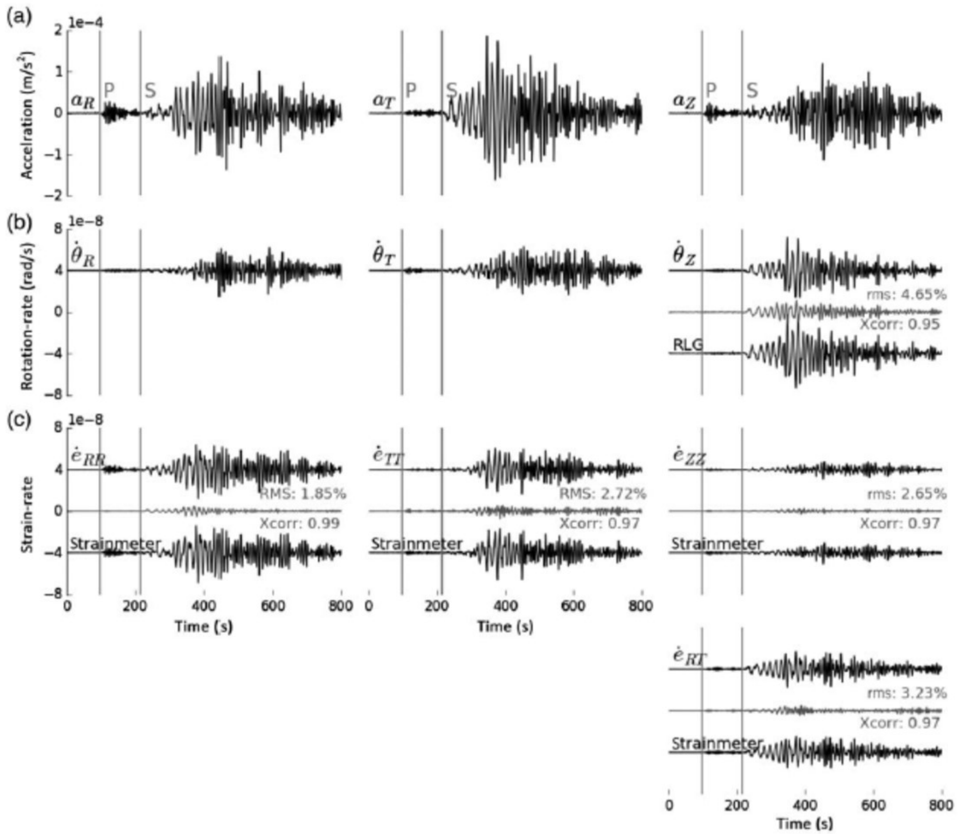
Quantitative Seismology

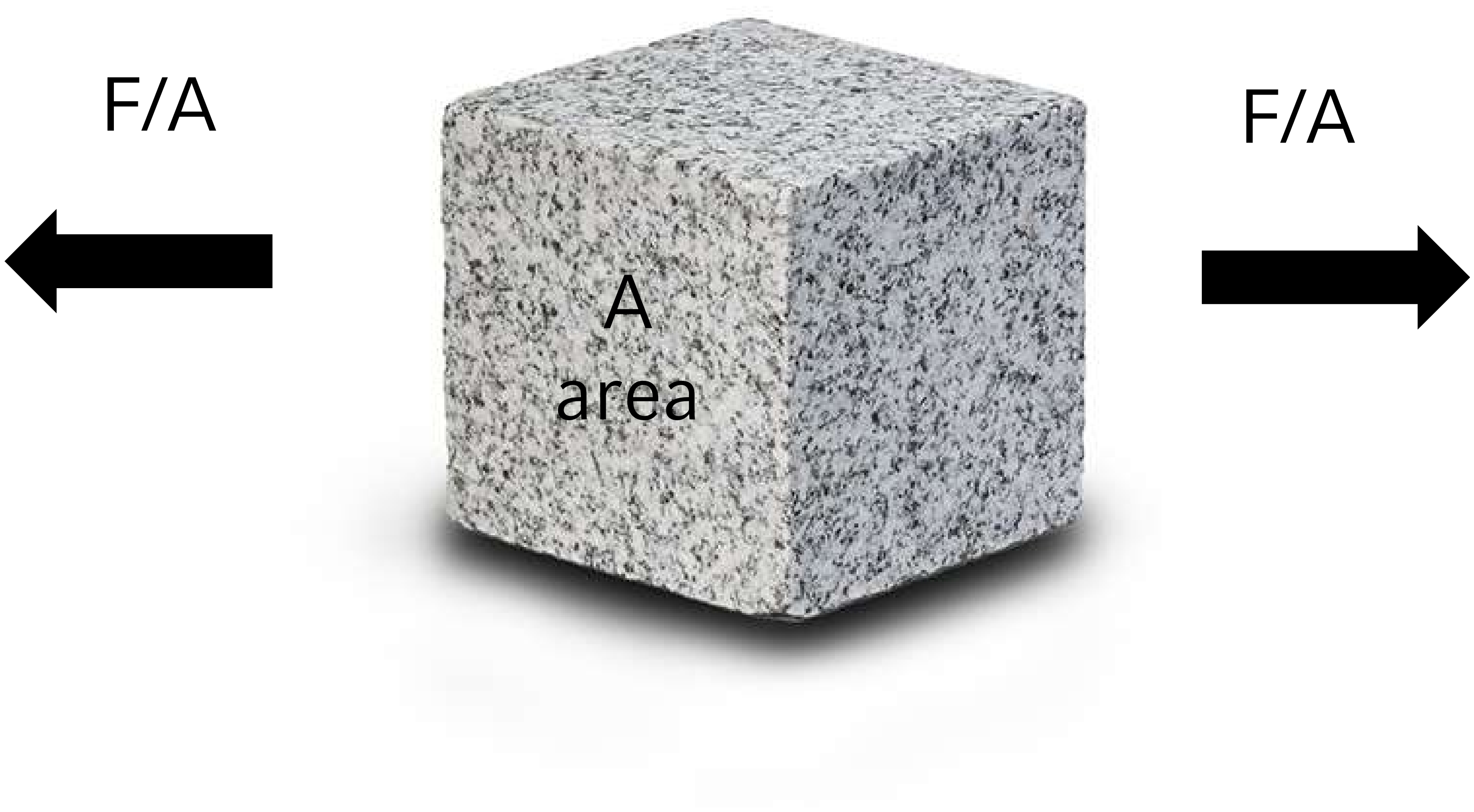
Wavefield gradient: displacement, strain, and rotation

- **Deformation** of an elastic body
- **Gradient**: strain and rotation tensor
- What is a **tensor, tensor rotation**
- Motion of a measurement point
- **Plane wave** analysis
- **Array-derived** wavefield gradient
- Applications:
  - Tomography
  - Complete ground motion
  - Earthquake physics
  - Planetary seismology

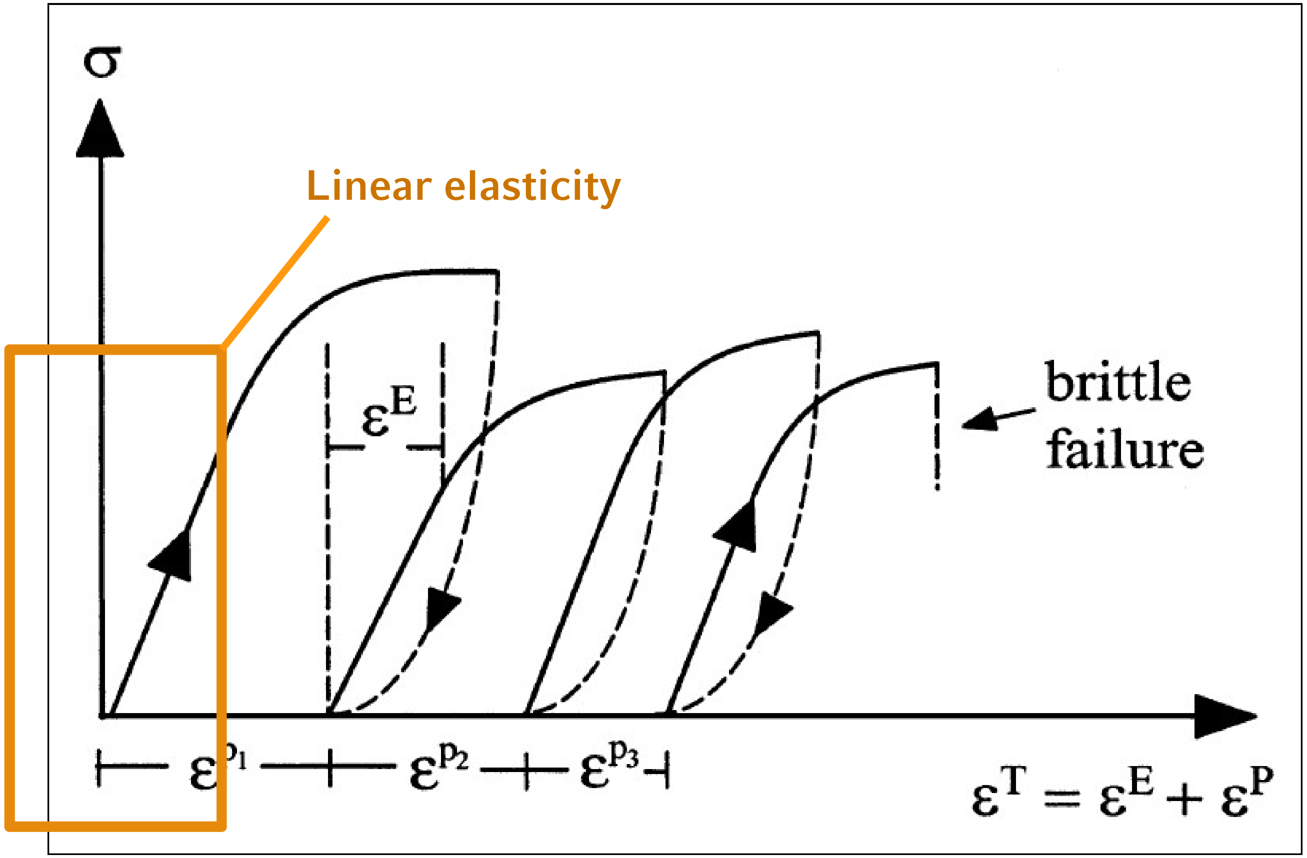


From Donner et al., SRL, 2017

Stress and strain



Damage, linearity, plasticity

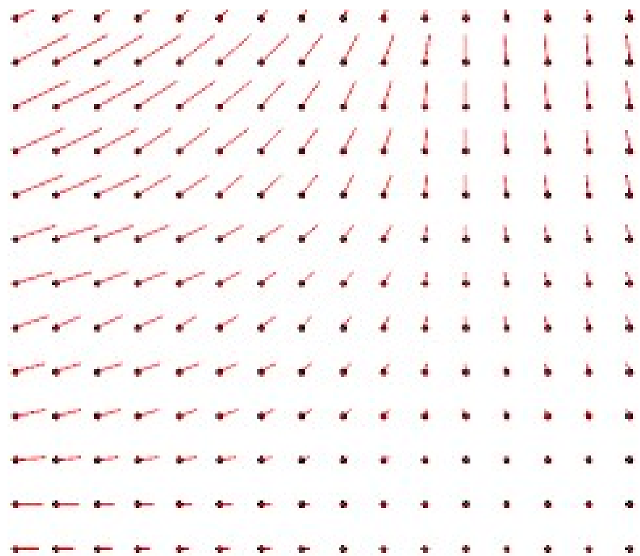


ε	strain
σ	stress
P	plasticity
T	total
E	elastic

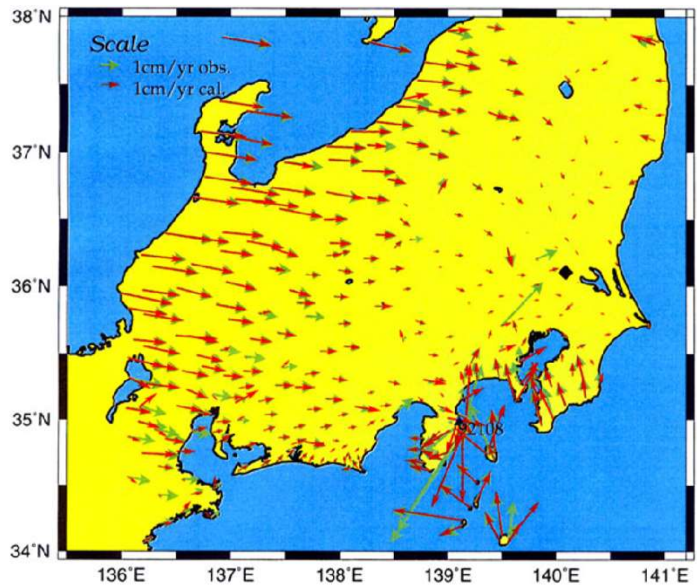
Vector fields (here: displacement)

$u_i(x,y,z)$

Theory

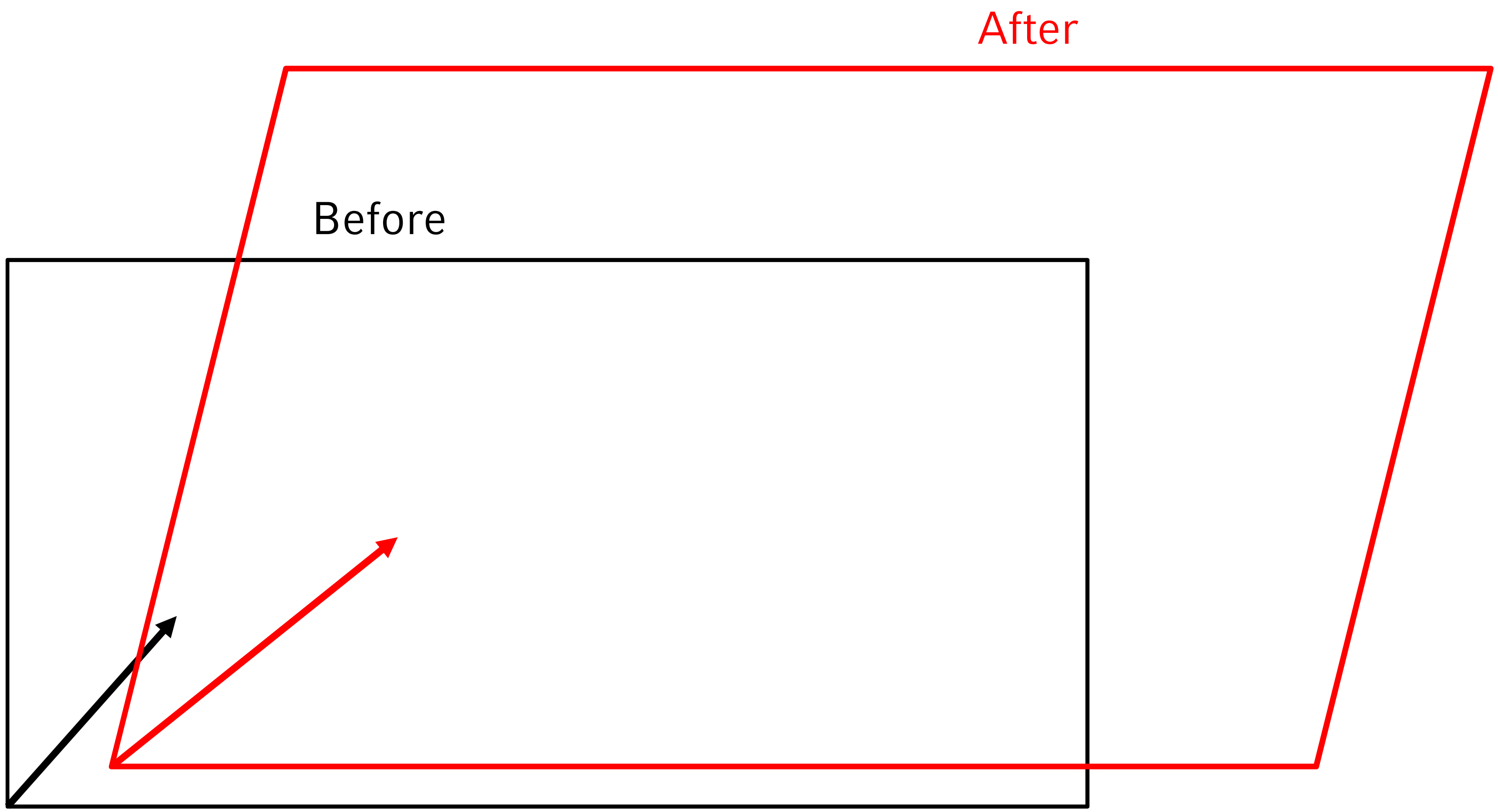


Observation



Is there a way to describe the amount and direction of deformation at a point?

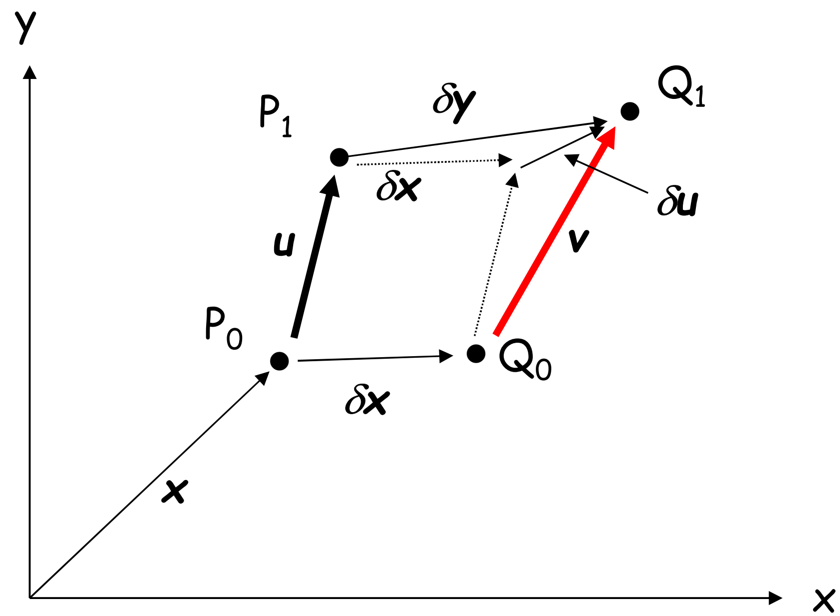
Strain



What operation transforms the **black vector** into the **red vector** ?

Incremental deformation

A point P_0 at position r is displaced to the point Q_0 by amount ***δx* after the deformation** ***(bold face*** denotes ***vector quantities)***



The vector ***u*** (before deformation) is now vector **v** (after deformation)

- Different origin
- Different direction
- Different orientation

We are interested in the DIFFERENCE between u and v!

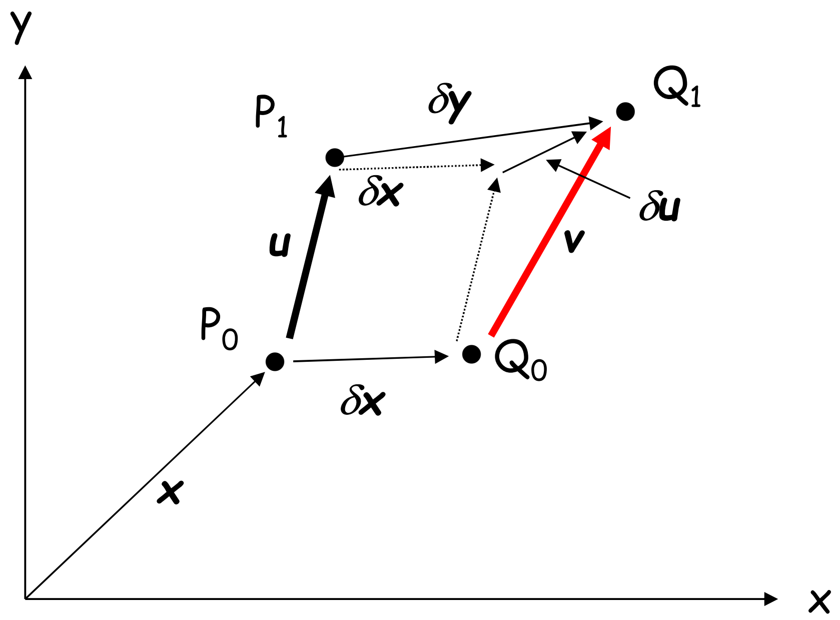
$\delta\mathbf{u} = \mathbf{v} - \mathbf{u}$ ***or*** $\mathbf{v} = \mathbf{u} + \delta\mathbf{u}$

$\mathbf{v} = \mathbf{u}(\mathbf{x}+\delta\mathbf{x})$ for small deformations

$\delta\mathbf{u} = \mathbf{u}(\mathbf{x}+\delta\mathbf{x}) - \mathbf{u}(\mathbf{x})$

Incremental deformation

Linearization



Taylor expansion

$\delta\mathbf{u} = \mathbf{u}(\mathbf{x}+\delta\mathbf{x}) - \mathbf{u}(\mathbf{x})$

we change to index notation
and obtain in 3D

$$\boxed{\delta u_i = \frac{\partial u_i}{\partial x_k}\delta x_k}$$

What does this equation mean?
Let's go back to **1D**

Taylor and strain

Taylor expansion:

high-order terms

$$u(x+dx) = u(x) + \frac{\partial u}{\partial x} dx + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} dx^2 \dots \frac{1}{n!}\frac{\partial^n u}{\partial x^n} dx^n$$

linearization

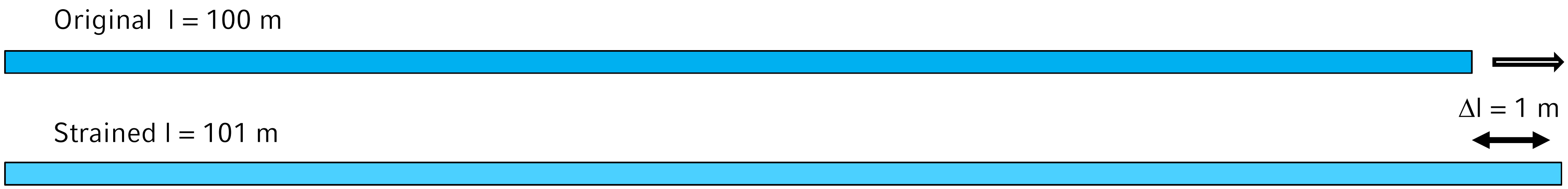
$$u(x+dx) \approx u(x) + \frac{\partial u}{\partial x} dx$$

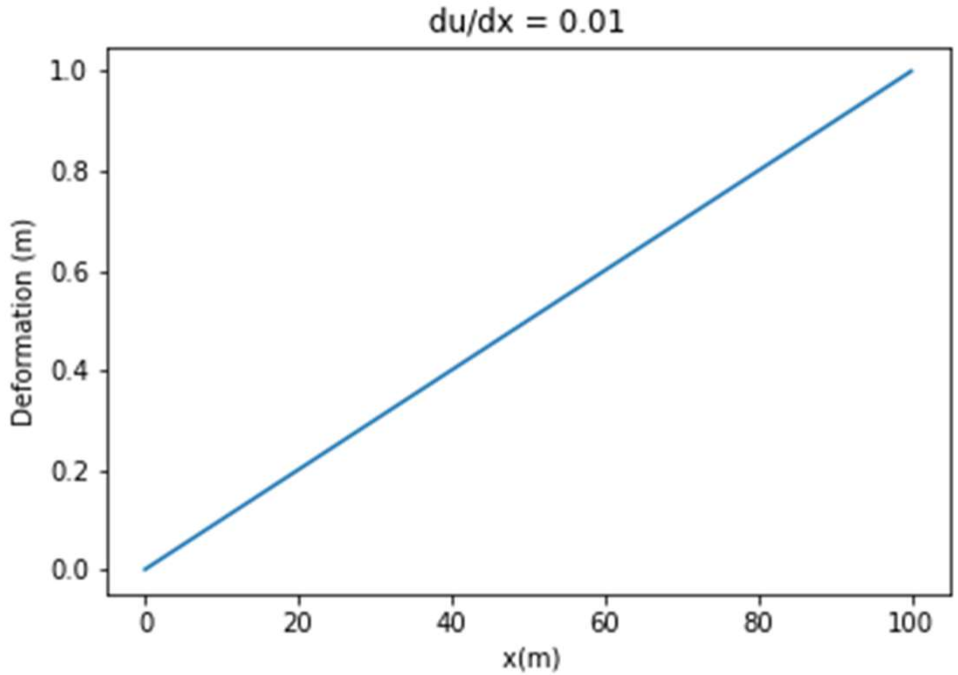
$$u(x+dx) - u(x) = \frac{\partial u}{\partial x} dx$$

deformation

$$\boxed{\delta u_i = \frac{\partial u_i}{\partial x_k}\delta x_k}$$

1D example





Let’s call

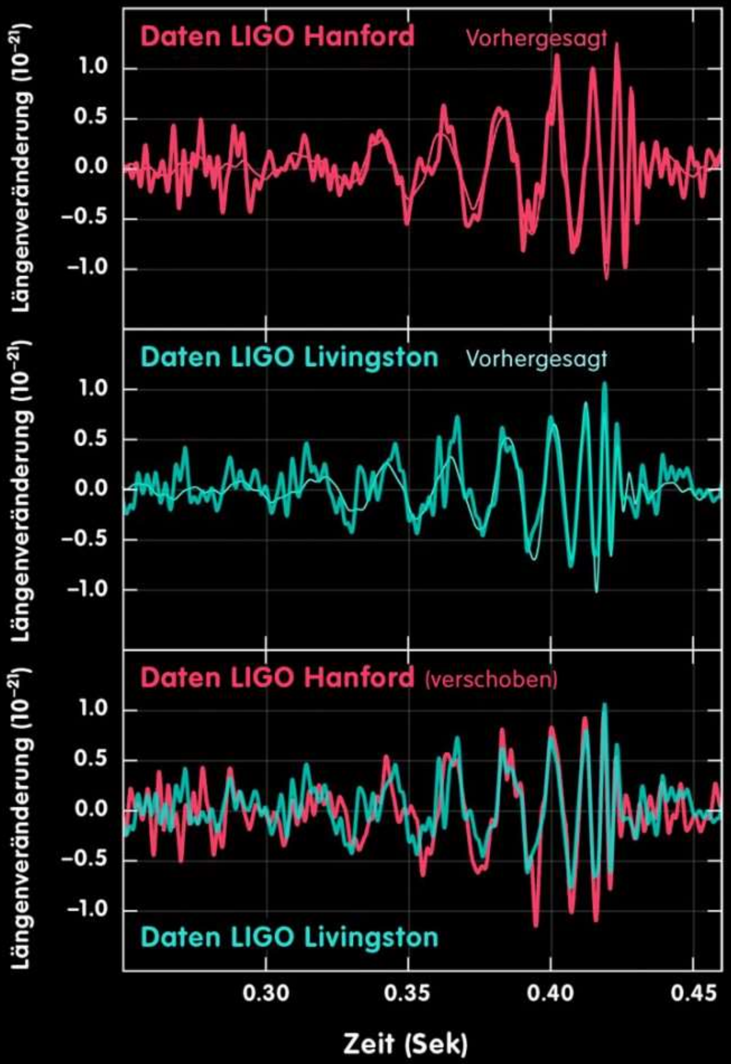
$\varepsilon = du/dx$

longitudinal strain

Its meaning is the **relative change of length**, the absolute change of length is

$\Delta l = \varepsilon\, l$

Small Exercise



LIGO

The peak **longitudinal strain amplitude is 10^{-21}**. The arm length of the LIGO sensor is 4km. What is the change in length of one arm?

The distance to the Moon is 384.400 km. Along that distance, what would be the length change? How does it relate to the diameter of an H atom?

5 mins.

3D deformation

The story in 3D

$$\delta u_i = \underbrace{\frac{\partial u_i}{\partial x_k}}\delta x_k$$

$$\begin{pmatrix} \partial_x u_x & \partial_x u_y & \partial_x u_z \\ \partial_y u_x & \partial_y u_y & \partial_y u_z \\ \partial_z u_x & \partial_z u_y & \partial_z u_z \end{pmatrix}$$

$\frac{\partial u_y}{\partial x} \rightarrow \partial_x u_y$

trace

Matrix symmetries

For any matrix A

$$A = \underbrace{\frac{1}{2}\left(A + A^T\right)}_{\text{symmetric}} + \underbrace{\frac{1}{2}\left(A - A^T\right)}_{\text{antisymmetric}}$$

Thus

$$\frac{\partial u_i}{\partial x_j} = \overbrace{\underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}_{\varepsilon_{ij}}}^{\text{symmetric}} + \overbrace{\underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)}_{r_{ij}}}^{\text{antisymmetric}}$$

The strain tensor

$$\varepsilon_{ij} = \frac{1}{2}\left(\partial_i u_j + \partial_j u_i\right)$$

symmetric

$$\varepsilon_{ij} = \begin{pmatrix} \partial_x u_x & \frac{1}{2}(\partial_x u_y + \partial_y u_x) & \frac{1}{2}(\partial_x u_z + \partial_z u_x) \\ & \partial_y u_y & \frac{1}{2}(\partial_y u_z + \partial_z u_y) \\ & & \partial_z u_z \end{pmatrix}$$

diagonal elements

off-diagonal elements

What is a *tensor?*

v	u_i	ε_{ij}	C_{ijkl}
scalar	vector	matrix	matrix
rank 0	rank 1	rank 2	rank 4

in our cases $i = 1, 2, 3$ (same for j, k, l) ....

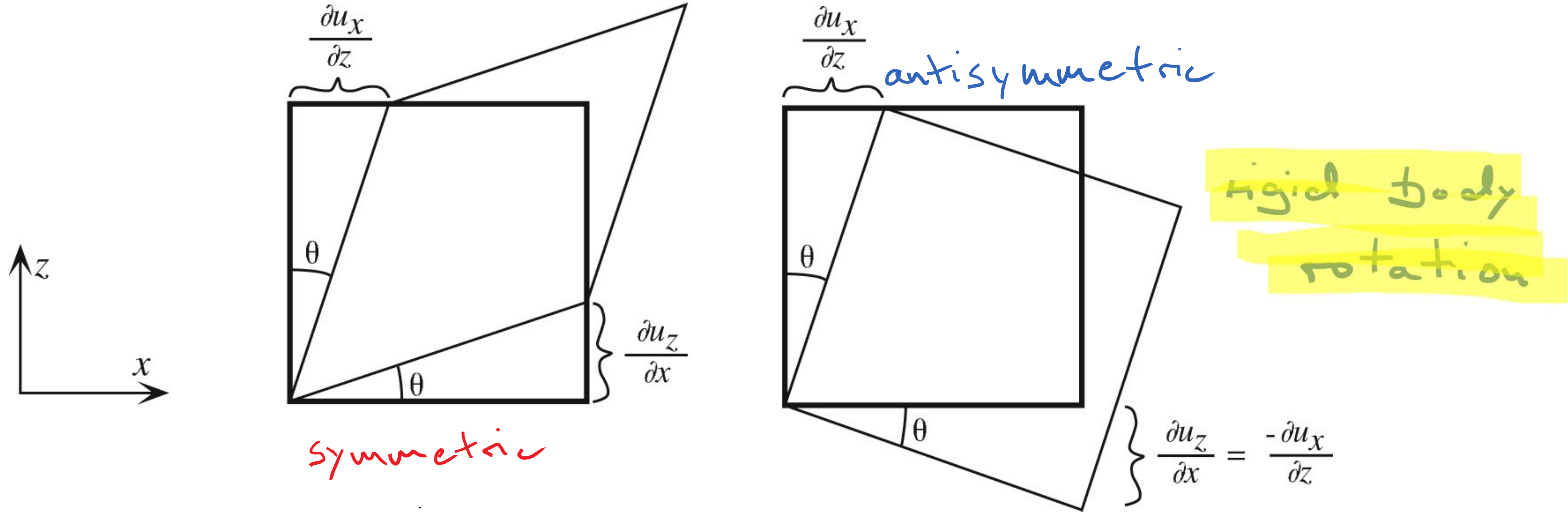
so the dimensionality is 3 !

Tensors and matrices are not the same thing !

Tensors preserve physical properties (coordinate changes)

cool explanation: https://youtu.be/bpG3gqDM80w

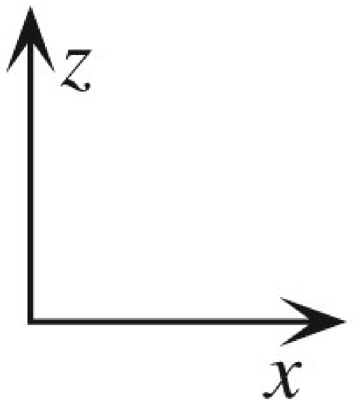
strain tensor

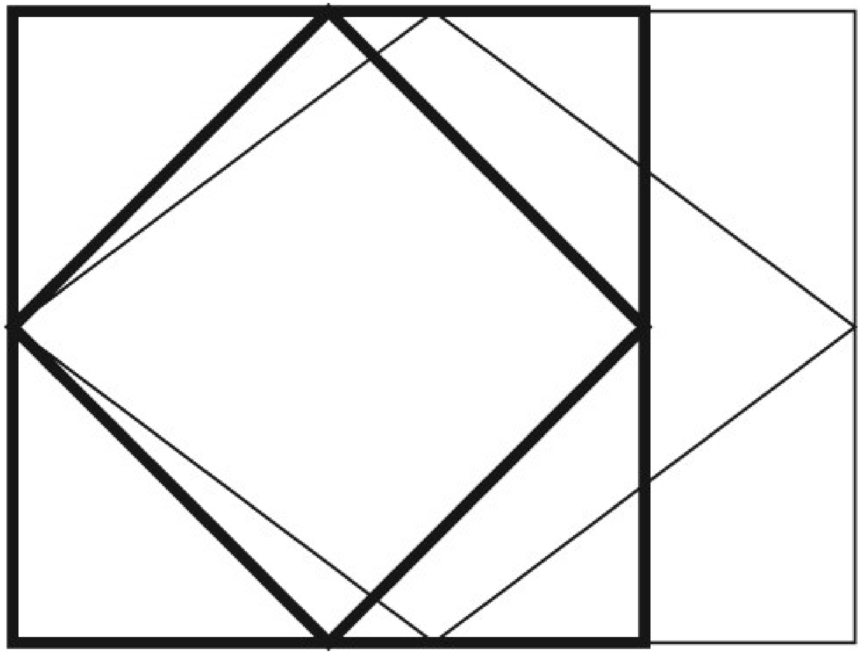


$\tan\theta = \frac{\Delta u_z}{\Delta x} \rightarrow$ small angles $\theta \approx \frac{\Delta u_z}{\Delta x}$

$\varepsilon_{xz} = \frac{1}{2}(\partial_x u_z + \partial_z u_x)$ is an angle

Extension and shearing





Extension of an elastic body in one direction leads to shearing! Remember

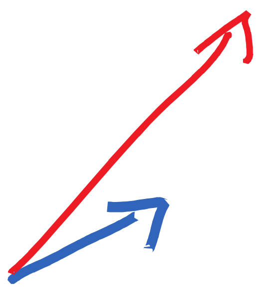
$$V_P = \sqrt{\frac{2\mu + \lambda}{\rho}}$$

A compressional P wave involves shearing

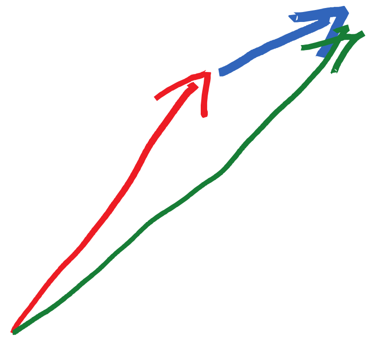
Matrix-vector operations

Remember: a matrix acting on a vector changes its **orientation** and **length**

$$\underline{du} = \underline{\underline{\varepsilon}}\,\underline{u}$$



After deformation



A point has moved to

$$\underline{u} + \underline{\underline{\varepsilon}}\,\underline{u} = \underline{u}\left(\underline{\underline{I}} + \underline{\underline{\varepsilon}}\right)$$

Other descriptions (Müller)

$$z_i = u_i + du_i = \underset{\text{translation}}{u_i} + \underset{\text{deformation}}{\epsilon_{ij} y_j} + \underset{\text{rotation}}{\xi_{ij} y_j}$$

$$\begin{aligned}
\epsilon_{ij} &= \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \qquad \xi_{ij} := \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_j}\right) \\
\epsilon_{ij} &= \epsilon_{ji} \\
\xi_{ij} &= -\xi_{ji} \qquad (\Rightarrow \xi_{11} = \xi_{22} = \xi_{33} = 0)\,.
\end{aligned}$$

Deformation

2.1.4 Deformation component of displacement

After separating out the rotation term, only the deformation term is of interest since it describes the forces which act in a body. The deformation is described completely by the six components ϵ_{ij} which are, in general, different. These dimensionless components will now be interpreted physically.

The starting point is $du_i = \epsilon_{ij} y_j$, i.e., we assume no rotation.

a) During this transformation, a line remains a line, a plane remains a plane, a sphere becomes an ellipsoid and parallel lines remain parallel.

b) *Deformation components* $\epsilon_{11}, \epsilon_{22}, \epsilon_{33}$

Coordinate origin at P and special selection of Q : $y_1 \neq 0, y_2 = y_3 = 0$.

Relative length change

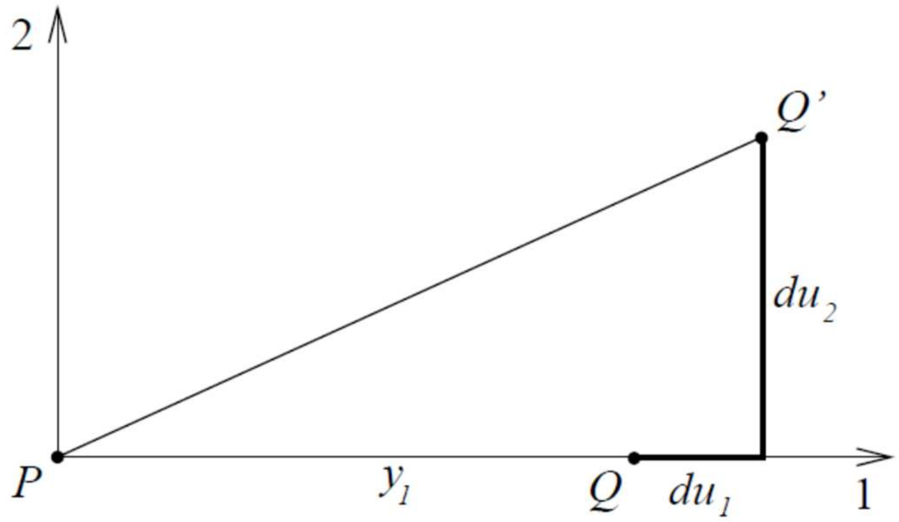
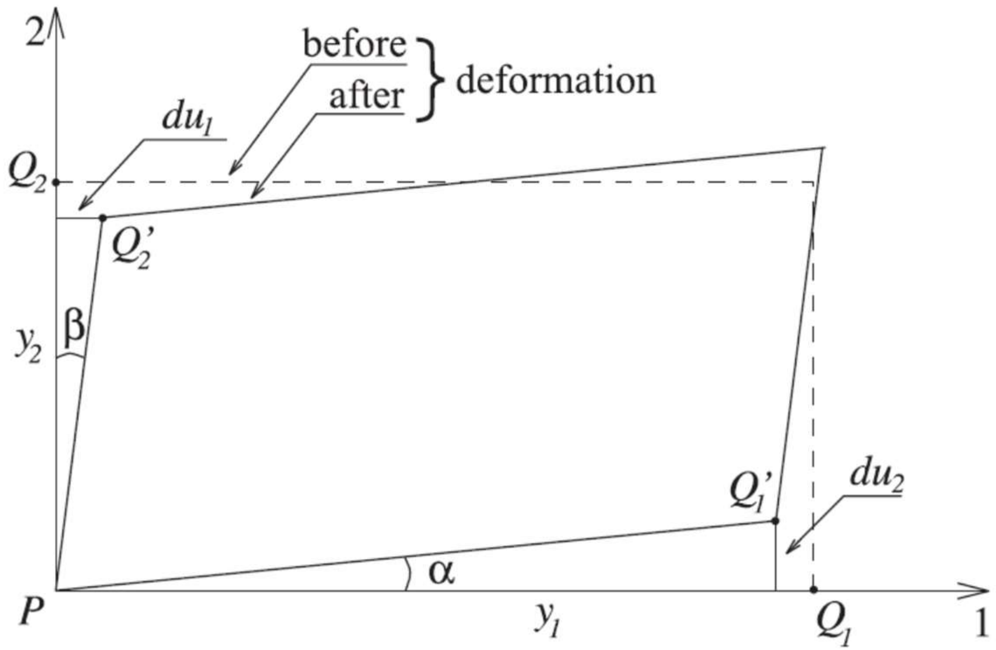


Fig. 2.3: Sketch for deformation components.

$$\begin{aligned} du_1 &= \epsilon_{11} y_1 \\ du_2 &= \epsilon_{21} y_1 \\ du_3 &= \epsilon_{31} y_1 = 0 \qquad (\text{assumption}: \quad \epsilon_{31} = 0). \end{aligned}$$

Shear components

Shear components $\epsilon_{12}, \epsilon_{13}, \epsilon_{23}$

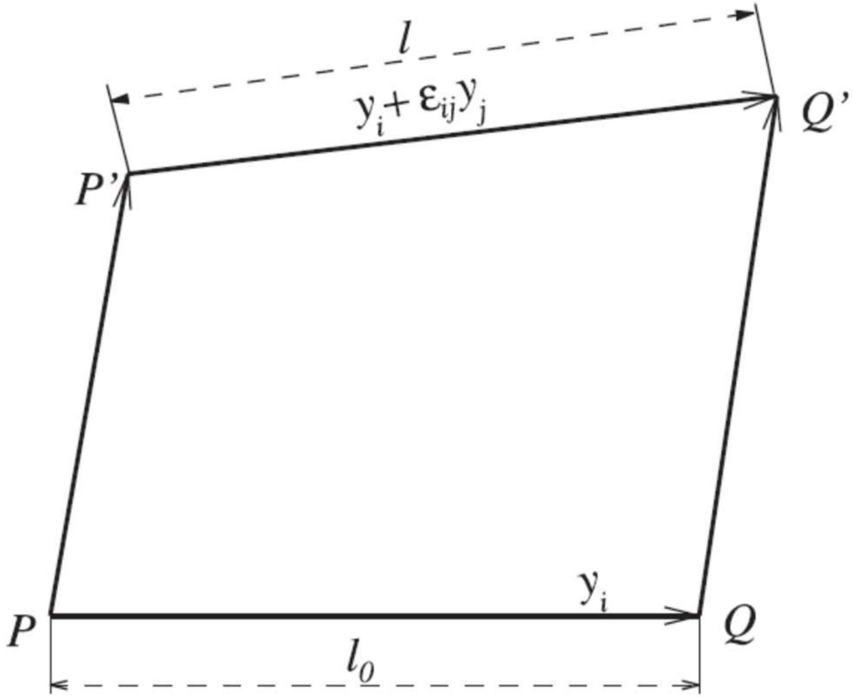


$$
\begin{aligned}
Q_1 \to Q_1' \; : \; du_2 &= \epsilon_{21} y_1 = \epsilon_{12} y_1 \\
Q_2 \to Q_2' \; : \; du_1 &= \epsilon_{12} y_2
\end{aligned}
$$

$$
\begin{aligned}
\tan\alpha \simeq \alpha \simeq \frac{du_2}{y_1} &= \epsilon_{12} \\
\tan\beta \simeq \beta \simeq \frac{du_1}{y_2} &= \epsilon_{12}.
\end{aligned}
$$

Length changes of PQ

Length changes of distance $\overline{PQ}$



$$\overline{PQ} = l_0 = \left\{\sum_{i=1}^{3} y_i^2\right\}^{1/2}$$

$$\overline{P'Q'} = l = \left\{\sum_{i=1}^{3} (y_i + \epsilon_{ij} y_j)^2\right\}^{1/2} =$$

$$= \left\{\sum_{i=1}^{3} y_i^2 + 2\epsilon_{ij} y_i y_j + \underbrace{\boxed{\sum_{i=1}^{3} (\epsilon_{ij} y_j)^2}}_{\text{dropped}}\right\}^{1/2}.$$

$$l = l_0 \left(1 + \frac{2}{l_0^2}\epsilon_{ij} y_i y_j\right)^{\frac{1}{2}} = l_0 + \frac{1}{l_0}\epsilon_{ij} y_i y_j.$$

Relative length changes

$$\frac{l - l_0}{l_0} = \epsilon_{ij}\frac{y_i y_j}{l_0^2} = \epsilon_{ij} n_i n_j \text{ (SC twice; quadratic form in } n_k)$$

$$n_i = \frac{y_i}{l_0} = \text{unit vector in direction of } y_i.$$

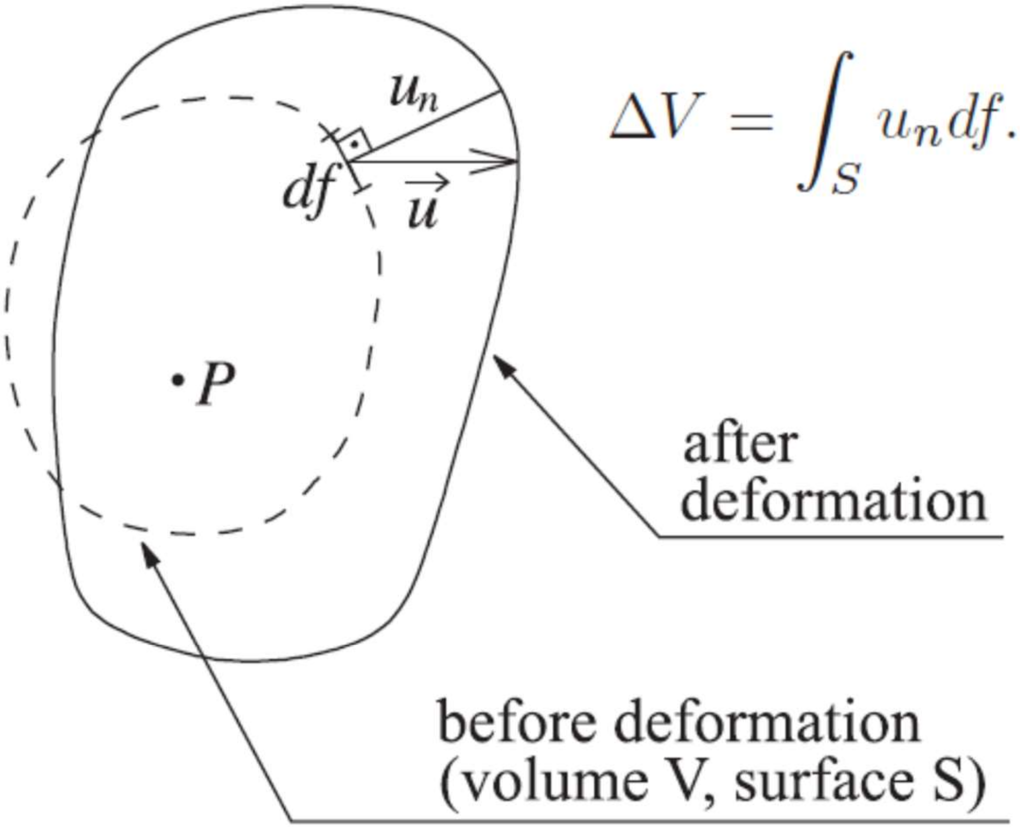
Rotation (*rigid body*)

$$\xi_{ij} := \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_j}\right)$$

$$\begin{pmatrix} 0 & \xi_{12} & \xi_{13} \\ -\xi_{12} & 0 & \xi_{23} \\ -\xi_{13} & -\xi_{23} & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} \xi_{12}y_2 + \xi_{13}y_3 \\ -\xi_{12}y_1 + \xi_{23}y_3 \\ -\xi_{13}y_1 - \xi_{23}y_2 \end{pmatrix} = \overrightarrow{\xi} \times \overrightarrow{y}$$

$$\text{with} \quad \overrightarrow{\xi} = (-\xi_{23}, \xi_{13}, -\xi_{12}) = \frac{1}{2}\nabla \times \overrightarrow{u}.$$

Volume Change



$$\Delta V = \int_S u_n df.$$

$$\Delta V = \int_V \nabla \cdot \vec{u}\, dV,$$

$$\frac{\Delta V}{V} = \frac{1}{V}\int_V \nabla \cdot \vec{u}\, dV.$$

$$\lim_{V\to 0} \frac{\Delta V}{V} = \Theta.$$

$$\Theta = \nabla \cdot \vec{u} := \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}$$

$$= \epsilon_{11} + \epsilon_{22} + \epsilon_{33}$$

Let's play with it!

seismic waves are

displacement
strain
stress
rotation

waves

So, what is the strain of seismic waves?

Plane waves

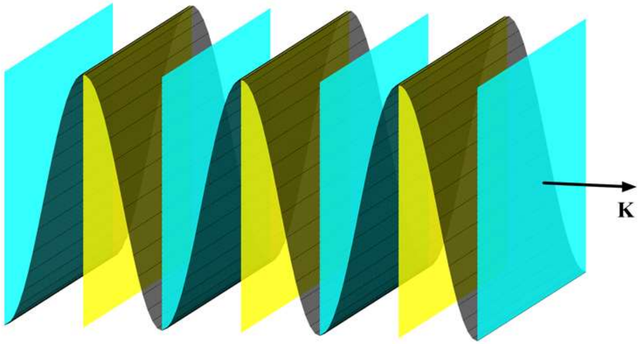
$u_z \uparrow \; \rightarrow x$

$c = \frac{\omega}{k}$

Amplitude

$$u_z = A \sin(kx - \omega t)$$

wavenumber — angular frequency



strain: $\varepsilon_{xz} = \frac{1}{2}(\partial_x u_z + \underbrace{\partial_z u_x}_{=0})$

$$= \frac{1}{2} A k \cos(kx - \omega t)$$

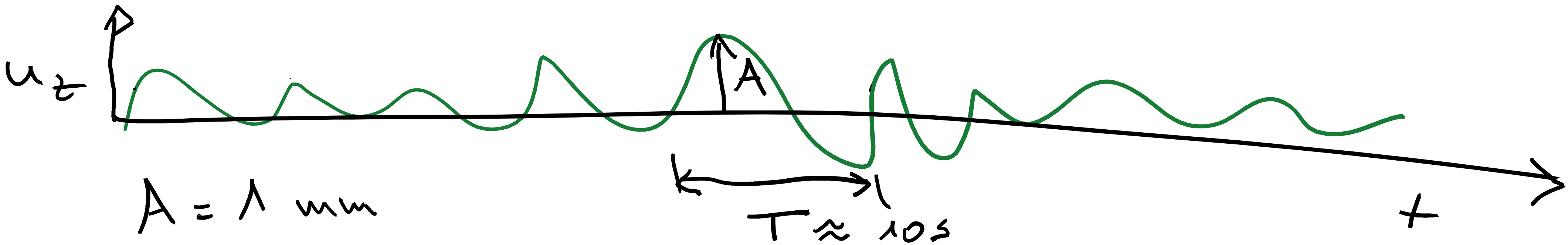
$$= \frac{1}{2} A \frac{2\pi}{\lambda} \cos(kx - \omega t)$$

$$c = \frac{\lambda}{T} = \lambda f = \frac{2\pi f}{\frac{2\pi}{\lambda}} = \frac{\omega}{k}$$

Remember

$$\varepsilon_{xz}^{max} = \pi \frac{A}{\lambda}$$

Simple example - theory



$A = 1$ mm

What is the maximum strain?

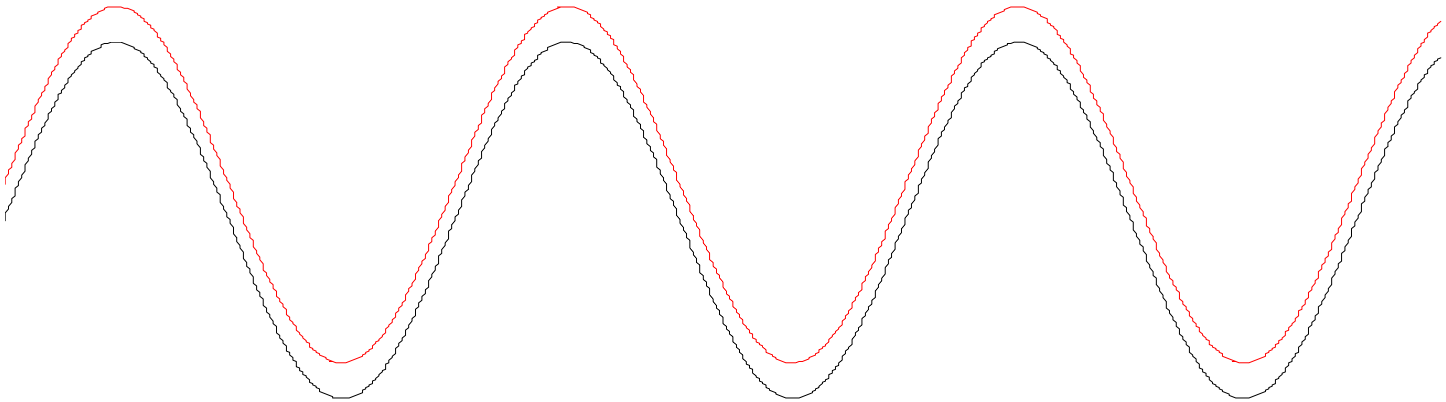
Assuming $c = 4000 \frac{m}{s}$ (Rayleigh wave)

$$\varepsilon_{xz}^{max} = \pi \frac{A}{\lambda} = \pi \frac{A}{cT} = \pi \frac{10^{-3}\ \cancel{m}}{4 \cdot 10^{3} \cdot 10\ \cancel{m}} = \boxed{\pi \frac{1}{4} 10^{-7}}$$

... shear waves ...

Plane transversely polarized (S or Love) wave propagating in x-direction with phase velocity c

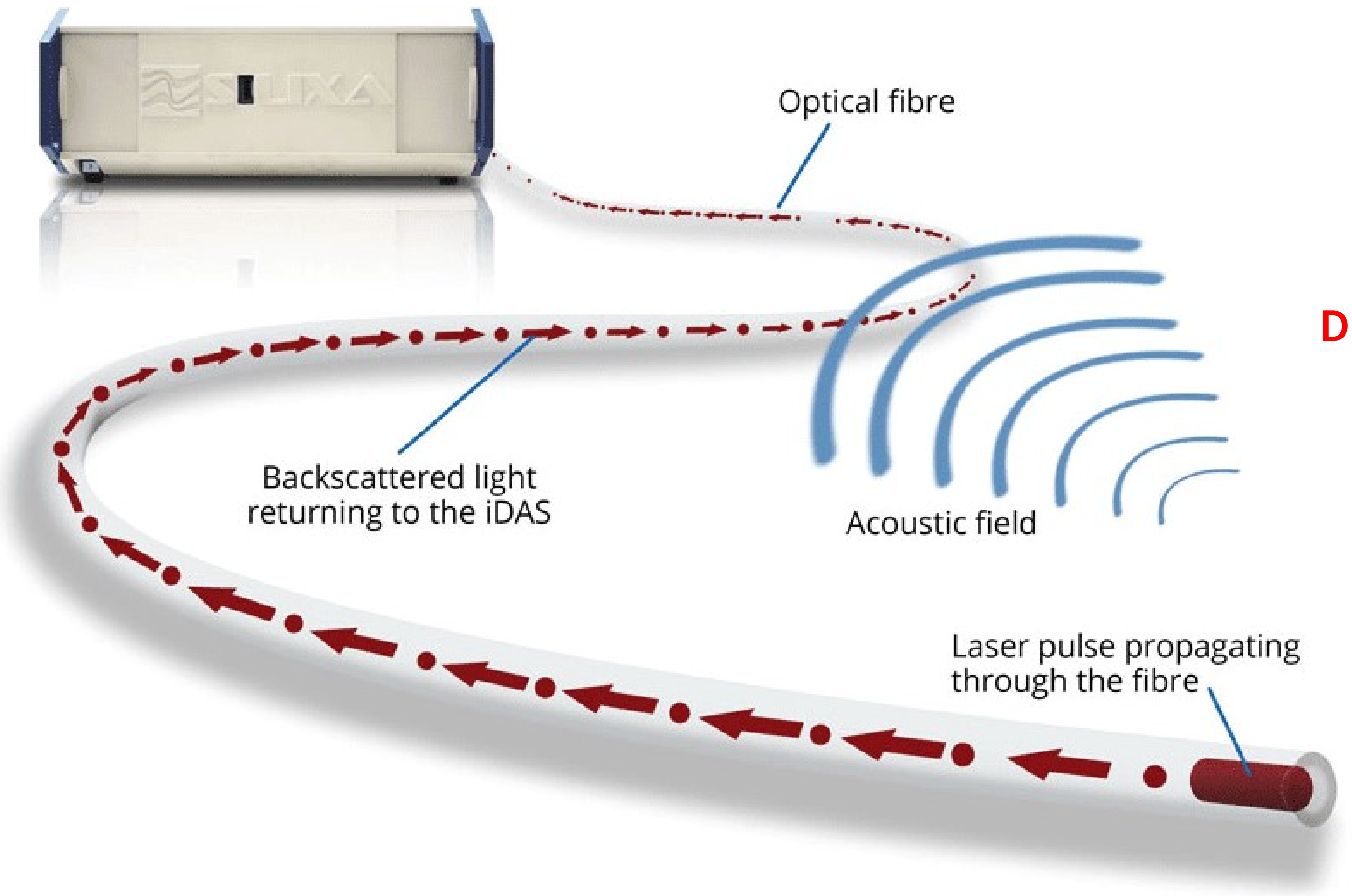
rotation rate – transverse acceleration



$$\frac{\ddot{u}_y(\mathrm{x},\mathrm{t})}{\Omega_z(\mathrm{x},\mathrm{t})} = -2\mathrm{c}$$

Rotation rate and acceleration should be **in phase** and the amplitudes scaled by **two times the horizontal phase velocity**

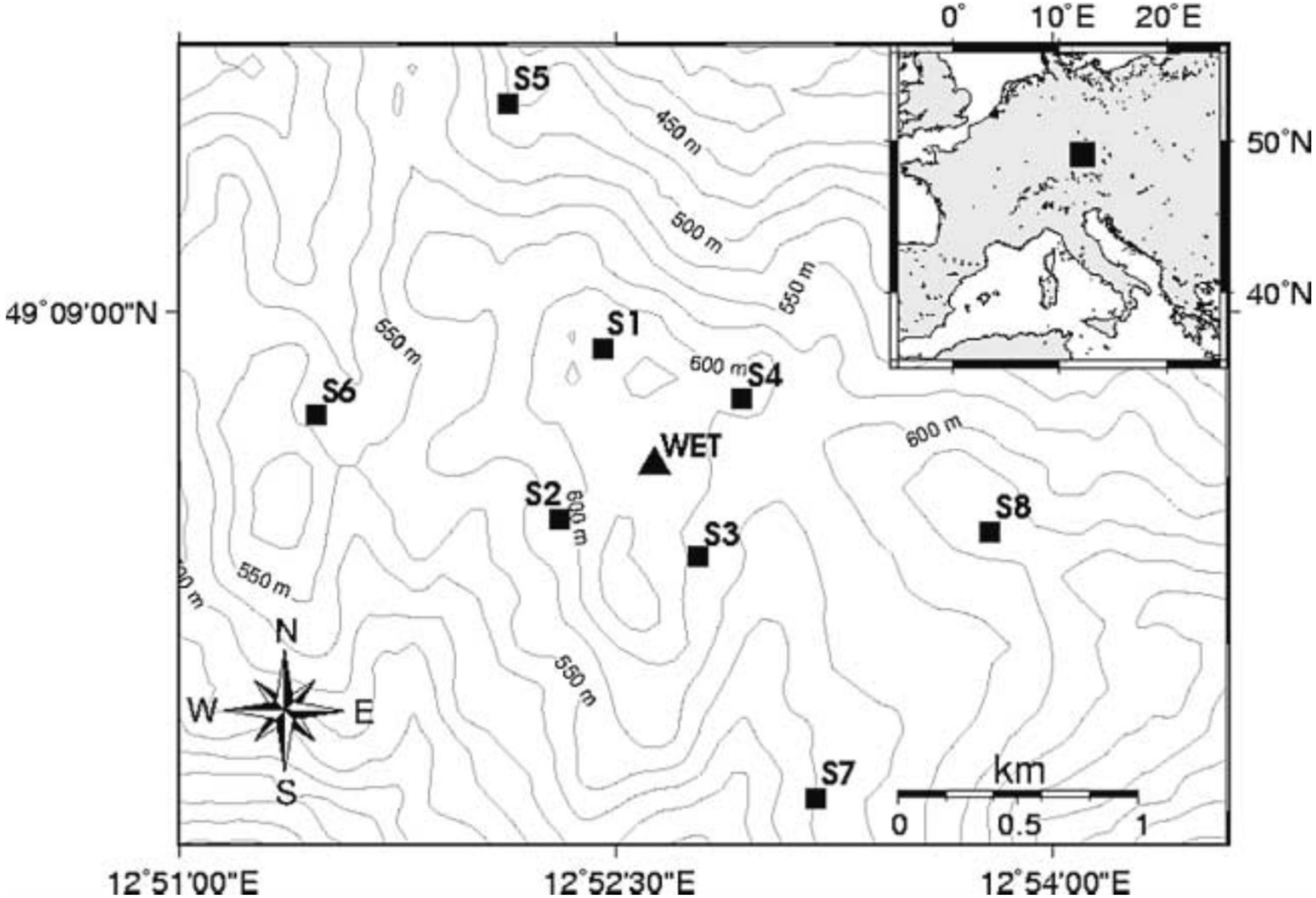
The hype with strain today!



DAS

Distributed Acoustic Sensing

Array-derived gradients



Curl of velocity field

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \frac{1}{2} \nabla \times v = \frac{1}{2} \begin{pmatrix} \partial_y v_z - \partial_z v_y \\ \partial_z v_x - \partial_x v_z \\ \partial_x v_y - \partial_y v_x \end{pmatrix}.$$

Array-derivation

$$\begin{aligned} d_i &= GR_i \\ &= \begin{pmatrix} \partial_x u_x & \partial_y u_x & \partial_z u_x \\ \partial_x u_y & \partial_y u_y & \partial_z u_y \\ \partial_x u_z & -\partial_z u_y & -\eta(\partial_x u_x + \partial_y u_y) \end{pmatrix} R_i, \end{aligned} \qquad (2)$$

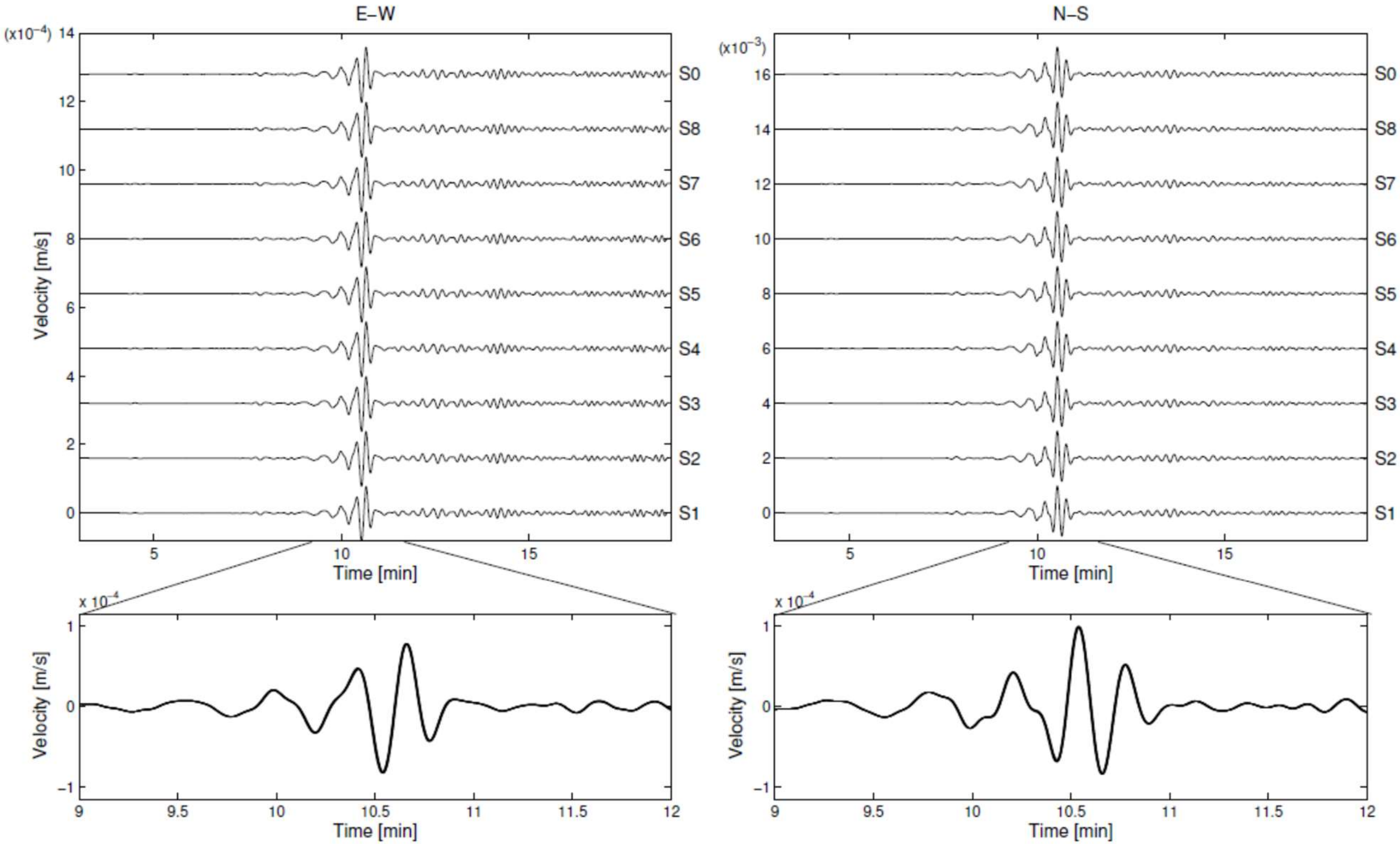
where, $\eta = \lambda\,(\lambda + 2\mu)$, λ and μ are the Lamé parameters, $d_i = u_i - u_0$, $R_i = r_i - r_0$, u_i, r_i, and u_0, r_0 are the displacements at the coordinates of the ith station and the reference station (subscript 0), respectively. At least three stations must be used to determine the horizontal-displacement gradient using this method.

Array-derived rotation

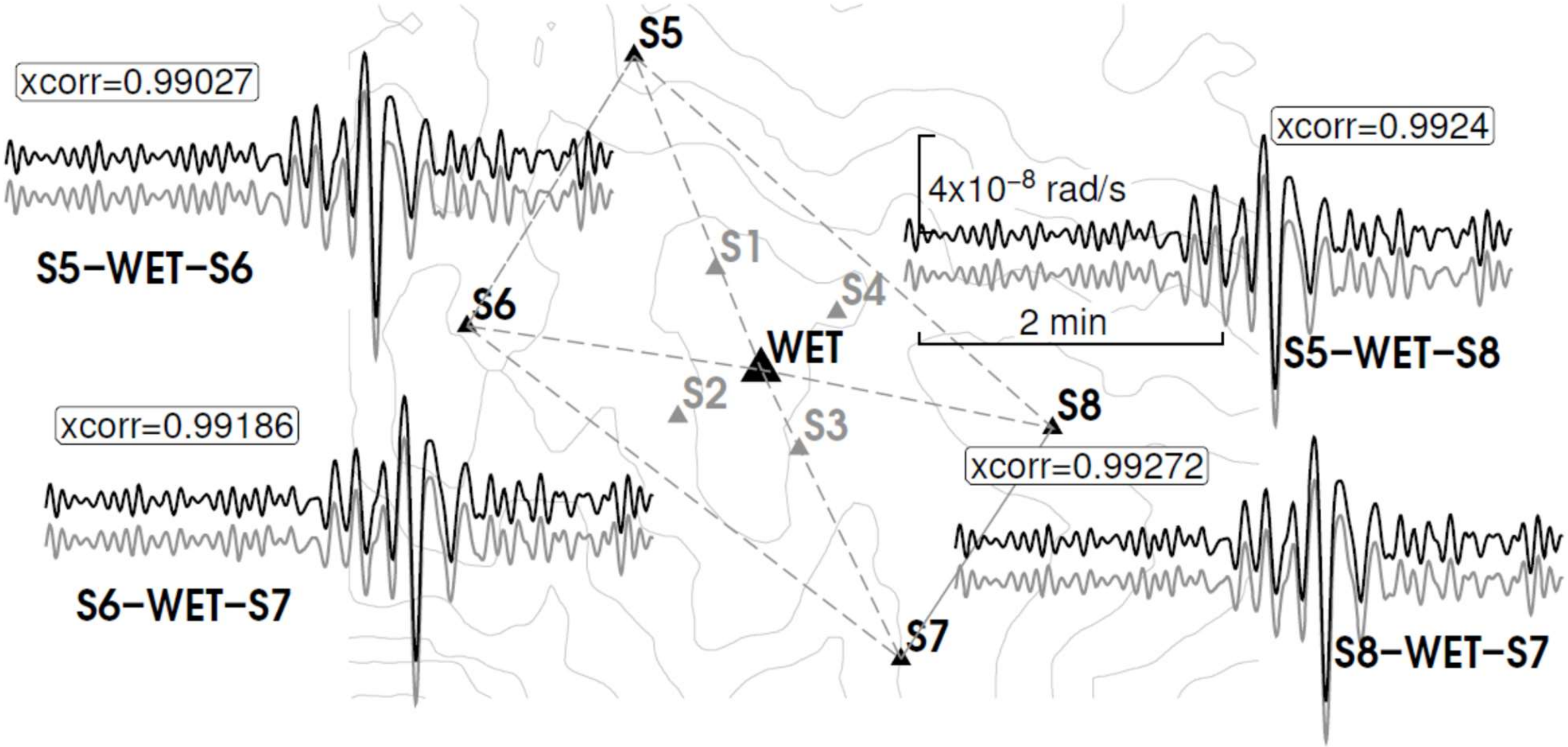
$$\omega_z = \frac{1}{2A}\left([b_i v_y^i + b_j v_y^i + b_k v_y^k] - [c_i v_x^i + c_j v_x^i + c_k v_x^k]\right), \quad (3)$$

where v^i is the velocity vector at the ith station, A is the area bounded by the station S_i, S_j, and S_k, $b_i = (y_k — y_j)/2$, and $c_i = (x_k — x_j)/2$, and b_j and c_j obtained by index circular permutation. Here, (x_i, y_i), (x_j, y_j), and (x_k, y_k) are coordinates of stations S_i, S_j, and S_k, respectively. When more than three

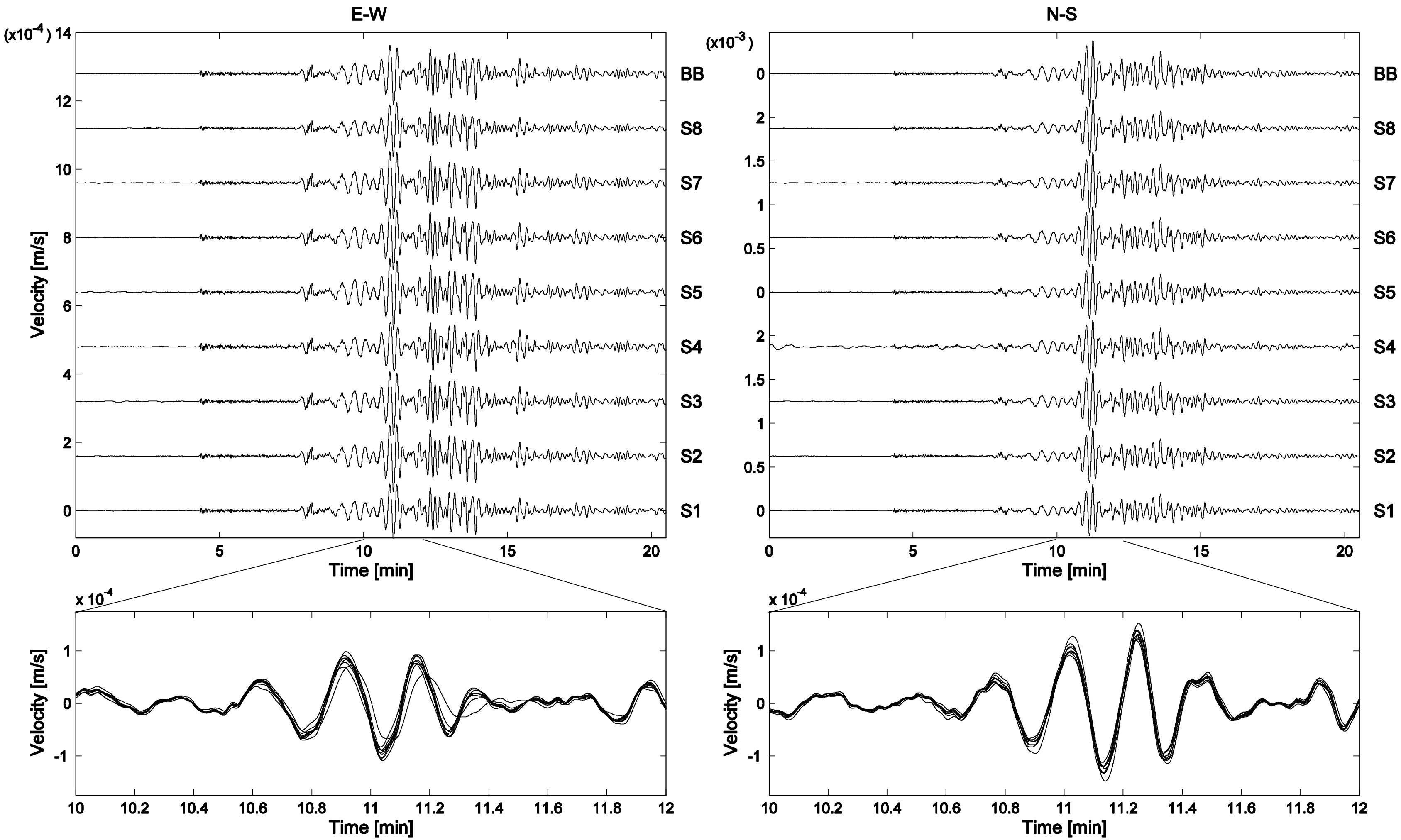
Array synthetics



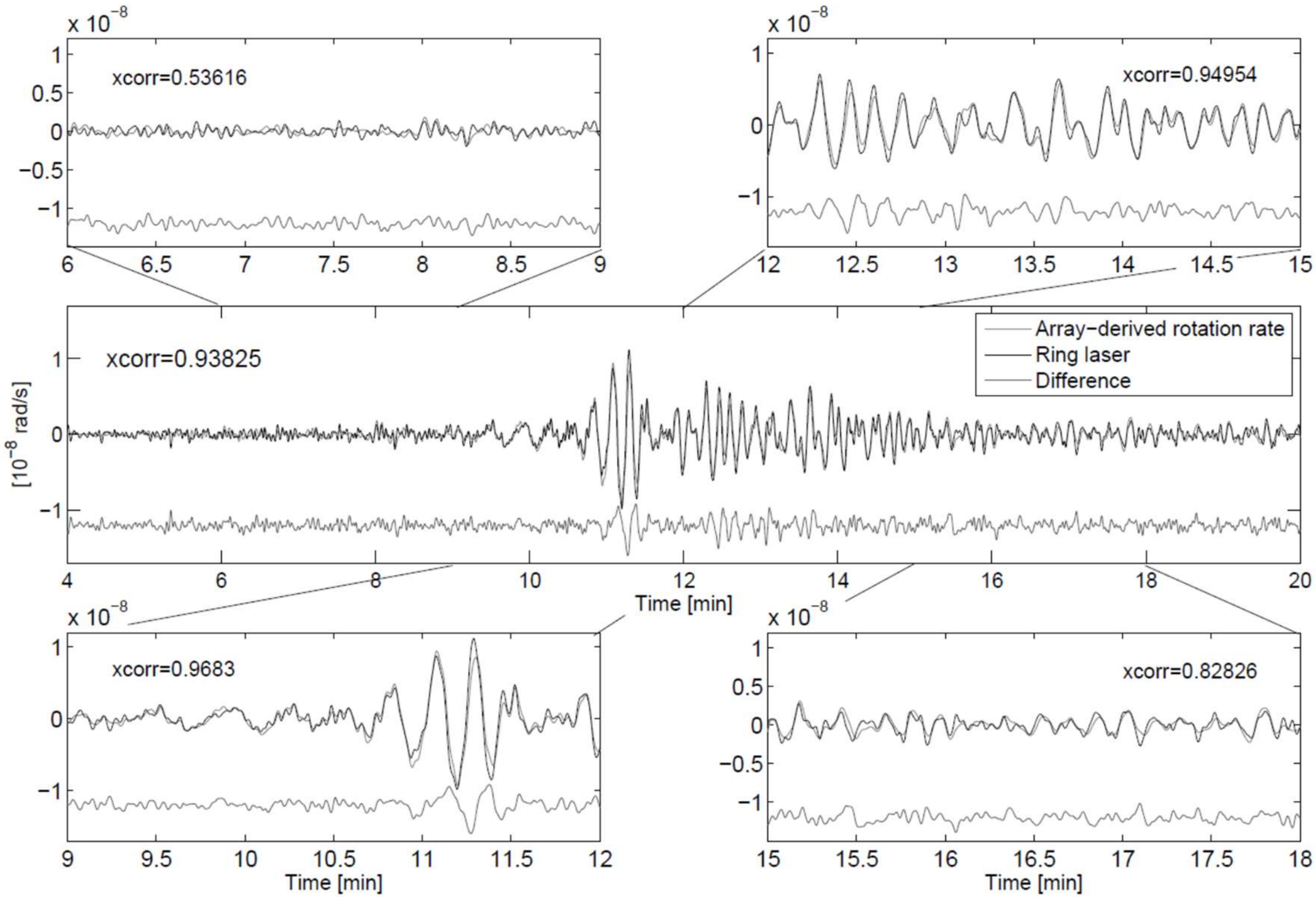
Synthetic tests



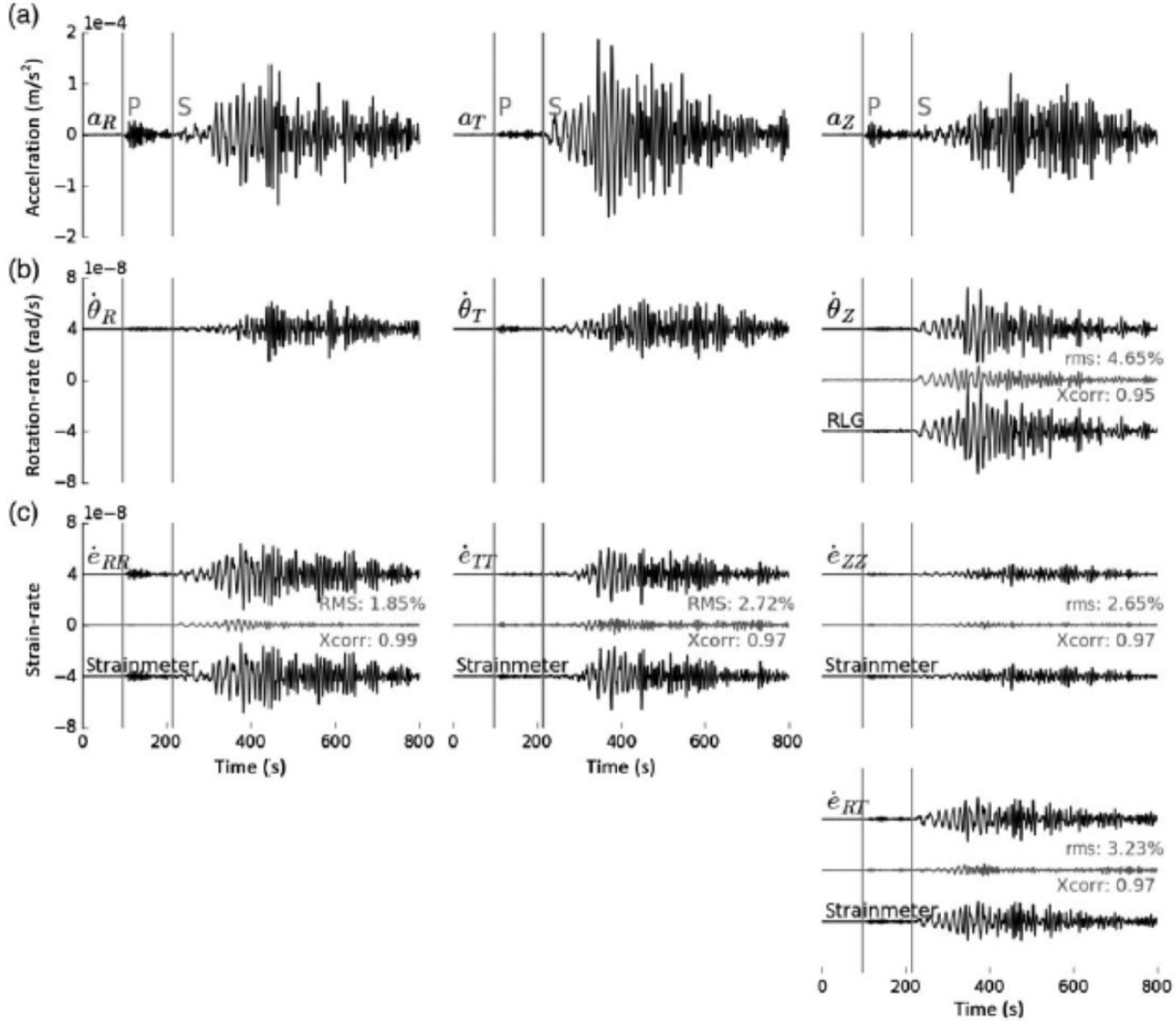
Observations



Ring laser (solid) – array-derived (gray)



Multicomponent seismology (direct vs. array-derived)



… blueseis.com

BLUESEIS-3A
PRELIMINARY TECHNICAL SPECIFICATIONS

PERFORMANCE

Sensor self-noise	10^{-3} Hz	10^{-2} Hz	10^{-1} Hz	1 Hz	10 Hz	100 Hz
in rad/s/√Hz	1.10^{-7}	2.10^{-8}	2.10^{-8}	2.10^{-8}	2.10^{-8}	1.10^{-7} [1]

Angular Random Walk	$< 15.\ 10^{-9}$ rad/s/√Hz (50 μ°/√h)
Passband	Flat from DC to 100Hz
DC rotation rate accuracy	< 5 μrad/s (1 °/h)
Heading	< 4° x secant(lat) [2]
Scale factor stability	< 1% guaranted for life
Calibration	Not needed
Settling time	< 1 minute

OPERATING RANGE / ENVIRONMENT

Operating / storage temperature	-10 to 50°C / -40 to 80°C
Rotation rate dynamic range	100 000 μrad/s
Operational tilt range	Any
Acceleration susceptibility	None
Pressure susceptibility	None
MTBF	100,000 hours

PHYSICAL CHARACTERISTICS

Ingress protection	IP66
Dimensions (L x W x H)	300 x 300 x 280 mm
Weight	20 kg

INTERFACES

Hardware interfaces	Ethernet + RS232/422 + 1 TTL input pulse for PPS
Output format	miniSEED (TCP/UDP)
Input format	NMEA (ZDA) / NTP / PTP for time stamping
Data output rate	Up to 200 Hz
Power supply / consumption	24 VDC / < 20 W
Man Machine Interface (MMI)	Web-based interface for configuration

BLUESEIS-3A ROTATIONAL SEISMOMETER

BROABAND & HIGH-GRADE 3-COMPONENT ROTATIONAL SEISMOMETER FOR LAND APPLICATIONS

iXBlue offers now to geosciences the possibility to explore rotational ground motion. Recognized throughout the industry for its mastery of Fiber Optic Gyroscope (FOG), the **iXBlue** group stands as a global leader in several high-grade applications such as inertial navigation, hydrography and satellite gyroscopes. Based on its 30 years' unchallenged expertise, **iXBlue** revolutionizes geosciences by offering a brand-new product that seismology has always been looking for. **BlueSeis-3A** is today the best and most reliable answer to the rotational seismometer need: 3-axis, broadband, low-noise, high dynamic range and flat passband solution with "geosciences-ready" interfaces including digitizer and time stamping.

FEATURES

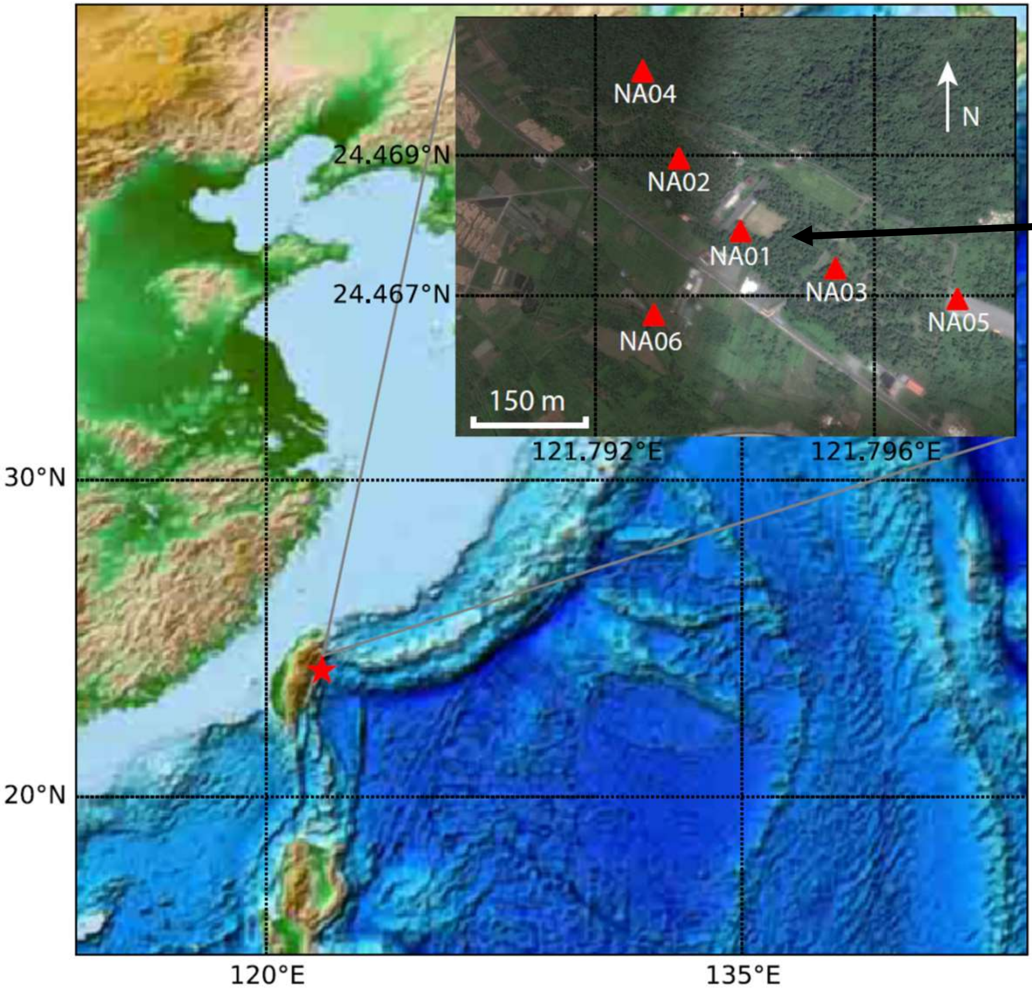
- 3 Interferometric Fiber Optical Gyroscope (I-FOG) for low self-noise and broadband measurement
- DC signal for absolute rotation measurement
- High dynamic range
- Embedded digitizer and GNSS time stamping
- Field-proven technology

BENEFITS

- Rotation as a new observable in seismology!
- Easy to deploy: no calibration, no tilt range limitation, insensitive to enviromental conditions
- Heading provided by the system
- 2-in-1: "weak motion" low-noise + "strong motion" dynamic
- Plug and play interfaces

APPLICATIONS • Seismic tomography • Volcanology • Earthquake physics • Geophysical exploration

Comparison with array-derivation, Taiwan

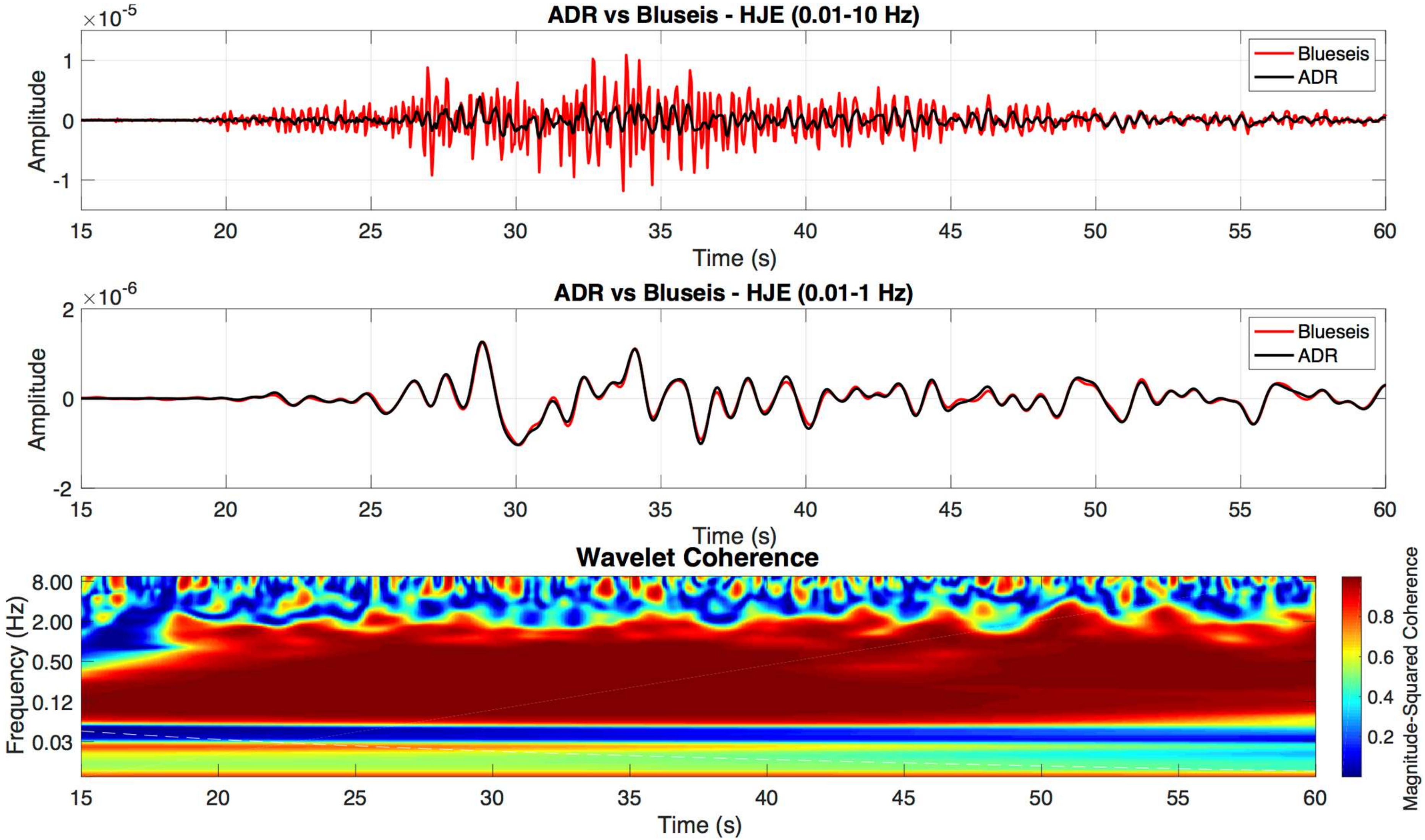


BLUESEIS at centre of seismic array

Data provided by Chin Jen Lin, Taipeh

Yuan et al., BSSA, 2020

Array-derived vs. blueSeis - East Component



Current Research on Strain and Rotation

- **Observations: rotation vs. translation**
- Observations: strain vs. translation
- Sensitivities of joint observables
- Sensitivities of gradient related quantities w.r.t. heterogeneities
- Strain rotation coupling
- Tracking seismic sources with 6C

2019-05-14T12:58:41.200000Z

Region: New Ireland Region, P.N.G.

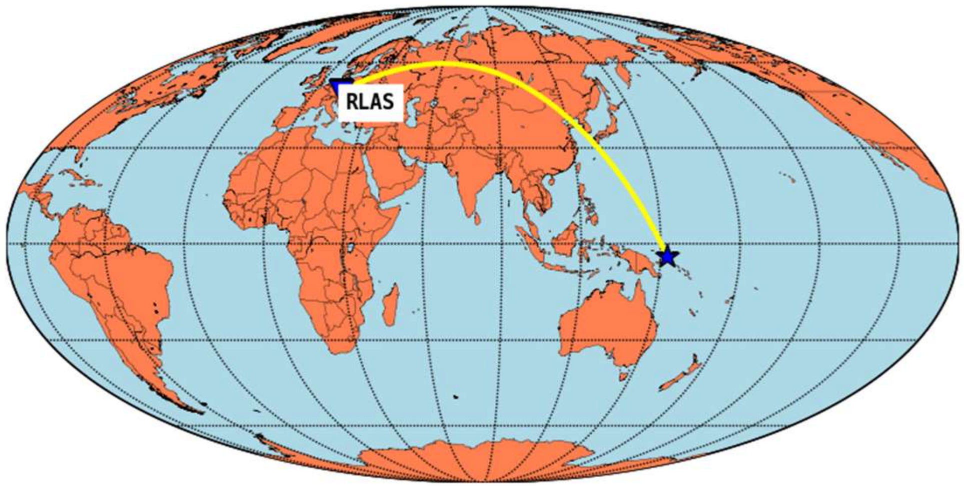
Magnitude: 7.6 Mwc

Distance: 13721.11 [km], 50.46 [°]

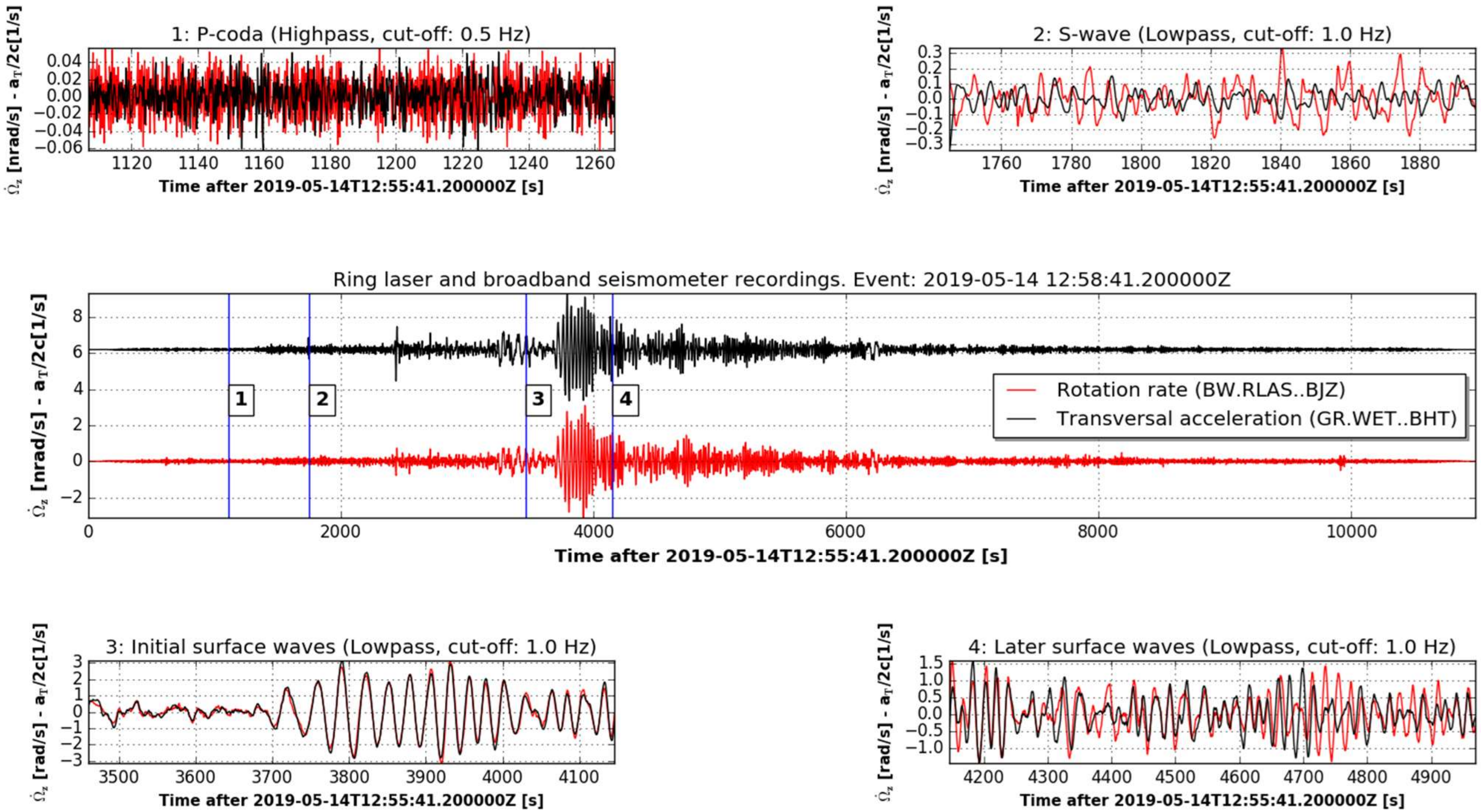
Depth: 18.8 [km]

Event Information:
Global Centroid-Moment-Tensor Catalog (GCMT)

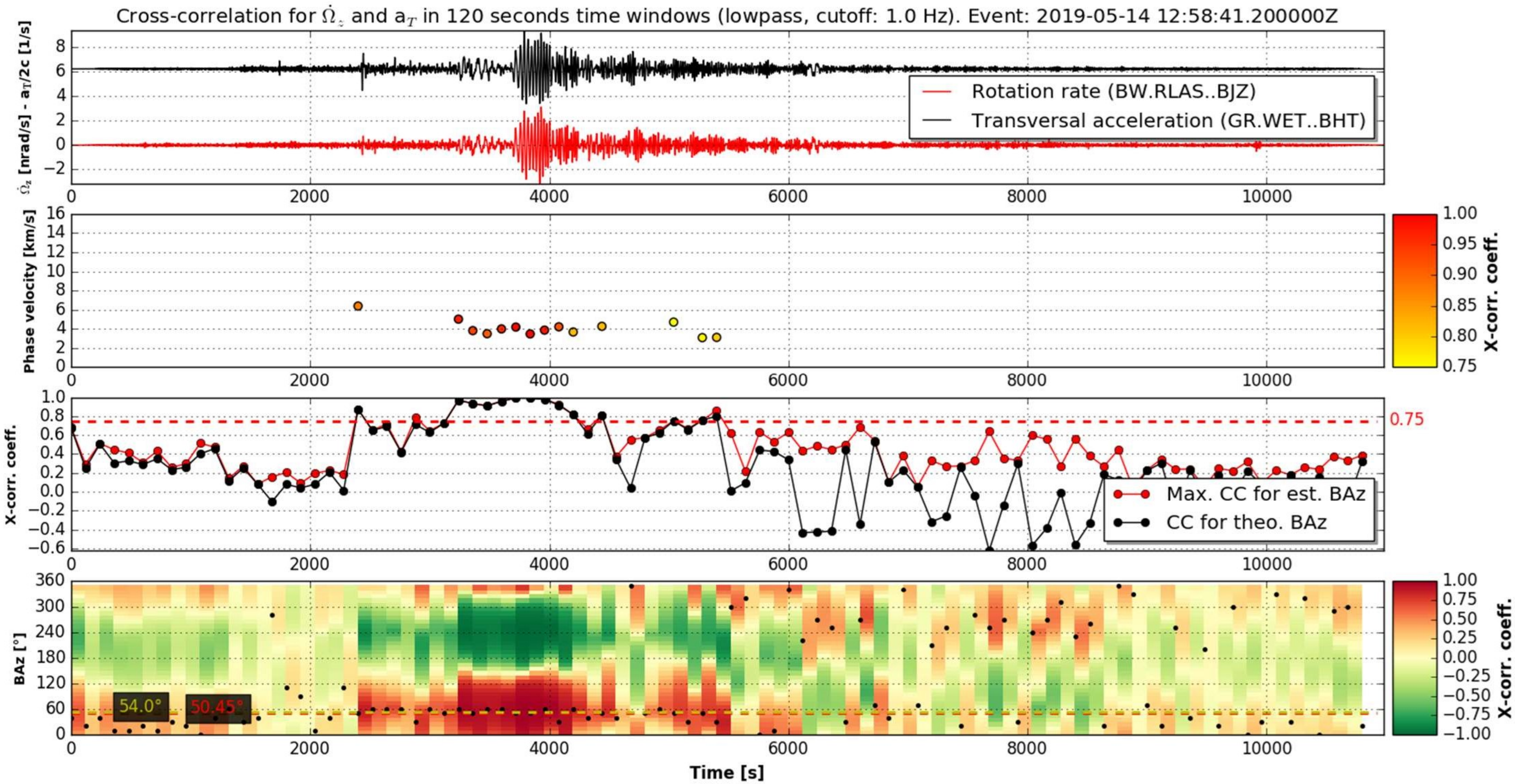
Processing Date:
2019-05-15



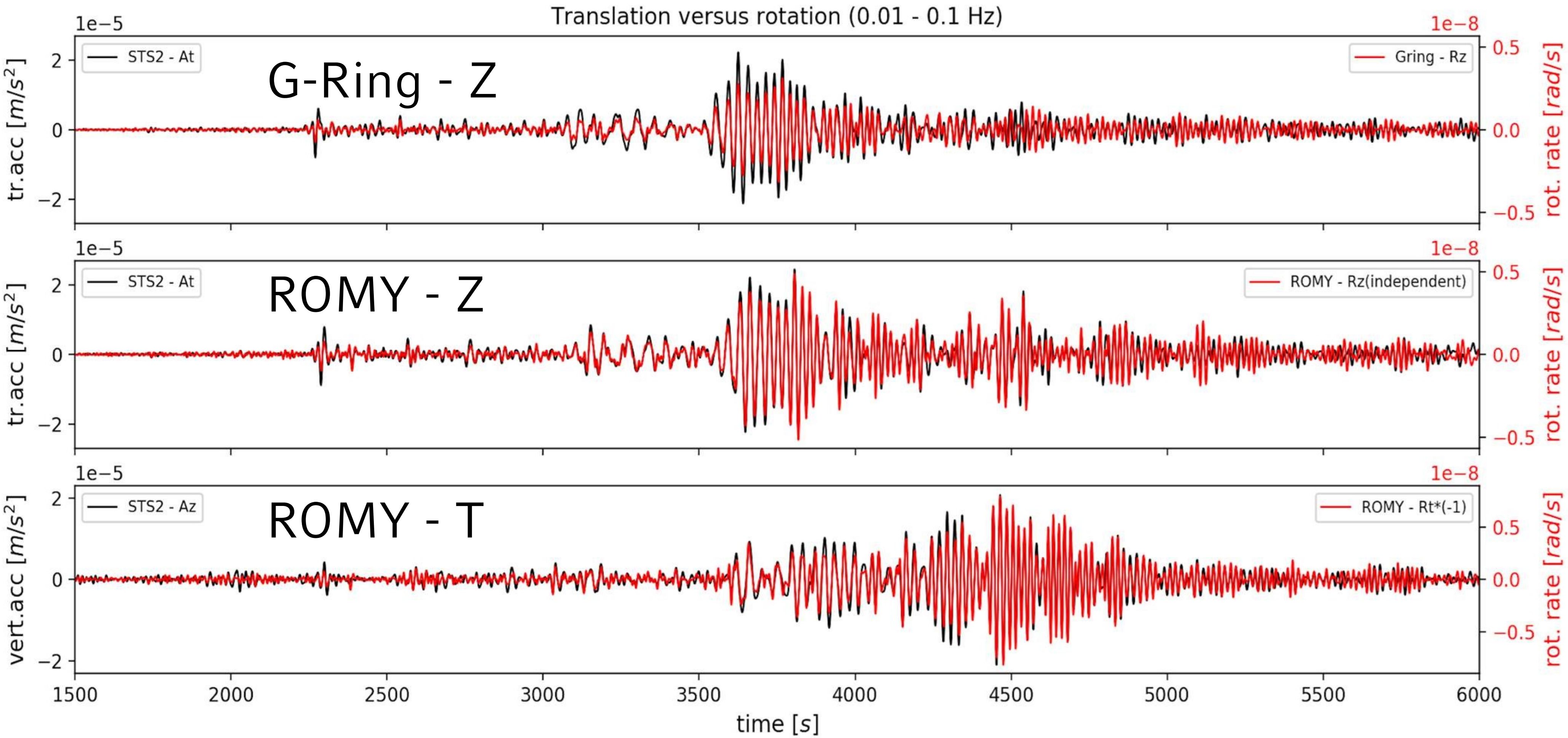
G Ring Observations Wettzell



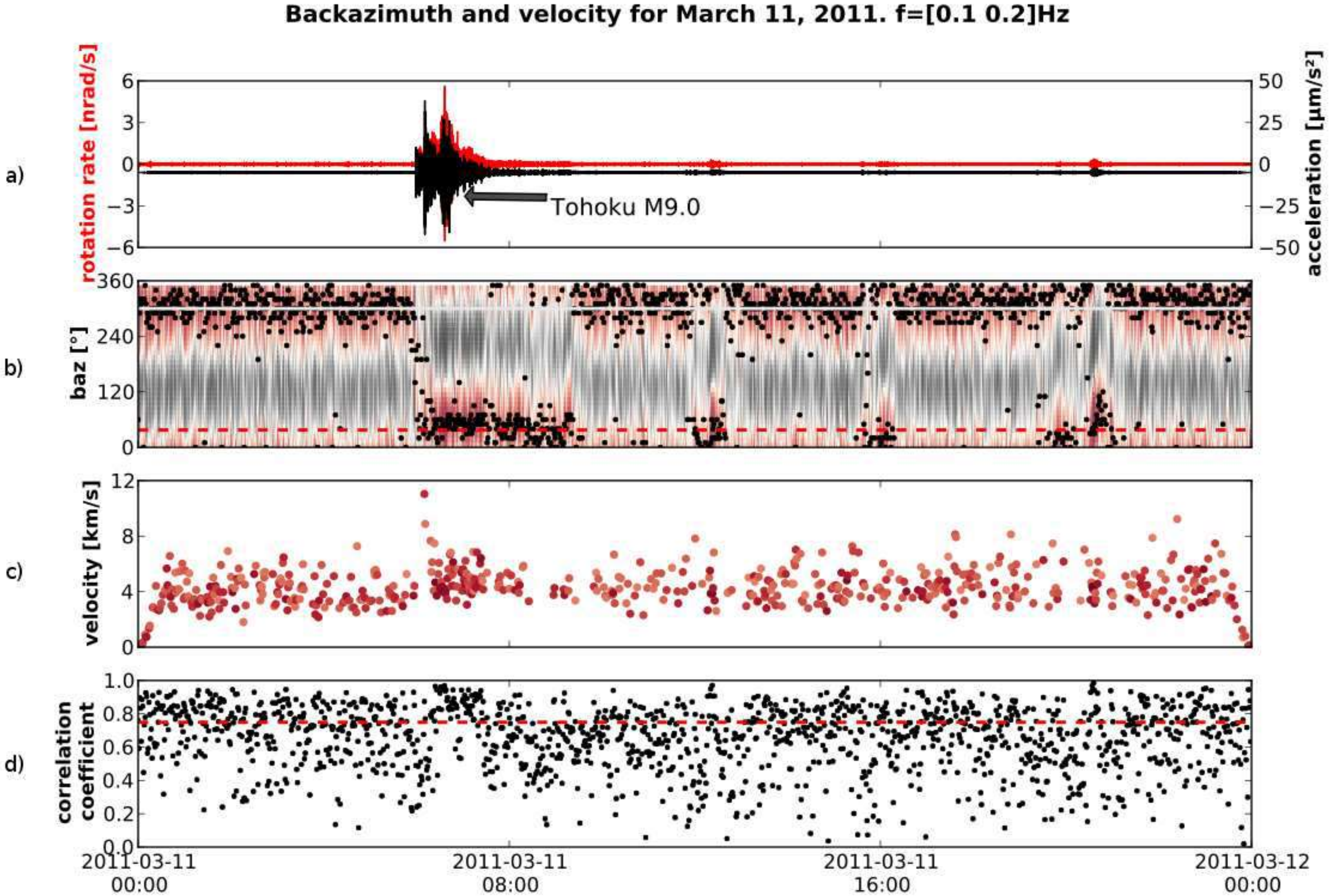
G Ring Observations Wettzell



Most Accurate 6 DoF Earthquake record to date!



Processing joint observations (translations, rotations)



Hadziioannou et al., 2012, J. Seis.

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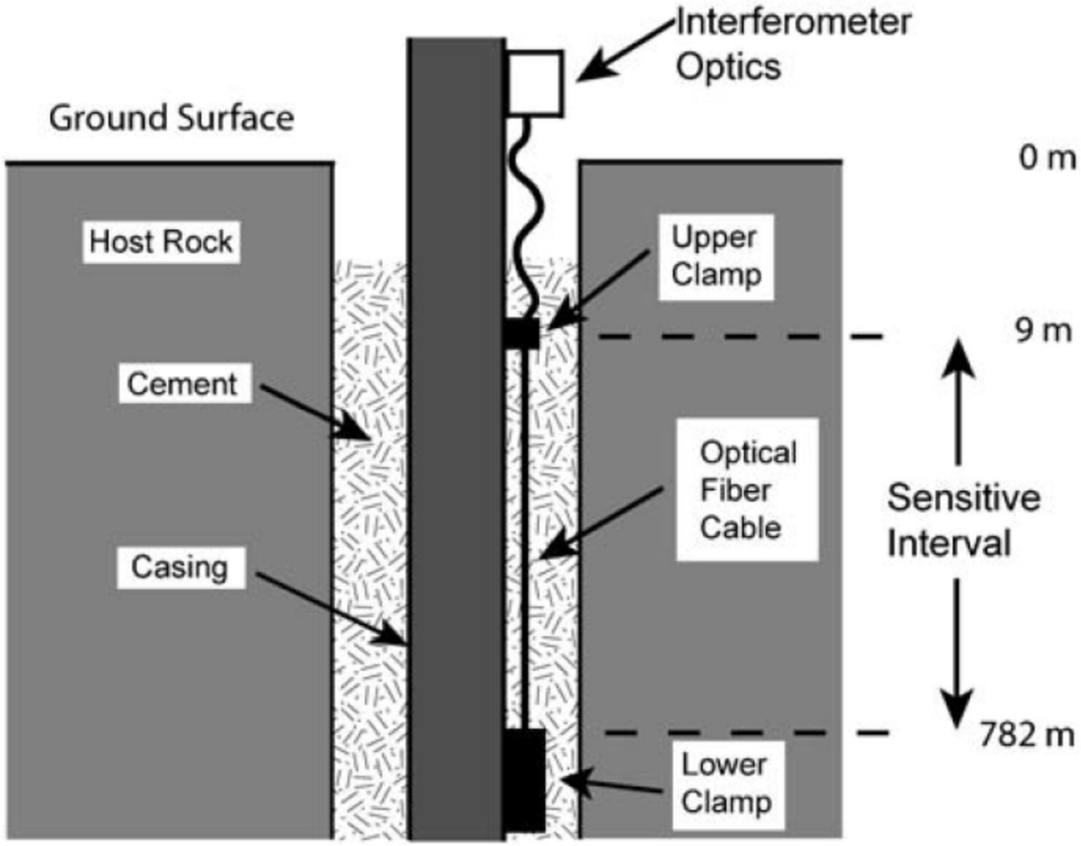
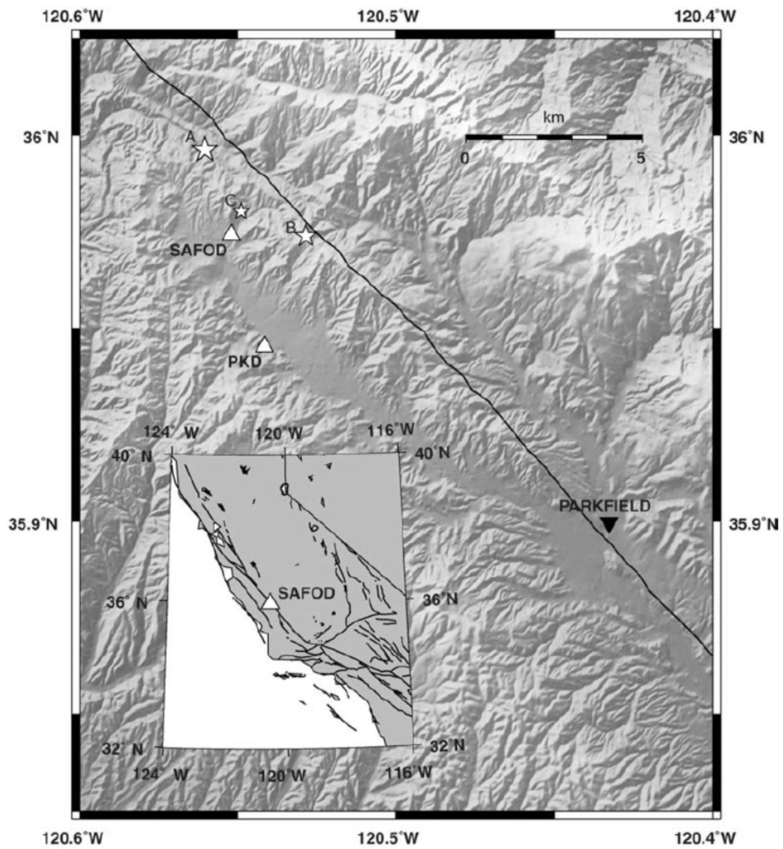
SAFOD


Interferometer Optics
Ground Surface
0 m
Host Rock
Upper Clamp
9 m
Cement
Optical Fiber Cable
Sensitive Interval
Casing
782 m
Lower Clamp


120.6°W
120.5°W
120.4°W
36°N
35.9°N
km
0
5
A
B
C
SAFOD
PKD
PARKFIELD
124° W
120°W
116°W
40° N
36° N
32° N

Synthetic seismograms

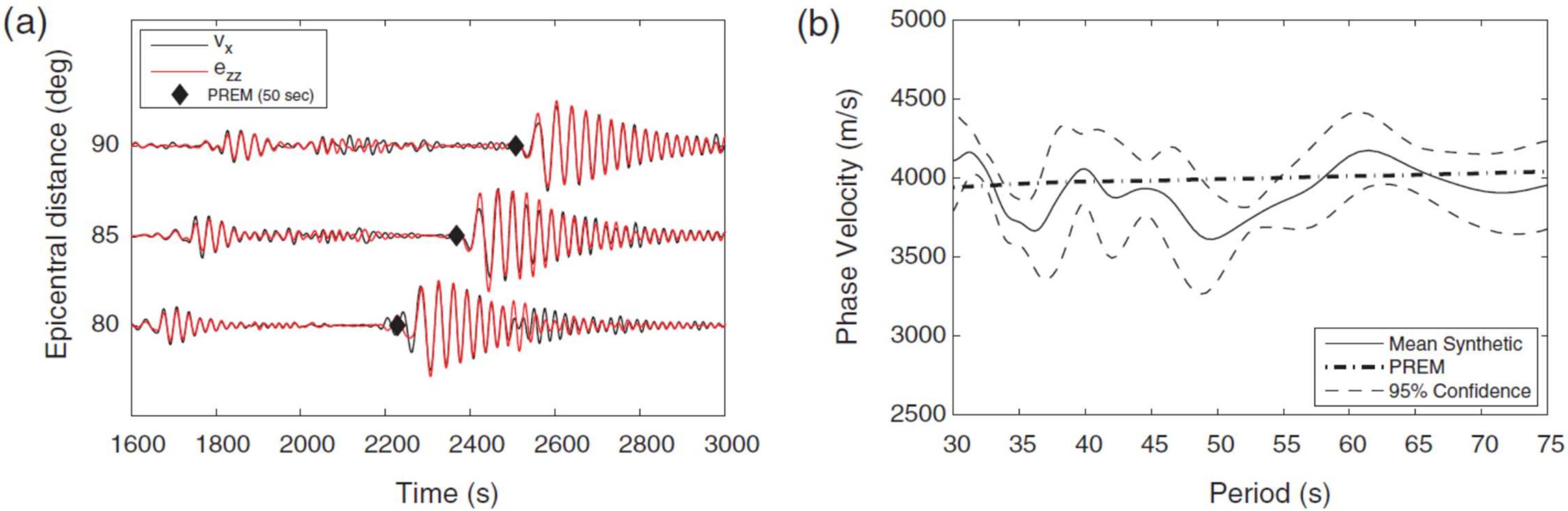
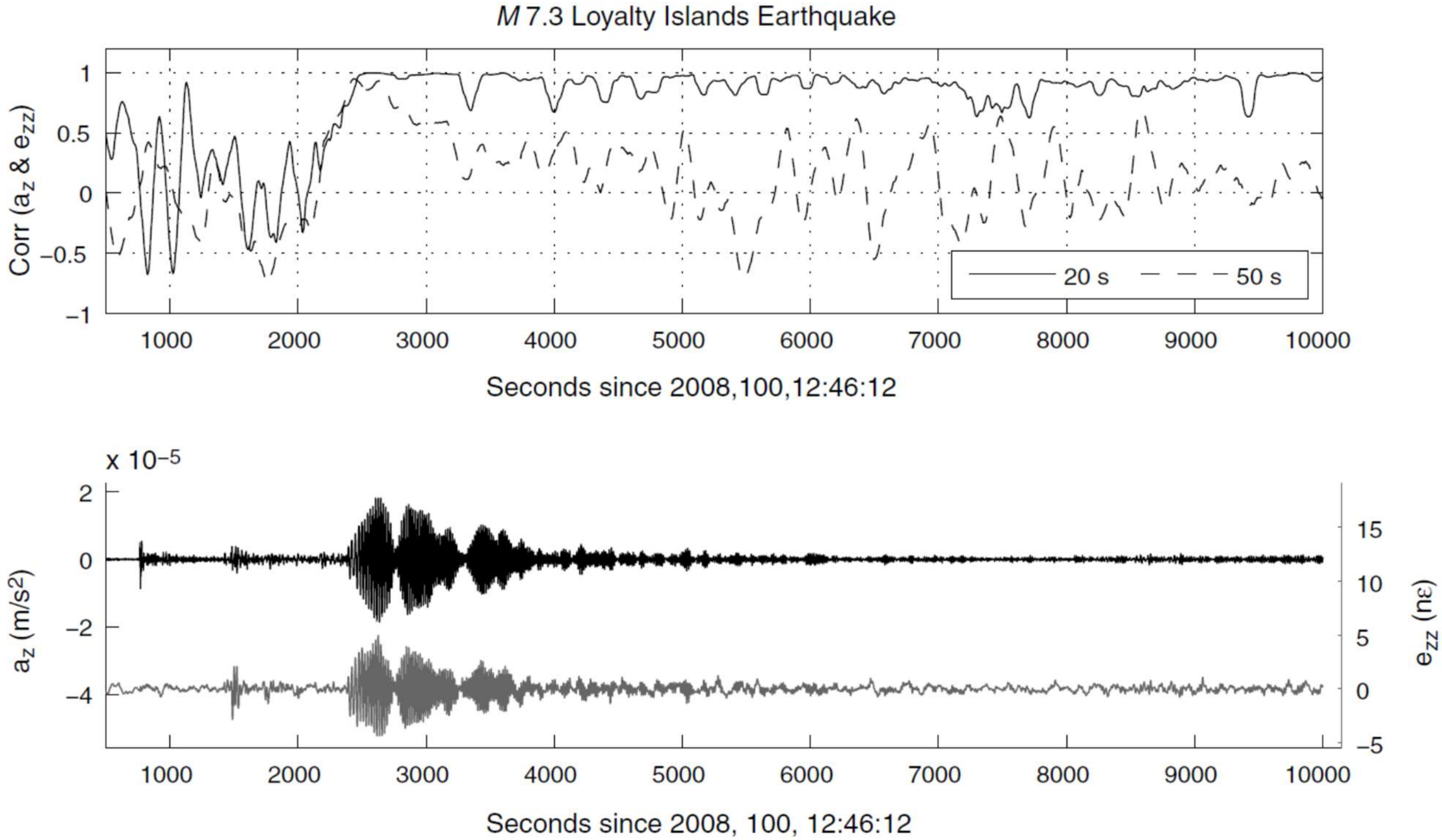


Figure 5. (a) Synthetic waveforms for a_z and e_{zz} recorded at epicentral receiver distances of 80, 85, and 90 degrees. Black diamonds denote the expected Rayleigh-wave arrival time for the PREM model at periods of 50 seconds. Waveforms were low-pass-filtered with a cutoff period of 30 seconds. (b) Phase velocities were estimated from synthetic spectra for 21 events; 95% confidence limits assume a normal distribution of errors.

Observations



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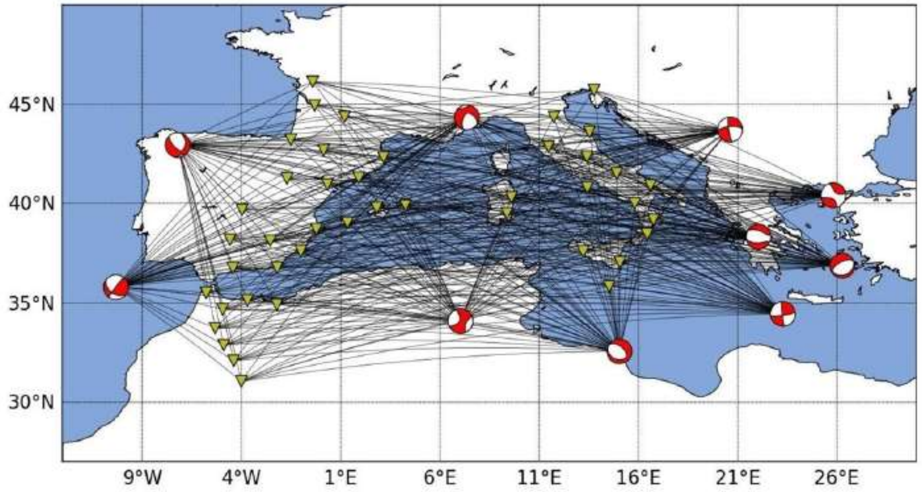
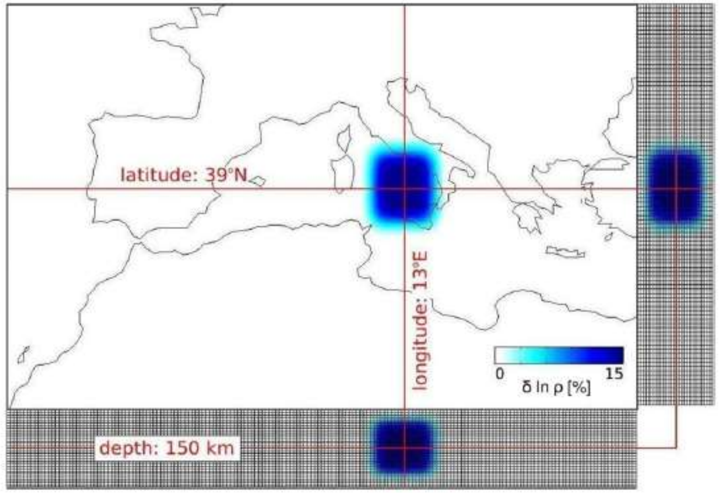
Sensitivities





Rotation

$$\frac{|\dot{\boldsymbol{u}}(\boldsymbol{x}^r)|}{|curl\ \boldsymbol{u}(\boldsymbol{x}^r)|} = \beta$$

Definition: apparent S wave speed

$$\beta_a(\boldsymbol{x}^r) \overset{\text{def}}{=} \frac{\|\dot{\boldsymbol{u}}(\boldsymbol{x}^r)\|_2}{\|curl\ \boldsymbol{u}(\boldsymbol{x}^r)\|_2}$$

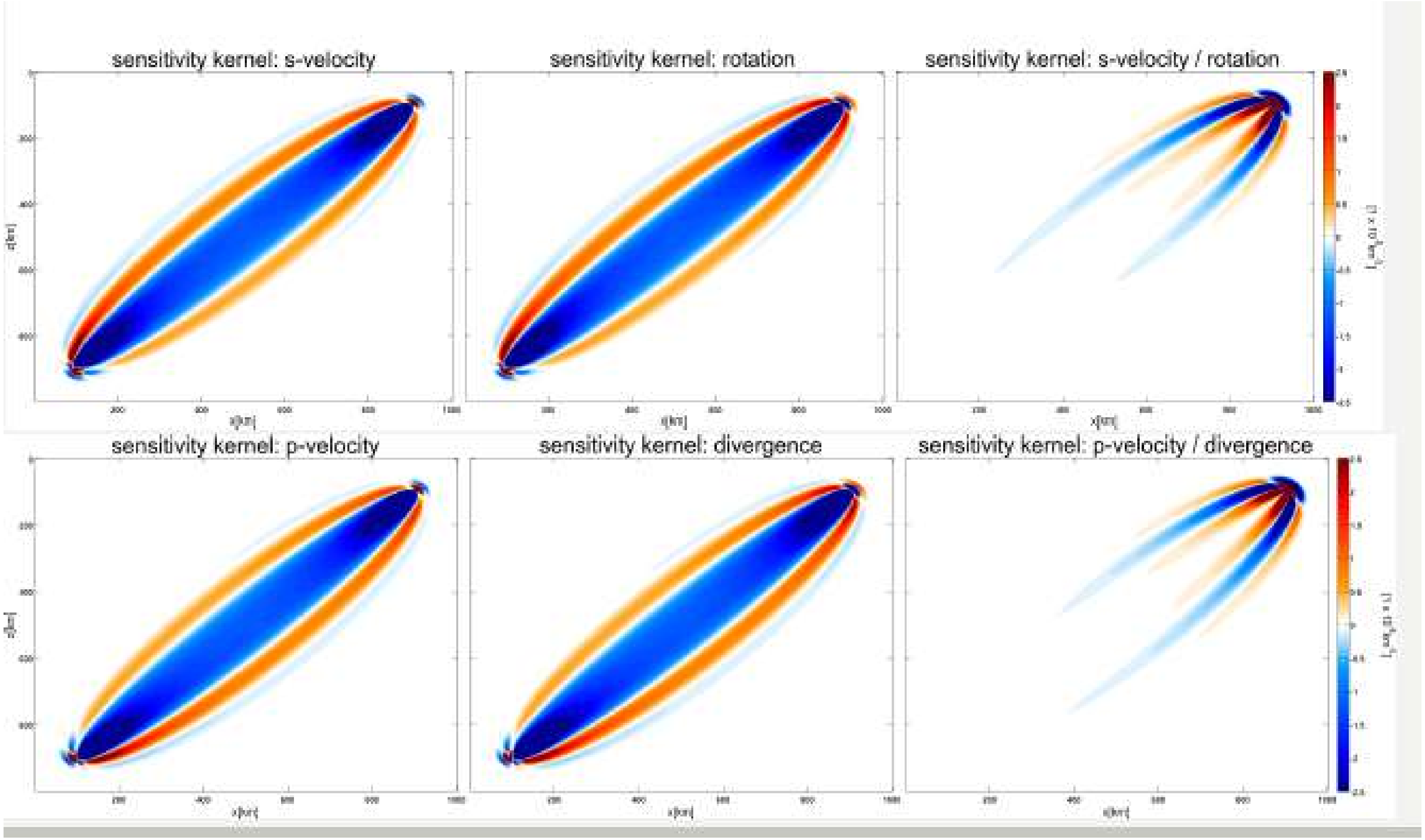
Strain

$$\frac{|\dot{\boldsymbol{u}}(\boldsymbol{x}^r)|}{|div\ \boldsymbol{u}(\boldsymbol{x}^r)|} = \alpha$$

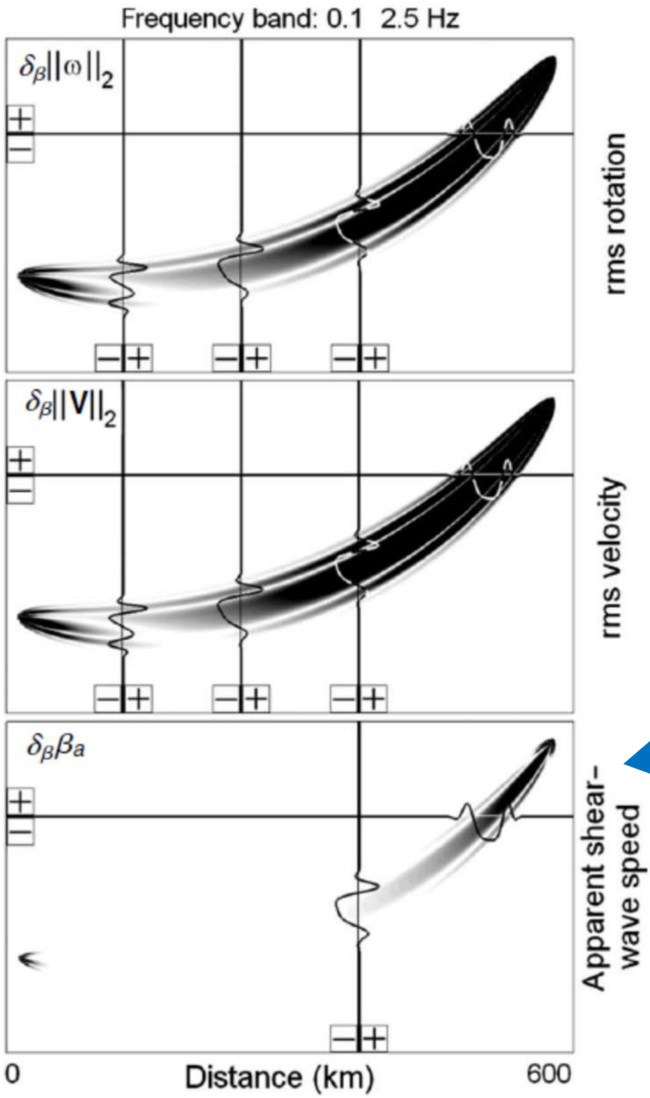
Definition: apparent P wave speed

$$\alpha_a(\boldsymbol{x}^r) \overset{\text{def}}{=} \frac{\|\dot{\boldsymbol{u}}(\boldsymbol{x}^r)\|_2}{\|div\ \boldsymbol{u}(\boldsymbol{x}^r)\|_2}$$

… works also with divergence …



Local Sensitivity



Local sensitivity to structure below receiver

$$\delta_\beta \ln \beta_a(\mathbf{x}) = -2\frac{\rho(\mathbf{x})}{\beta(\mathbf{x})}\mathbf{A}(\mathbf{x})\boxed{[\mathbf{K}^v(\mathbf{x}) - \mathbf{K}^\omega(\mathbf{x})]}$$

$$\mathbf{K}^v(\mathbf{x}) = \mathbf{A}^v(\mathbf{x}) \int \dot{s}^v(T_r(\mathbf{x}) - t)\, \dot{s}(t - T_s(\mathbf{x}))\, dt$$

$$\mathbf{K}^\omega(\mathbf{x}) = \mathbf{A}^\omega(\mathbf{x}) \int \dot{s}^\omega(T_r(\mathbf{x}) - t)\, \dot{s}(t - T_s(\mathbf{x}))\, dt$$

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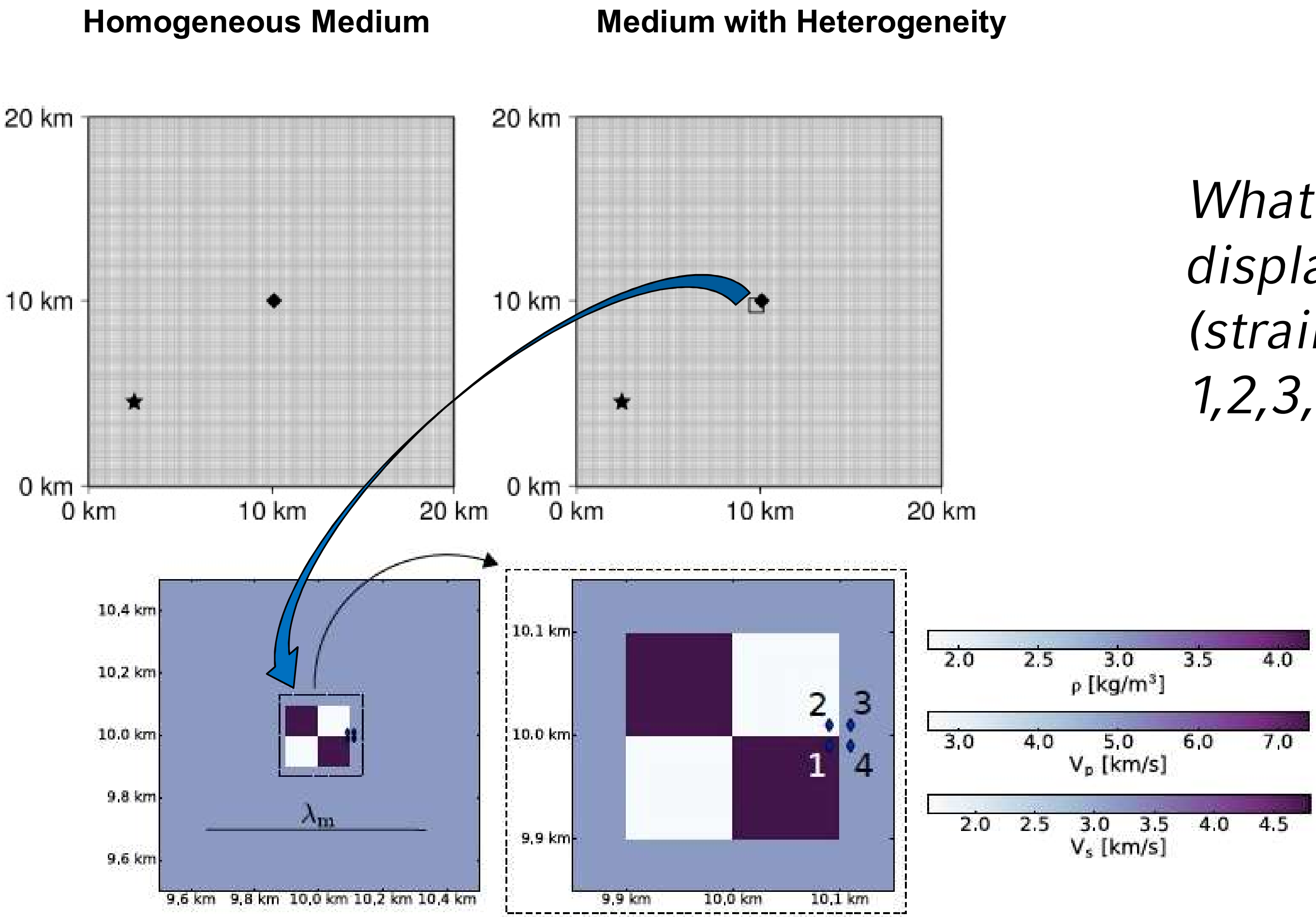
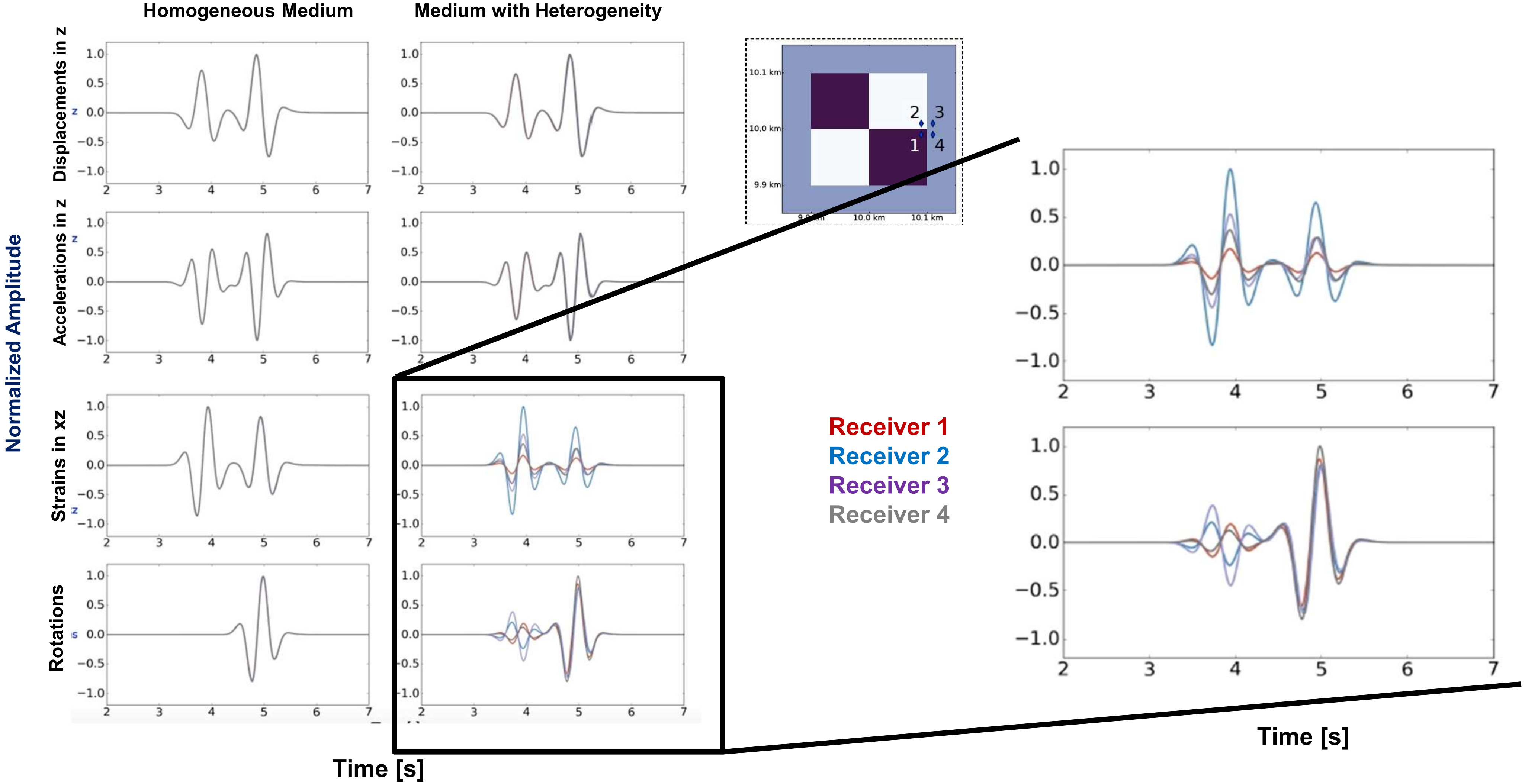
Heterogeneities - Homogenization



What do we measure as displacements and rotation (strain) at receiver locations 1,2,3, and 4?

Singh S., Capdeville Y., & Igel H., (2019, GJI)

Heterogeneities



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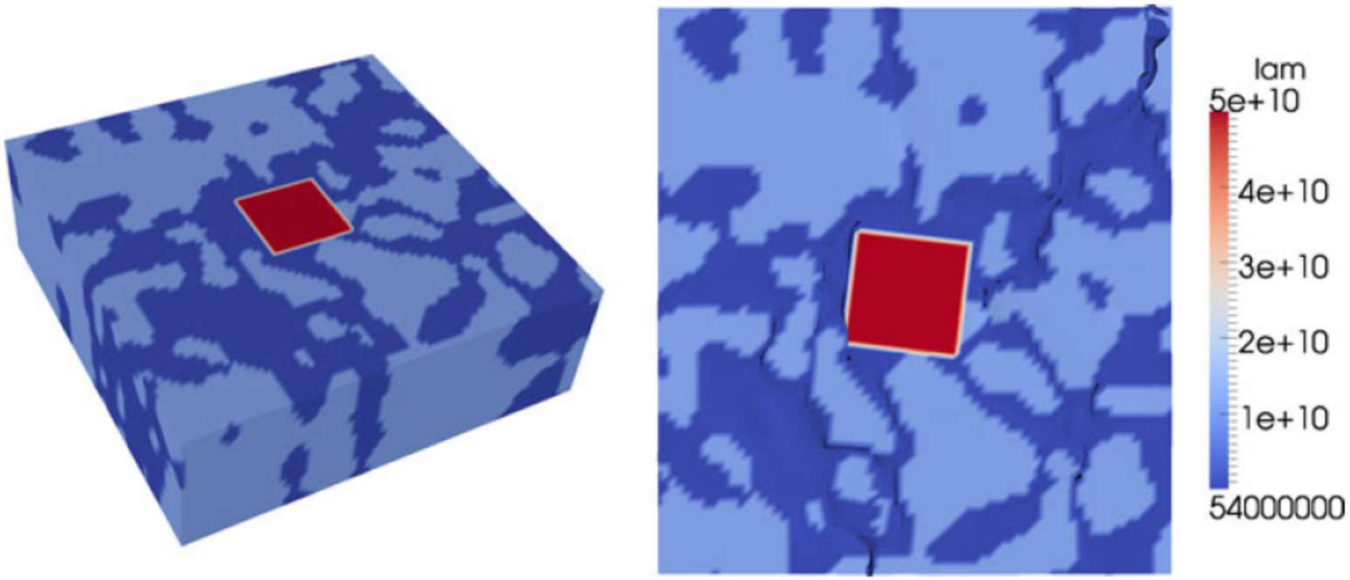
Strain-rotation coupling

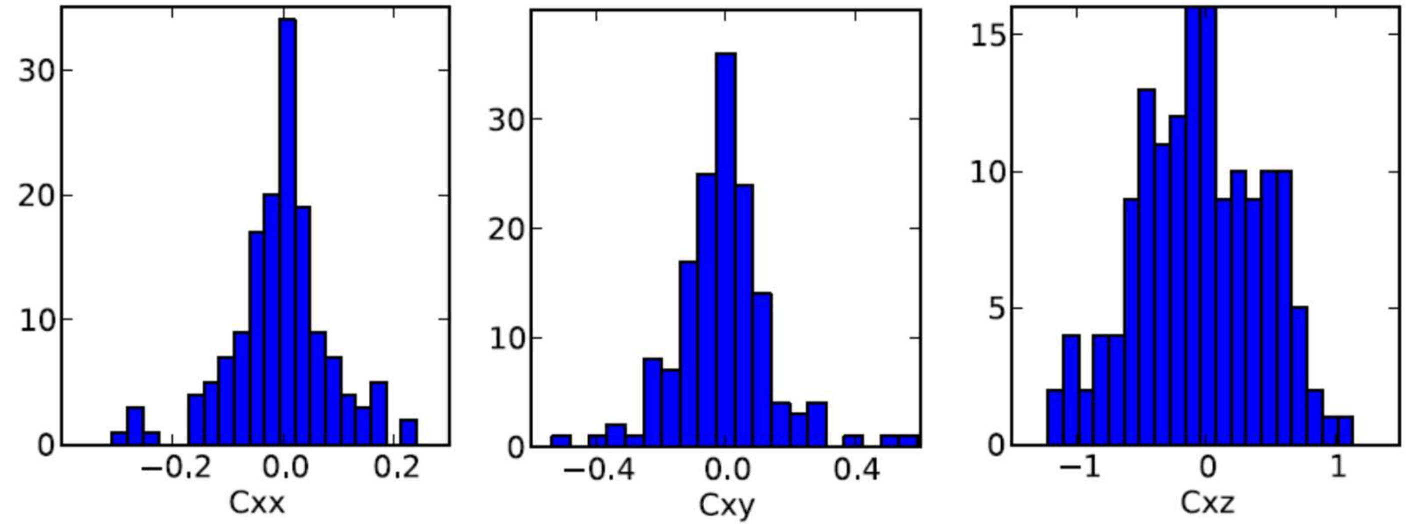
$$c_{ij} = \frac{\omega_j}{\epsilon_i} = \frac{\text{strain-induced rotation around } j\text{-axis}}{\text{strain component } i}$$

Fig. 1 Sample 3D random model $(3 \times 3 \times 1\ \mathrm{m}^3)$, *right*: imposed normal strain $\epsilon_{xx} = 10^{-6}$, deformation exaggerated by a factor of 10^5, local rotations caused by inhomogeneities



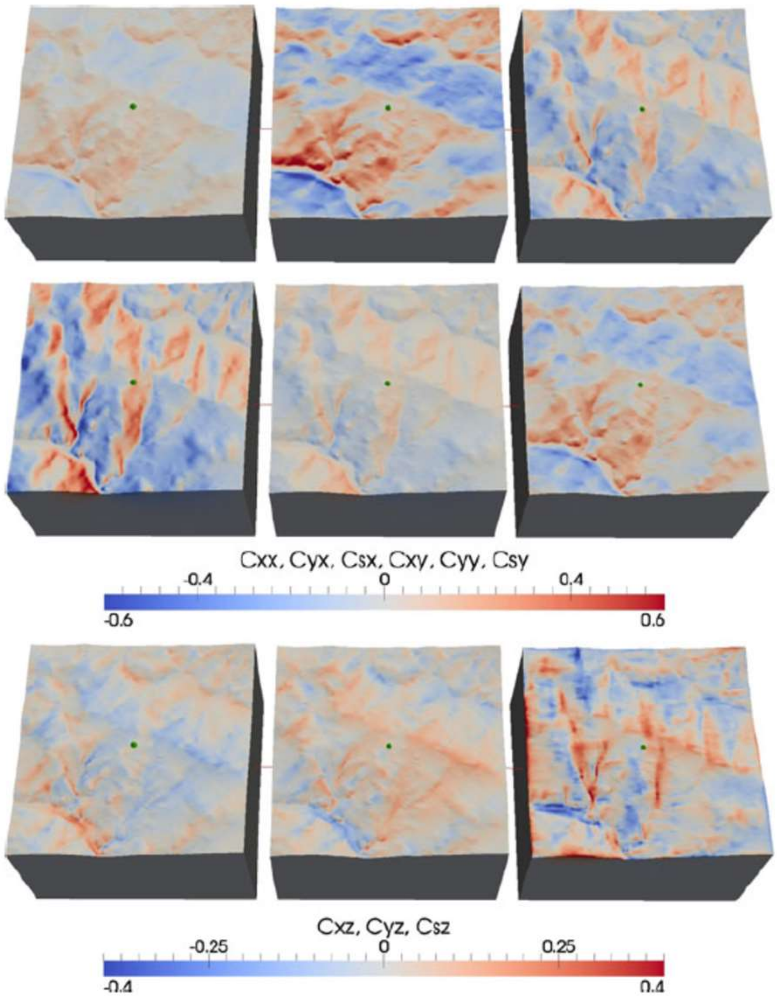
Strain-rotation coupling (heterogeneities)

Fig. 2 Histograms of coupling constants for 150 random models as in Fig. 1, strain in x-direction and rotation around x, y, and z, respectively, standard deviations are 0.09, 0.15, and 0.48

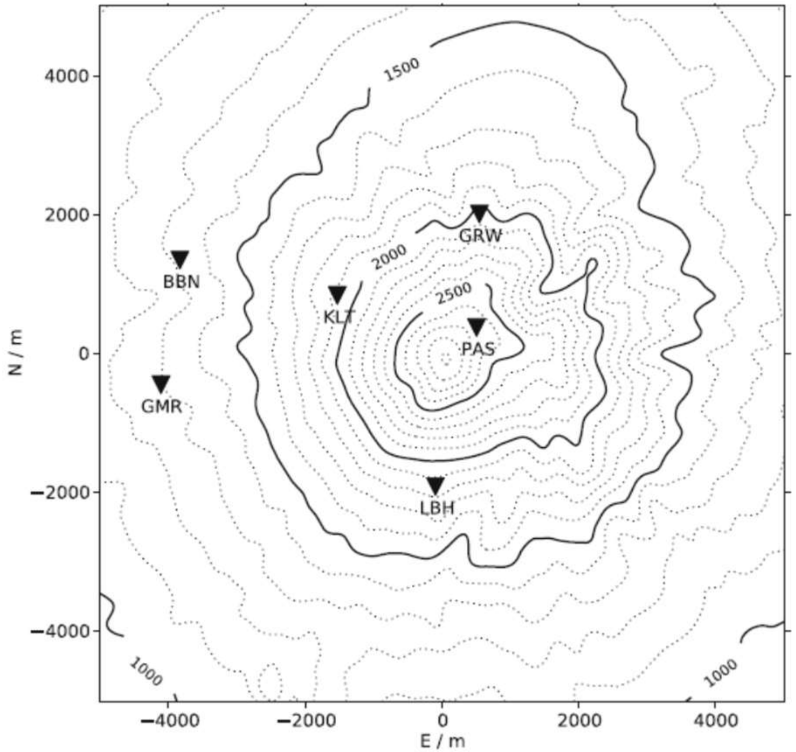


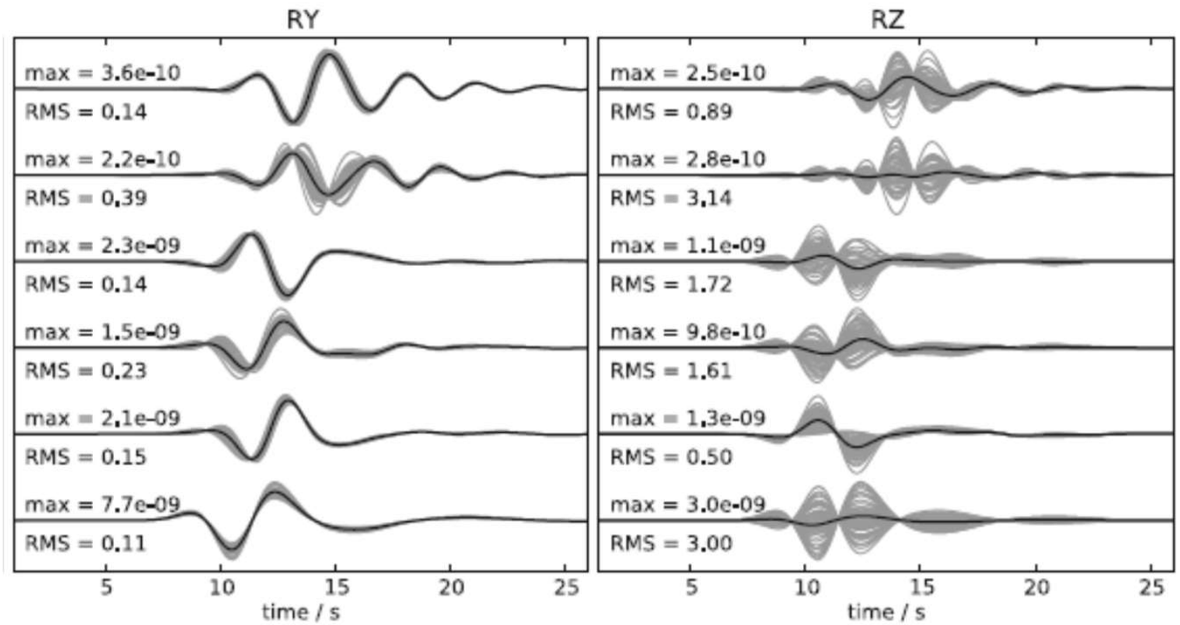
Strain-rotation coupling (topography)

Fig. 4 Wettzell area, 5 × 5 km: Strain rotation coupling constants c_{ij} as defined in Eq. 1 for normal strain in x (east–west, *left*), y (north–south, *center*) and horizontal shear strain (*right*) coupling into rotation around x-, y-, and z-axis (*top* to *bottom*) simulated for a homogeneous medium with topography (25 m resolution, DGM-D model of the German Federal Agency for Cartography and Geodesy), ring laser site is marked with *green cones*. *Upper colorbar* for the upper six models, *lower colorbar* for the lower three



Strain-rotation coupling (topography)





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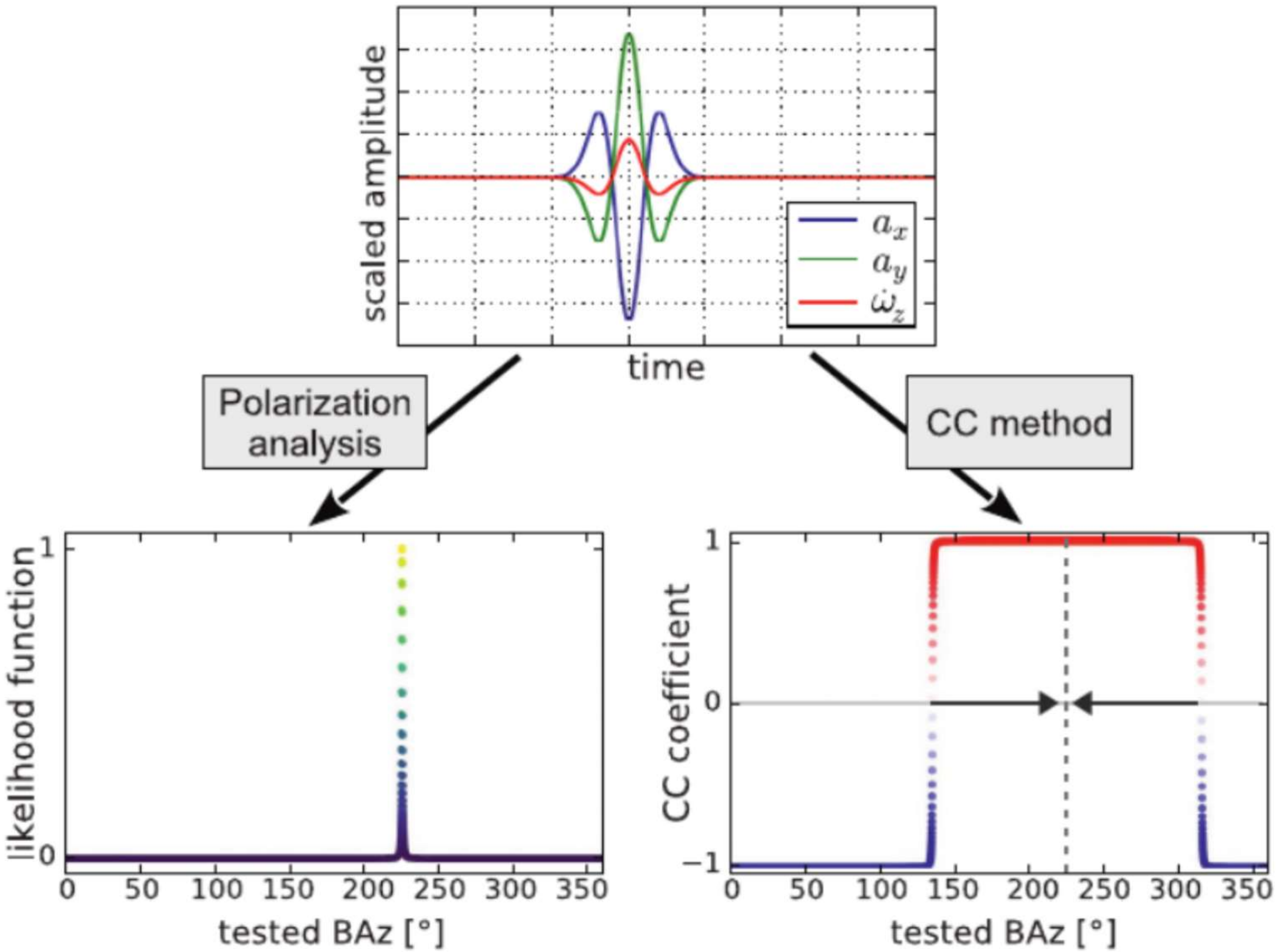
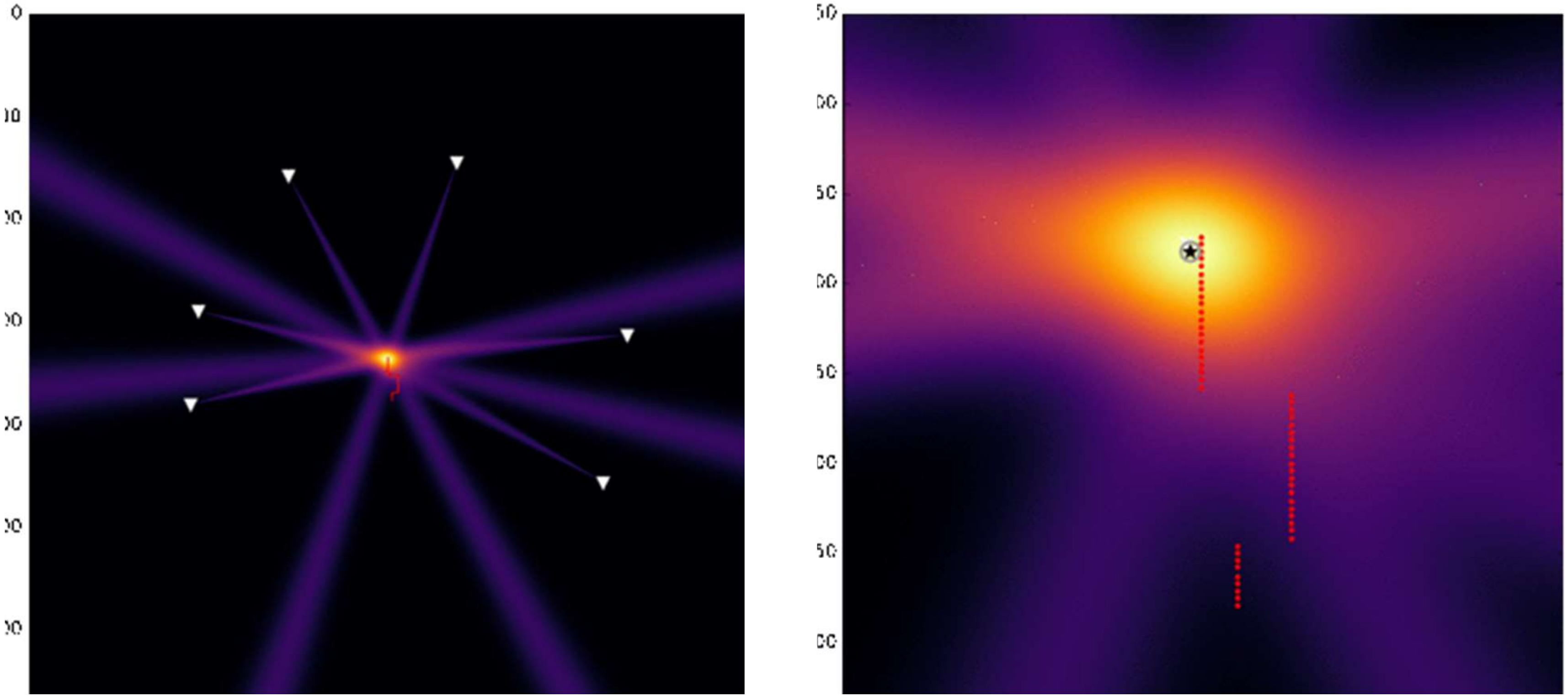
Propagation direction with 6C



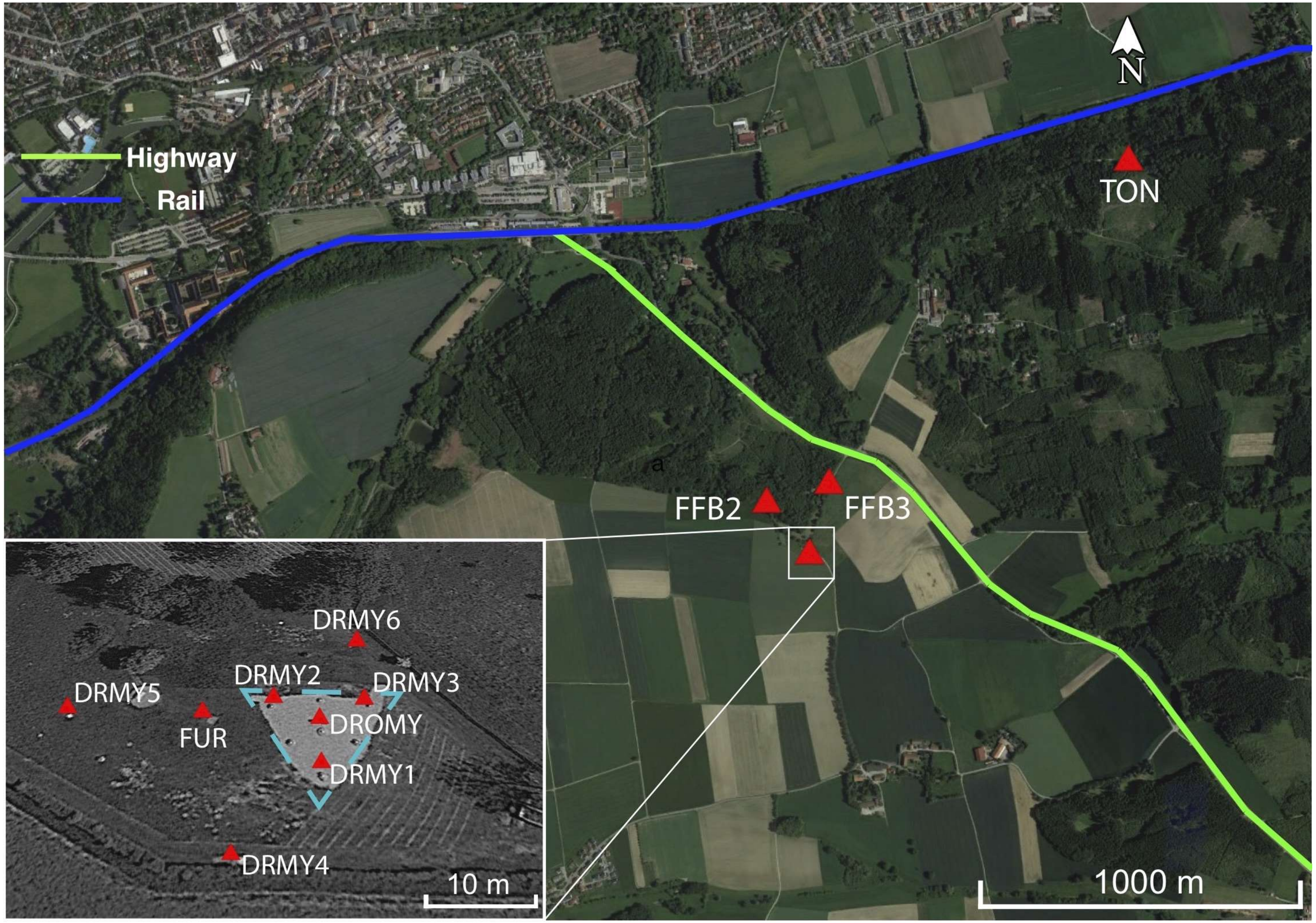
Seismic source tracking



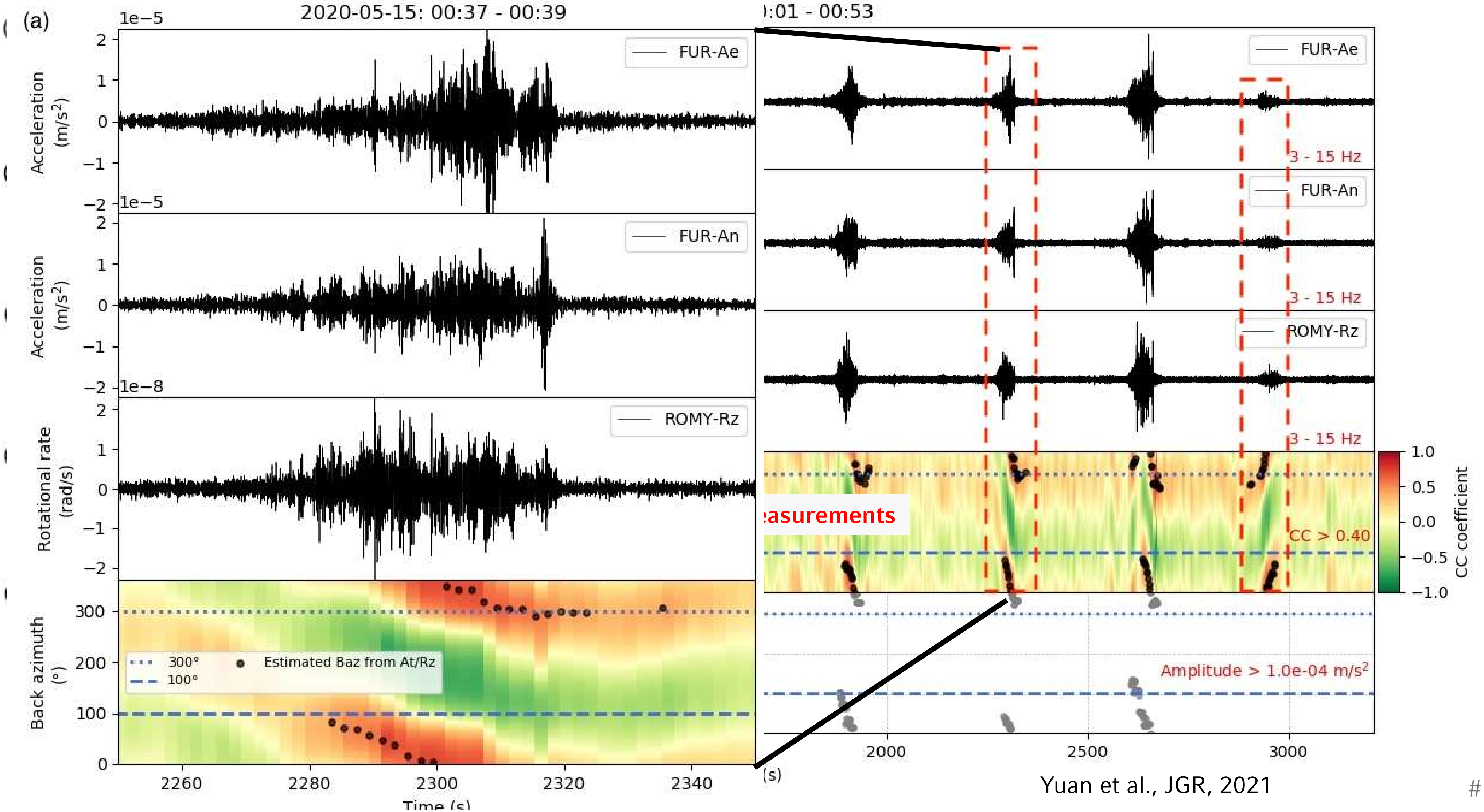
S wave back-azimuth estimations from 6 DoF point measurements

(Gessele, MSc thesis, 2019)

Singe-station speed control



Singe-station speed control (Yuan et al., JGR, 2020, subm.)



Yuan et al., JGR, 2021

Some conclusions

- Ground motion is **more than just $\mathbf{u}$**
- In fact it is $\mathbf{u}$ and $\nabla\mathbf{u}$
- But $\nabla\mathbf{u}$ is separated into ε and ξ
- **ε and ξ are gradient related quantities**
- ε measurements involve a spatial scale
- For rotations we observe $\nabla\times\mathbf{u}$
- $\nabla\times\mathbf{u}$ can (now) be directly measured (ring laser, blueSeis)
- **ε and ξ can be estimated from seismic arrays** (why bother measuring directly?)
- ε and ξ are classical linearized quantities and only work for small deformations
- Gradient observations have **increased near-receiver sensitivity**
- Systematic joint observations are just emerging
- Joint observations provide direct access to **phase velocities and propagation directions**

PFO seismic map