HOJA Nº 1
FECHA

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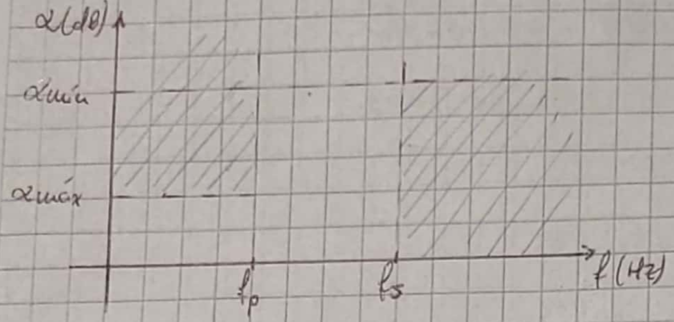
▸ Tarea semanal 3



$\alpha_{máx}$ [dB]	$\alpha_{mín}$ [dB]	f_p [Hz]	f_s [Hz]
1	12	1500	3000

$$\Omega_\omega = 2\pi f_p$$

$$\omega_s = \frac{2\pi f_s}{\Omega_\omega} = \frac{f_s}{f_p} = 2$$

$$\omega_p = \frac{2\pi f_p}{\Omega_\omega} = 1$$

1)
$$\varepsilon^2 = 10^{\alpha_{máx}/10} - 1 = 0{,}25892$$

$$\alpha_{mín_n} = 10 \log(1 + \varepsilon^2 \omega_s^{2n}) = 10 \log(1 + \varepsilon^2 2^{2n})$$

$$\alpha_{mín_2} = 7{,}11 \text{ dB}$$

$$\alpha_{mín_3} = 12{,}44 \text{ dB} \rightarrow \text{El filtro será de orden } n = 3$$

$$|T(\omega)|^2 = \frac{1}{1+\varepsilon^2 \omega^{2\cdot 3}} = \left.\frac{1}{1+\varepsilon^2\omega^6}\right|_{\omega = \frac{s}{j}} = T(s)\,T(-s)$$

$$|T(s)|^2 = \frac{1}{1+\varepsilon^2 \frac{s^6}{j^3 j^3}} = \frac{1}{1-\varepsilon^2 s^6} = \frac{\frac{1}{\varepsilon^2}}{\frac{1}{\varepsilon^2} - s^6} = T(s)\,T(-s)$$

$$\frac{\frac{1}{\varepsilon^2}}{\frac{1}{\varepsilon^2} - s^6} = \frac{c}{s^3 + as^2 + bs + c}\;\frac{c}{-s^3 + as^2 - bs + c}$$

$$c^2 = 1/\varepsilon^2 \rightarrow \boxed{c = 1/\varepsilon}$$

$$-bs^4 + a^2 s^4 - bs^4 = 0 \rightarrow a^2 = 2b \rightarrow a = \sqrt{2b} \rightarrow \boxed{a = 2\,\varepsilon^{-1/3}}$$

$$acs^2 - b^2 s^2 + acs^2 = 0 \rightarrow ac = b^2 \rightarrow b^2 = \sqrt{2b}\,c$$

$$b^4 = 2b\,c^2$$

$$b^3 = 2c^2$$

$$b = \frac{2}{\varepsilon^{2/3}} \rightarrow \boxed{b = 2\varepsilon^{-2/3}}$$

NOTA

$$\begin{cases} a = 2\varepsilon^{-1/3} \\ b = 2\varepsilon^{-2/3} \\ c = 1/\varepsilon \end{cases}$$

$$T(s) = \frac{c}{s^3 + a s^2 + b s + c}$$

$$T(s) = \frac{1{,}96}{s^3 + 2{,}50\, s^2 + 3{,}14\, s + 1{,}96}$$

Con alguna calculadora hallo las raíces:

$S_1 = -1{,}252$

$S_2 = -0{,}626 + j1{,}084$

$S_3 = -0{,}626 - j1{,}084$

$T(s) = 1{,}96 \cdot T_1(s) \; T_2(s)$, donde:

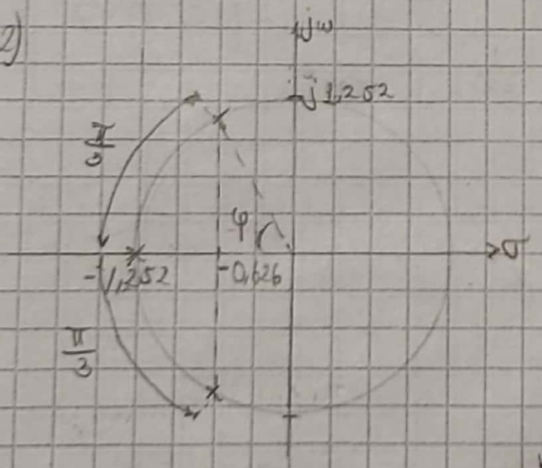
$$T_1(s) = \frac{1}{s + 1{,}252}$$

$$T_2(s) = \frac{1}{(s + 0{,}626 - j1{,}084)(s + 0{,}626 + j1{,}084)}$$

$$T_2(s) = \frac{1}{s^2 + 1{,}252\, s + 1{,}567}$$

$$T(s) = \frac{1{,}96}{(s + 1{,}252)(s^2 + 1{,}252\, s + 1{,}567)} \quad ; \quad \omega_0 = 1{,}252$$

2)



$$T(\omega) = T(s)\Big|_{s=j\omega} = \frac{1{,}96}{(1{,}252 + j\omega)(-\omega^2 + 1{,}567 + j\,1{,}252\,\omega)}$$

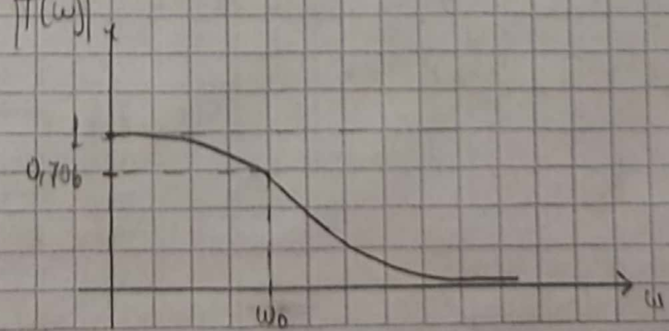
$$T(0) = \frac{1{,}96}{(1{,}252)(1{,}567)} \simeq 1$$

$$T(\omega_0) = 0{,}706$$

$$T(\omega \to \infty) = 0$$

$$Q = \frac{1}{2\cos(\varphi)} = \frac{1}{2\cos(\pi/3)}$$

$Q = 1$



NOTA

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3) $T(s) = T_1(s) \cdot T_2(s)$, donde ahora:

$$T_1(s) = \frac{\omega_0}{s+\omega_0}$$

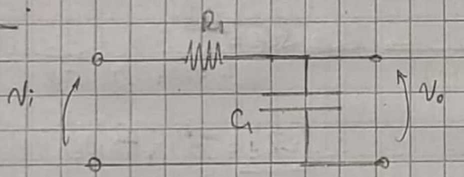
$$T_2(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

siendo $\omega_0 = 1,25$

$$T(s) = \frac{1,25}{s+1,25} \quad \frac{\frac{1,96}{1,25}}{s^2+1,25s+1,56}$$

$$T(s) = \frac{1,25}{(s+1,25)} \quad \frac{1,56}{(s^2+1,25s+1,56)}$$

$T_1(s)$:



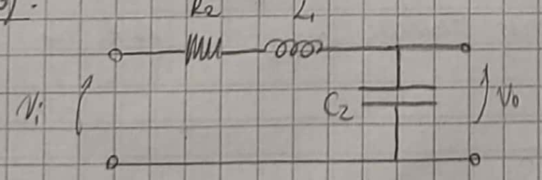
$$V_o = V_i \frac{1/sC_1}{1/sC_1 + R_1}$$

$$T_1(s) = \frac{1/R_1C_1}{s + 1/R_1C_1} = \frac{\omega_0}{s+\omega_0} = \frac{1,25}{s+1,25}$$

$\omega_0 = \frac{1}{R_1C_1} = 1,25$ normalizo en impedancia : $\Omega_Z = R_1 = 1 \rightarrow \frac{1}{C_1} = 1,25$

$$\begin{cases} C_1' = 0,8 \\ R_1' = 1 \end{cases}$$

$T_2(s)$:



$$V_o = V_i \frac{1/sC_2}{1/sC_2 + sL_1 + R_2} = V_i \frac{1}{s^2L_1C_2 + sC_2R_2 + 1}$$

$$T_2(s) = \frac{1/C_2L_1}{s^2 + s\frac{R_2}{L_1} + \frac{1}{C_2L_1}} = \frac{1,56}{s^2+1,25s+1,56}$$

$$\begin{cases} \omega_0^2 = \frac{1}{C_2L_1} = 1,56 \\ \frac{R_2}{L_1} = 1,25 \end{cases}$$

Normalizo en impedancia :

$\Omega_Z = R_2 = 1 \rightarrow L_1 = \frac{1}{1,25} = 0,8$

$C_2 = \frac{1}{1,56\, L_1} = 0,8$

$$\Rightarrow \begin{cases} R_2' = 1 \\ L_1' = 0,8 \\ C_2' = 0,8 \end{cases}$$

NOTA

Desnormalizo para simular:

$$C_1 = \frac{0{,}8}{\Omega_Z\,\Omega_\omega} = \frac{0{,}8}{1\;2\pi f_p} = \frac{0{,}8}{2\pi}$$

$$R = R'\,\Omega_Z = 1 \qquad = 1$$

↳ para verlo en una frecuencia unitaria

$$C_2 = \frac{0{,}8}{2\pi}$$

$$L_1 = 0{,}8\,\frac{1}{2\pi f_p} = \frac{0{,}8}{2\pi}$$

4) Tengo que hallar la nueva Ω_Z:

$$100n = \frac{0{,}8}{\Omega_Z\,.\,\Omega_\omega} = \frac{0{,}8}{\Omega_Z(2\pi\,f_p)} \rightarrow \Omega_Z = \frac{0{,}8}{100n\;2\pi\;1{,}5k}$$

$$\boxed{\Omega_Z = 848{,}83\,\Omega}$$

$$C_1 = C_2 = 100nF$$

$$R_1 = R_2 = R'\,\Omega_Z = 848{,}83\,\Omega$$

$$L_1 = L'\,\frac{\Omega_Z}{\Omega_\omega} = 0{,}8\,\frac{848{,}83}{2\pi\cdot 1{,}5k} = 72{,}05\,mHy$$

5) Sigo con el RC serie con salida en el capacitor para el polo simple, la configuración que me va a dar los polos complejos conjugados puede ser un Ackerberg-Mossberg.

$$T(s) = T_{A-M}(s)\,.\,T_{RC}(s) = \underbrace{-\frac{R_3}{R_1}}_{K}\left(\frac{\omega_0^2}{s^2 + s\,\frac{\omega_0}{Q} + \omega_0^2}\right)\left(\frac{\omega_0}{s+\omega_0}\right)$$

Para que sea igual a la transferencia anterior; del A-M se tiene:

$$|K| = 1 \rightarrow \boxed{R_3 = R_1}$$

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$$\frac{\omega_0}{Q} = \frac{1}{R_2 C} \xrightarrow{\boxed{Q=1}} \omega_0 = \frac{1}{R_2 C} = 1{,}25 \rightarrow \boxed{C = \frac{1}{1{,}25 R_2}}$$

$$\omega_0^2 = \frac{1}{R_3^2 C^2} = 1{,}56 \rightarrow \frac{1}{R_3 C} = 1{,}25 \Rightarrow \boxed{R_3 = R_2}$$

$$\Rightarrow \boxed{R_1 = R_2 = R_3 = R_4}$$

Del RC serie:

$$\omega_0 = \frac{1}{R_5 C_5} = 1{,}25 \rightarrow C_5 = \frac{1}{1{,}25 R_5} \underset{R_5 = R_2}{=} \frac{1}{1{,}25 R_2} = C$$

$$\begin{cases} R_1 = R_2 = R_3 = R_4 = R_5 \\ C = C_5 \end{cases} \Rightarrow \Omega_Z = R_2 \quad \begin{cases} R_1 = R_2 = R_3 = R_4 = R_5 = 1 \\ C = C_5 = 0{,}8 \frac{1}{R_2} \Omega_Z = 0{,}8 \end{cases}$$

Desnormalizando con $\Omega_Z = 848{,}83\ \Omega$ porque $C = 100\ nF$ $\rightarrow \begin{cases} R = 848{,}83\ \Omega \\ C = C_5 = 100\ nF \end{cases}$

*** Bonus:**

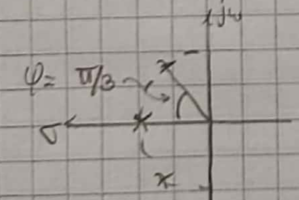
"Hago pasar" un filtro de MP como Butterworth:

$$|T(\omega)|^2 = \frac{1}{1+\varepsilon^2 \omega^{2n}} = \frac{1}{1+\left(\frac{\omega}{\varepsilon^{1/n}}\right)^{2n}} = \frac{1}{1+\omega_B^{2n}}$$

y lo encaro como un Butterworth porque "$\varepsilon^2 = 1$"

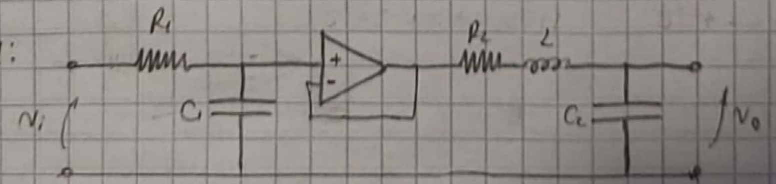
$$\Omega_\omega = \omega_p \ ; \quad \Omega_\omega' = \omega_p\, \varepsilon^{-1/n} = \omega_B \ ; \quad \omega_B = 1 \cdot (0{,}5)^{-1/3} = 1{,}25 \text{ rad/seg}$$

Del diagrama de P y Z:



$$T(s) = \frac{1}{(s+1)(s^2 + s\, 2\cos(\varphi) + 1)} = \frac{1}{(s+1)(s^2+s+1)} \ , \quad \omega_0 = 1 \quad \frac{\omega_0}{Q} = 1$$

Misma red:



$$T(s) = \frac{1/R_1C_1}{s + 1/R_1C_1} \cdot \frac{1/C_2L_1}{s^2 + s\frac{R_2}{L_1} + \frac{1}{C_2L_1}}$$

$$\omega_0 = \frac{1}{R_1 C_1} = 1 \rightarrow \boxed{C_1 = \frac{1}{R_1}}$$

NOTA

$$\frac{\omega_0}{Q} = \frac{R_2}{L_1} = 1 \rightarrow \boxed{R_2 = L_1}$$

Propongo $R_1 = \Omega_Z$ y $R_1 = R_2$

$$\omega_0^2 = \frac{1}{C_2 L_1} = 1 \rightarrow \boxed{C_2 = \frac{1}{L_1} = \frac{1}{R_2}}$$

$$\begin{cases} C_1' = 1 \\ C_2' = 1 \\ L_1' = 1 \end{cases}$$

De antes, la Ω_Z impuesta por los capacitores de 100 nF, $\Omega_Z = 848,83\ \Omega$

Desnormalizo, dividiendo también por la ω_B para el reescalamiento del eje frecuencial.

$$R_1 = R_2 = 848,83\ \Omega$$

$$L_1 = \frac{L'}{\Omega_\omega} \Omega_Z \frac{1}{\omega_B} = \frac{1}{2\pi\, 1,5k} \; 848,83 \; \frac{1}{1,25} = 72,05\ mHy$$

$$C_1 = C_2 = 100\ nF$$

Como se puede ver, el valor de los componentes es el mismo.