$$S_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

$$S_x = \sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$$y_c = (y_1 - \bar{y}, \dots, y_i - \bar{y}, \dots, y_n - \bar{y})'$$
$$x_c = (x_1 - \bar{x}, \dots, x_i - \bar{x}, \dots, x_n - \bar{x})'$$

$$\Longrightarrow r_{xy} = \frac{x_c' y_c}{\|x_c\| \|y_c\|} \overset{*}{=} \frac{S_{xy}}{S_x S_y}$$

$$a'b = (a_1, \dots, a_n) \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = \sum_{i=1}^{n} a_i b_i$$

$$\|a\| = \sqrt{a'a} = \sqrt{\sum_{i=1}^{n} a_i^2}$$

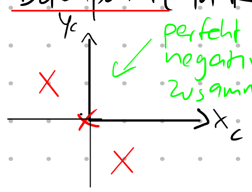
$$\Longrightarrow \begin{aligned} x_c' y_c &= S_{xy} \\ \|x_c\| &= S_x \end{aligned} \quad *$$

Geometrische Intuition:

Daten:

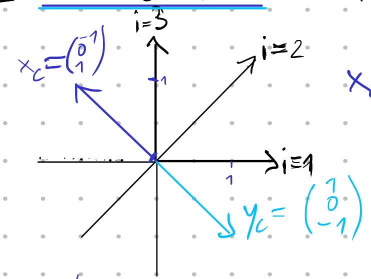
i	x_c	y_c
1	-1	1
2	0	0
3	1	-1

$\Longrightarrow$ n Datenpunkte in $\mathbb{R}^2$



perfekt linearer negativer Zusammenhang

$\Longrightarrow$ 2 Merkmalsvektoren in $\mathbb{R}^n$



$x_c \| y_c$

$$x_c' y_c = -1 \cdot 1 + 0 \cdot 0 + 1 \cdot -1 = -2$$

$$\|x_c\| = \|y_c\| = \sqrt{1^2 + 0^2 + (-1)^2} = \sqrt{2}$$

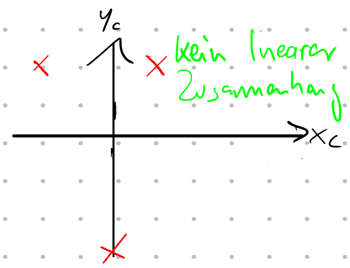
$$\Longrightarrow \frac{x_c' y_c}{\|x_c\| \|y_c\|} = \frac{-2}{\sqrt{2}\sqrt{2}} = -1$$

$$\Longrightarrow \measuredangle(x_c, y_c) = 180°$$

Daten:

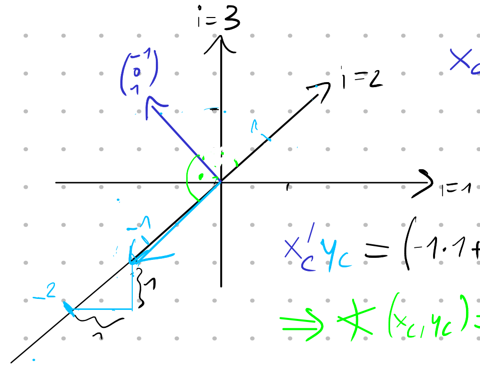
i	x_c	y_c
1	-1	1
2	0	-2
3	1	1

$\Longrightarrow$



kein linearer Zusammenhang

$\Longrightarrow$



$x_c \perp y_c$

$$x_c' y_c = (-1 \cdot 1 + 0 \cdot -2 + 1 \cdot 1) = 0$$

$$\Longrightarrow \measuredangle(x_c, y_c) = 90°$$

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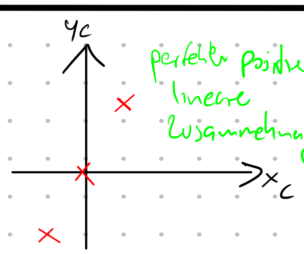
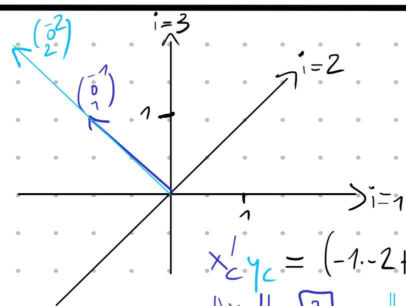
i	x_c	y_c
1	-1	-2
2	0	0
3	1	2

$\Longrightarrow$



perfekter positiver linearer Zusammenhang

$\Longrightarrow$



$x_c \| y_c$

$$x_c' y_c = (-1 \cdot -2 + 0 \cdot 0 + 1 \cdot 2) = 4$$

$\|x_c\| = \sqrt{2}$ $\quad$ $\|y_c\| = \sqrt{8}$

$$\Longrightarrow \frac{x_c' y_c}{\|x_c\| \|y_c\|} = \frac{4}{\sqrt{16}} = 1$$

$$\Longrightarrow \measuredangle(x_c, y_c) = 0°$$