_3) + R}{s^2L_1L_3R}$$

$\operatorname{sgn}(i-j) = \operatorname{sgn}(4-3) = 1$

$\operatorname{sgn}(m-n) = \operatorname{sgn}(1-3) = -1$

$$\frac{V_{43}}{V_{13}} = \frac{V_o}{V_i} = (-1)\frac{(-1/s^2L_1L_3)}{\dfrac{s^3L_1L_3C_2 + s^2L_1C_2R + s(L_1+L_3) + R}{s^2L_1L_3R}}$$

$$T(s) = \frac{R}{s^3L_1L_3C_2 + s^2L_1C_2R + s(L_1+L_3) + R} = \frac{\dfrac{R}{L_1L_3C_2}}{s^3 + s^2\dfrac{R}{L_3} + s\dfrac{(L_1+L_3)}{L_1L_3C_2} + \dfrac{R}{L_1L_3C_2}}$$

verifica con la anterior ✓

- Bonus : impedancia de entrada con MAI.

$$Z_{mn} = \frac{V_{mn}}{I_{mn}} = \frac{Y_{mn}^{mn}}{Y_n^n}$$

filas

columnas

$m, n = 1, 3$

$$\hookrightarrow Y_{13}^{13} = \frac{s^3 L_1 L_3 C_2 + s^2 L_1 C_2 R + s(L_1 + L_3) + R}{s^2 L_1 L_3 R}$$

$$Y_3^3 = (-1)^6 \left[(1/sL_1) \frac{s^3 L_1 L_3 C_2 + s^2 L_1 C_2 R + s(L_1 + L_3) + R}{s^2 L_1 L_3 R} - (-1/sL_1)(-1/s^2 L_1 L_3 - 1/sL_1 R) \right]$$

$$Y_3^3 = \frac{1}{sL_1} \left[\frac{s^3 L_1 L_3 C_2 + s^2 L_1 C_2 R + s(L_1 + L_3) + R}{s^2 L_1 L_3 R} + \frac{-sL_3 - R}{s^2 L_1 L_3 R} \right]$$

$$Y_3^3 = \frac{1}{sL_1} \quad \frac{s^3 L_1 L_3 C_2 + s^2 L_1 C_2 R + sL_1}{s^2 L_1 L_3 R} = \frac{s^2 L_3 C_2 + s C_2 R + 1}{s^2 L_1 L_3 R}$$

$$Z_{13} = Z_e = \frac{\dfrac{s^3 L_1 L_3 C_2 + s^2 L_1 C_2 R + s(L_1 + L_3) + R}{s^2 L_1 L_3 R}}{\dfrac{s^2 L_3 C_2 + s C_2 R + 1}{s^2 L_1 L_3 R}}$$

$$Z_e = \frac{s^3 L_1 L_3 C_2 + s^2 L_1 C_2 R + s(L_1 + L_3) + R}{s^2 L_3 C_2 + s C_2 R + 1} \qquad \begin{cases} R = 1 \\ C_2 = 4/3 \\ L_1 = 1,5 \\ L_3 = 0,5 \end{cases}$$

$$Z_e = \frac{s^3 + 2s^2 + 2s + 1}{\frac{2}{3} s^2 + \frac{4}{3} s + 1}$$