Machine Learning in Geophysics

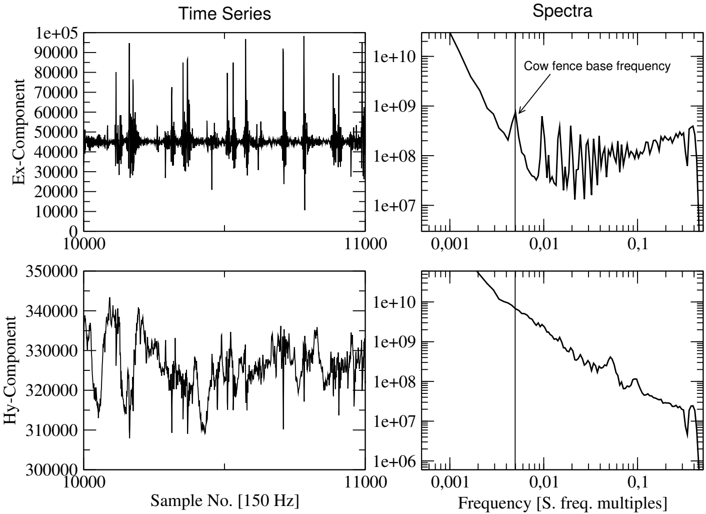
Lecture 8 – Neural network fundamentals

Magnetotellurics

$$\begin{pmatrix} E_x(\omega) \\ E_y(\omega) \end{pmatrix} = \frac{1}{\mu_0} \underbrace{\begin{pmatrix} Z_{xx}(\omega) & Z_{xy}(\omega) \\ Z_{yx}(\omega) & Z_{yy}(\omega) \end{pmatrix}}_{\text{complex impedance tensor}} \begin{pmatrix} B_x(\omega) \\ B_y(\omega) \end{pmatrix}$$

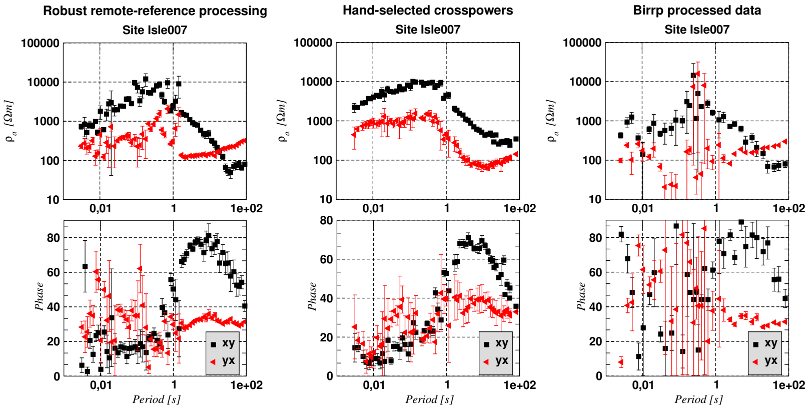
- Inductive, passive electromagnetic method
- Electric field and magnetic field are measured simultaneously
- Recorded data is windowed and transformed into frequency domain
- Local noise sources make far-field assumption invalid

Example of contaminated time series and spectra



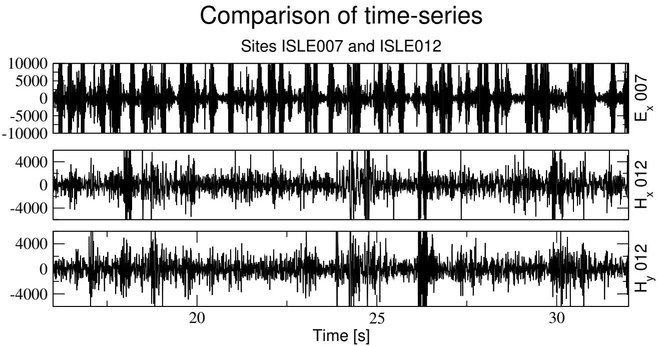
Strong cow-fence spikes in time series, spectra show strong

Remote-reference processing of contaminated data



Curves should be smooth with small error bars

Basic idea

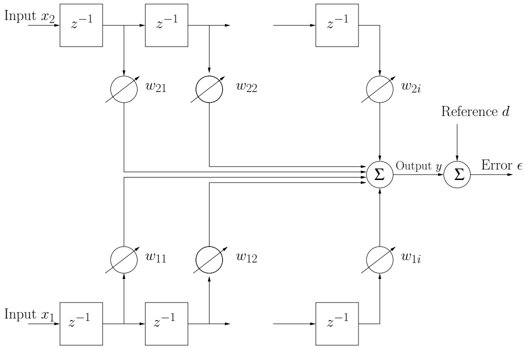


- We have several synchronously recording sites
- Noise free magnetic field is highly correlated but not identical
- Everything is affected by noise to a different degree
- Need to extract the correlated parts

Noise reduction using adaptive filters

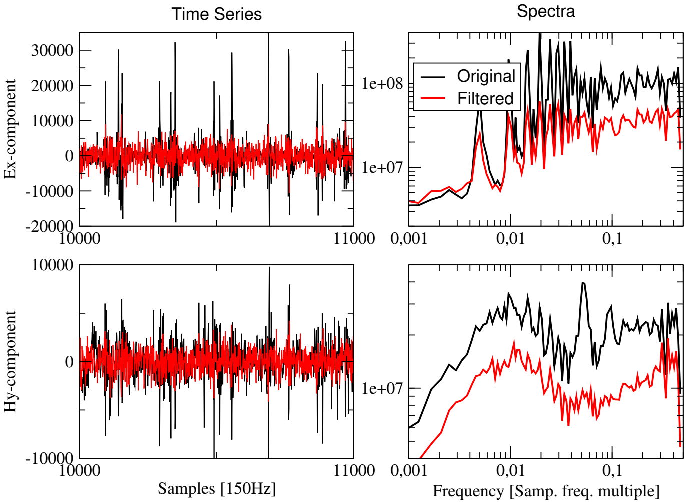
- Adaptive filters are widely used in a range of signal-processing tasks from electrical engineering to medicine
- Provide flexible design that can adjust to problem
- LMS-adaptive linear combiner simplest adaptive filter
- Assume stationary or slowly varying non-stationarity
- Can be extended to non-linear domain $\Rightarrow$ neural networks

Conceptional diagram of a two-channel LMS-Filter



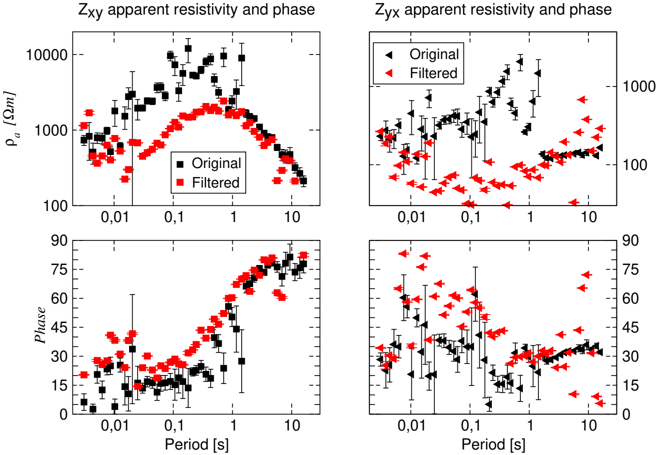
H_x and H_y of remote site used to clean up local data (E_x, E_y, H_x and H_y).

Application to a noise contaminated site



Filter removes spikes and reduces spectral signature

Comparison of transfer functions

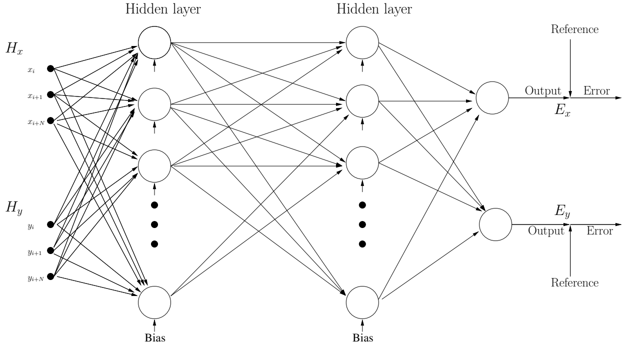


Improvement in Z_{xy}, maybe apparent resistivity Z_{yx}, suspicious phase Z_{yx}

Summary

- Adaptive combiner is linear part of neural network neuron
- Used as adaptive filter to remove noise
- Implementation relatively simple, some important tuning parameters
- Can be used in noise removal for geophysical time-series

Neural networks



$$net_i = \sum_{j=1}^{N} w_{ij} x_j + b_i$$

$$\varphi(net_i) = \tanh\left(\frac{1}{2} net_i\right)$$

Backpropagation algorithm

Prediction error

$$e_i(n) = d_i(n) - y_i(n). \tag{1}$$

$$\begin{aligned} w_{ij}^l(n+1) &= w_{ij}^l(n) + \mu x_j(n)\delta_i^l(n), \qquad (2) \\ b_i^l(n+1) &= b_i^l(n) + \mu \delta_i^l(n), \qquad (3) \end{aligned}$$

$$\delta_i^l(n) = \begin{cases} \varphi'\left(net_i^l\right)\ [d_i(n) - y_i(n)] & l = M \\ \varphi'\left(net_i^l\right)\ \sum_k w_{ki}\delta_k^{l+1}(n) & 1 \leq l < M \end{cases}. \tag{4}$$

Activation functions

Sigmoid:

$$\varphi(z) = \frac{1}{1 + \exp(-z)}$$

tanh:

$$\varphi(z) = \tanh\left(\frac{1}{2}z\right)$$

Relu:

$$\varphi(z) = \max(0, z)$$

Step:

$$\varphi(z) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$$