Exercises – Stress

1. **Symmetries:** With the definition of stress p_{ij} and traction P_i

$$P_i = p_{ij} n_j$$

Show that if $\overrightarrow{P}$ is the traction for direction $\overrightarrow{n}$, and $\overrightarrow{P'}$ for the direction $\overrightarrow{n'}$, it holds that $\overrightarrow{P}\overrightarrow{n'} = \overrightarrow{P'}\overrightarrow{n}$.

2. ***Principal stresses:*** The following stress tensor **σ** is given:

$$\begin{bmatrix} 1/4 & -5/4 & (\frac{1}{2})^{3/2} \\ -5/4 & 1/4 & -(\frac{1}{2})^{3/2} \\ (\frac{1}{2})^{3/2} & -(\frac{1}{2})^{3/2} & 3/2 \end{bmatrix}$$

Calculate the principal stresses (eigenvalues) and their directions. What is the direction of the largest stress (i.e., angels with the axes)? Hint: Best to do this with a Jupyter notebook. Try to plot the principle stress directions in 3D with arrows (or an ellipsoid).

3. Stress of seismic waves: What are realistic stresses for seismic waves inside the Earth? Formulate (see previous exercise sheet) a plane displacement wave (P and/or S) with Amplitude A, wave speed c, and frequency f. With density ρ=2500 kg/m^3, calculate the Lamé constants and the absolute value of the stress tensor components

$$\sigma_{ij} = \lambda \delta_{ij} \epsilon_{kk} + 2\mu \epsilon_{ij}$$

Hint: Write a Jupyter notebook. It may be instructive to fix amplitude and speed and plot the maximum stress as a function of frequency.