Ice and Climate: SIA simulation of an artic glacier

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1 Introduction

Glaciers constitute an important part of the global climate. A rising air temperature as a consequence of globabal warming may result in a reduction of glacier area worldwide.

In this report, we simulate the length, mass and flowline of a typical arctic glacier. For our simulation, we use the shallow-ice approximation (SIA) in combination with Weertman-sliding. The air temperature increase is modelled by a rise of the equilibrium altitude (ELA): The ELA is the altitude, where ablation and accumulation processes cancel each other out. With rising temperatures, the ELA is expected to increase.

2 Governing equations

The flux of the model is given by the following equations:

$$\begin{aligned} F &= H \cdot U \\ &= H \cdot (U_d + U_s) \\ &= H \cdot \left(\frac{2}{5} f_d H \tau_d^3 + f_s H^-1 \tau_d^3 \right) \\ &= \frac{2}{5} f_d \left(\rho g\right)^3 H^5 \left(\frac{\partial h}{\partial x} \right)^3 + f_s \left(\rho g\right)^3 H^3 \left(\frac{\partial h}{\partial x} \right)^3 \end{aligned} \tag{1}$$

Given the flux, the change in ice thickness can be calculated via:

$$\frac{\partial H}{\partial t} = \frac{\partial F}{\partial x} + \dot{b} \tag{2}$$

We adopt the following spatially discretized scheme to update the flux:

$$F_{i+1/2} = \frac{2}{5} f_d \left(\rho g\right)^3 \cdot H_{i+1/2}^5 \left(\frac{\partial h}{\partial x} \right)_{i+1/2}^3 \tag{3}$$

Here,

$$H_{i+1/2} \approx \frac{H_{i+1} - Hi}{2} \quad \text{and} \quad \left(\frac{\partial h}{\partial x} \right)_{i+1/2} \approx \frac{h_{i+1} - h_i}{\Delta x} \tag{4}$$

That is, we calculate the fluxes between the grid points using the averaged values of thickness of the two neighbouring points and approximate the gradient $\frac{\partial h}{\partial x}$ by central finite differences.

The ice thickness is updated on the grid points using a forward Euler scheme:

$$\begin{aligned} H_i^{t_0+\Delta t} &= H_i^{t_0} + \Delta t \cdot \left(\frac{\partial H}{\partial t}\right)_i^{t_0} + \mathcal{O}(\Delta t^2) \\ &= H_i^{t_0} + \Delta t \cdot \left(-\frac{\partial F}{\partial x} + \dot{b}\right)_i^{t_0} + \mathcal{O}(\Delta t^2) \end{aligned} \tag{5}$$

Here, the flux derivative is given by

$$\left(\frac{\partial F}{\partial x}\right)_i^{t_0} = \frac{F_{i+1/2}^{t_0} - F_{i-1/2}^{t_0}}{\Delta x}, \tag{6}$$

i.e. we use central finite differences with the flux values between grid points.

3 Simulation

In order to analyse the flow line of a real world Arctic glacier, as well as estimate the sensitivity of the glacier mass and response time to changes in the ELA (research question 8) first the bedrock (B) was adjusted to:

$$B = 1700 - 0.05 \cdot x + 100 \cdot e^{-0.001 \cdot x} \tag{7}$$

This corresponds to a moderate slope of the bedrock with a steep increase in altitude at the top, consistent with a real world arctic glacier with an ELA around 400 m below the highest point. The shape of the bedrock along with the simulation of the glacier formation can be seen in the animation attached.
All plots and the animation are simulated for a time frame of 5000 years. For the first 1000 years the ELA remains unchanged at 1400 m in order to reach an equilibrium for the glacier build-up. After these 1000 years the ELA is changed by 25 m every 500 years until the highest ELA of 1600 m is reached. Figure 1 shows the glacier volume over a 5000-year period as the ELA increases. The figure illustrates a decrease

ELA [m]	1425	1450	1475	1500	1525	1550	1575	1600
Mass Loss [10^6kg]	-302.9	-345.3	-347.4	-359.7	-365.4	-371.8	-392.9	-417.8

Table 1: The mass loss for an increasing ELA

in glacier volume i.e. mass with a rising ELA. As the ELA is changed there is an initial exponential drop-off of the glacier volume until it reaches a new equilibrium. Additionally, the figure indicates that the rate of mass loss accelerates as the ELA increases, resulting in greater mass loss at a higher ELA. This is quantified more clearly in table 1 where the mass loss for each ELA step is shown. That is, our simulation shows an accelerated mass loss of the arctic glacier due to the increase in ELA.

Figure 3 illustrates the response time for the ELA range discussed. The response times show some variation, but no significant trend can be identified. The average response time across the data is approximately 154 years. The flow line is visualized in figure 2 as a line from the highest to the lowest point of the glacier. By increasing the ELA, the flow line becomes steeper and its length decreases.

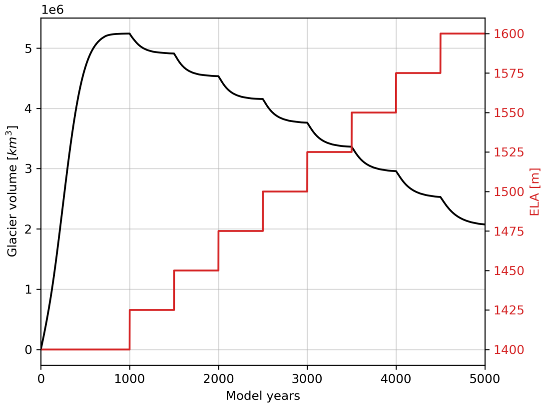

Figure 1: The volume of the glacier depending on simulated years is illustrated for changing values of the ELA

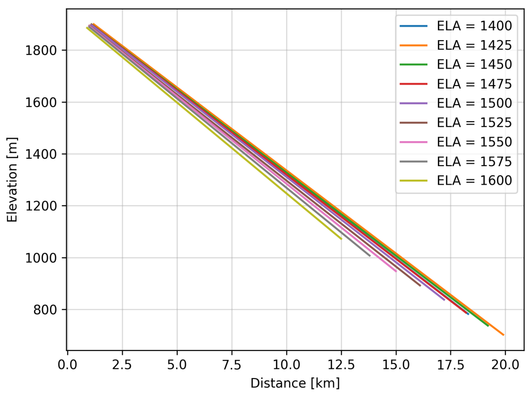

Figure 2: Flow line for a varying ELA

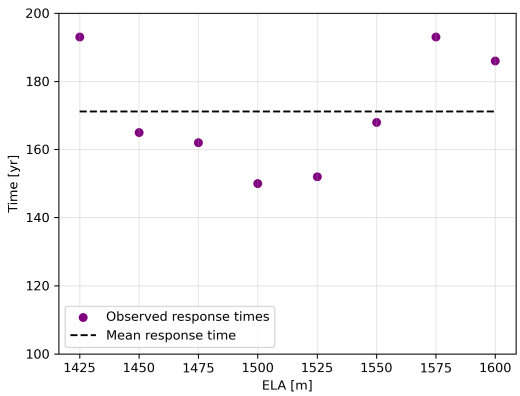

Figure 3: Response time for a varying ELA