Exercises – Strain and Rotation

Theory and Programming Exercises

1. Split the deformation tensor ε_{ij} into one part that is pure shear (no volume change) and into one part that is pure volume change. Show using Gauss' Law that the relative volume change is ε_{ii} (follow lecture).

2. The interior of a volcano deforms in three directions (N,E,Z) by $\Delta u = (0.3, -0.1, 0.4)$ m for a hexahedral body of side length (1.5km, 3.3km, 3km). Formulate displacement field **u**(**x**) and the elements of the corresponding strain tensor **ε** inside the body.

3. Write a Jupyter notebook that calculates the action of an arbitrary strain field in 2D (tensor) on a vector and visualize the result. Define vectors **a** and **b** and calculate with vector operations the area of the parallelogram it spans. Quantify the area change due to the strain field.

4. Using the results from the previous notebook compare for appropriate strain tensors the length change (before deformation after deformation) using vector operations (exact) and the linearization (l_0 is length of **y**, l is length of **y** after deformation). Show the error of the linearization as a function of absolute (mean) strain value.

$$\frac{l - l_0}{l_0} = \epsilon_{ij}\frac{y_i y_j}{l_0^2}$$

5. A coordinate system is defined such that x-E, y-N, z-vertical. A plane shear wave is propagating horizontally at an angle of 10° (counterclockwise) w.r.t. the x-axis. The amplitude is 1nm, the frequency is 0.1Hz and the phase velocity is 4km/s. Formulate a sinusoidal displacement field **u** and its gradient components $\partial_j u_i$. Derive the elements of the strain and rotation tensors. What is the action of the rotation tensor on an arbitrary vector inside the medium? Hint: You can also formulate it generally in terms of amplitudes, propagation direction etc in a Jupyter notebook. It might also be interesting to show the motions of all (non-zero) displacement, strain, and rotation components.

6. Derive the equations for strain and rotation tensors from the formulation of the wavefield gradient. Show that the rotation tensor applied to vector y: $\xi_{ij}y_j$ is equivalent to $1/2(\nabla \times \boldsymbol{u}) \times$ **y.**

$$\nabla\boldsymbol{u} = \frac{\partial u_i}{\partial x_j}$$

   With

$$\xi = (-\xi_{23}, \xi_{13}, -\xi_{12}) \qquad \text{(this is a vector!)}$$

   Show that $|\xi|$ is the absolute rotation angle of vector y.

7. The motion of Rayleigh waves in a homogeneous halfspace can be described by (c is phase velocity, k is wavenumber)

$$u_x = C(e^{-0.8475kz} - 0.5773e^{-0.3933kz})\sin k(ct - x)$$
$$u_z = C(-0.8475e^{-0.8475kz} + 1.4679e^{-0.3933kz})\cos k(ct - x)$$

Calculate analytically the strain components of this wavefield. Calculate the ratio of vertical acceleration $\ddot{u}_z$ and ε_{zz} and the ratio of horizontal velocity $\dot{u}_x$ and ε_{xx} velocity) and y-component of rotation rate.

8. The free surface condition

$$\sigma_{i3} = 0 \ (i = x, y, z).$$

implies that

$$\frac{\partial u_x}{\partial x_z} = -\frac{\partial u_z}{\partial x_x} \quad ; \quad \frac{\partial u_y}{\partial x_z} = -\frac{\partial u_z}{\partial x_y} \quad ; \quad \frac{\partial u_z}{\partial x_z} = -\frac{\lambda}{\lambda + 2\mu}\left(\frac{\partial u_x}{\partial x_x} + \frac{\partial u_y}{\partial x_y}\right)$$

Assuming an isotropic stress-strain relation

$$\sigma_{ij} = \lambda \epsilon_{kk} \delta_{ij} + 2\mu \epsilon_{ij}$$

Show that this implies at the free surface (geometrical meaning?)

$$\omega_x = \frac{\partial u_z}{\partial x_y} \quad ; \quad \omega_y = -\frac{\partial u_z}{\partial x_x}$$

Comprehension Questions

9. What could be the reason why array-derived strain or rotation and directly measured strain or rotation are rarely the same (but very close)? What information could be contained in the differential observation?

10. Normally three seismometer locations are required to determine the location of an earthquake in an elastic half space. Explain why – in principle – one 6-C (translation and rotation) record may be enough to locate an earthquake (assume that you record both surface waves and body waves.

11. Explain why the joint observation of rotation (strain) and translation has a strong near-receiver sensitivity. What are potential applications?

12. As you know standard seismometer records have permanent signals generate in the oceans. Do you expect those signals also to appear on all rotation sensors components? Give reasons.

13. What do you think is more reliable in seismology: measurements of travel times (phase) or absolute amplitudes (as used to estimate ratios of strain/rotation and translations)

14. DAS (distributed acoustic sensing) allows measuring along-cable strains. Is it possible to distinguish shear and P waves?