A Multi-Level, Distance-Weighted Microprice for Limit Order Books

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Abstract

We present a generalized microprice estimator for limit order books that accounts for multiple levels of depth and discounts their influence exponentially in distance. This construction extends the classical top-of-book microprice and provides a smoother, more robust measure of instantaneous fair value. We derive the estimator formally, discuss its properties, and illustrate its behavior on a realistic order book snapshot. Applications to market making, execution, and short-horizon alpha generation are briefly discussed.

1 Motivation and Applications

Microprice-based estimators play a central role in high-frequency trading and market making. The standard top-of-book microprice reacts only to the best bid and ask sizes, which may be susceptible to fleeting orders and spoofing. By incorporating additional depth information, a multi-level estimator yields a fair value that is less sensitive to short-lived quote updates while remaining highly responsive to genuine shifts in supply and demand.

Such fair value estimates are used in:

- **Market Making:** Centering bid and ask quotes around the microprice and adding inventory-dependent skew improves execution quality and inventory control.
- **Optimal Execution:** Deciding whether to join, improve, or step back from the touch can be guided by the position of the microprice relative to the mid-price.
- **Predictive Signals:** Deviations between microprice and mid-price often correlate with short-term price movements, producing alpha signals.

2 Classical Microprice and Its Limitations

Let B_1 and A_1 denote the best bid and ask prices with volumes V_1^b and V_1^a. The classical microprice (Cartea, Jaimungal, and Penalva, 2015) is defined as

$$\mathrm{MP}_{\mathrm{TOB}} = \frac{V_1^a B_1 + V_1^b A_1}{V_1^a + V_1^b}. \tag{1}$$

This construction weights each side's price by the *opposite* side's volume, reflecting the probability that the next trade occurs at that price. When the ask side is heavy, $\mathrm{MP}_{\mathrm{TOB}}$ tilts toward B_1, anticipating a likely downward trade.

However, (1) ignores liquidity beyond the best level. Large resting volume one or two ticks away often stabilizes prices and should shift fair value in its direction. The multi-level estimator below addresses this.

3 Multi-Level, Distance-Weighted Microprice

Consider L bid levels (B_ℓ, V_ℓ^b) sorted $B_1 > B_2 > \cdots > B_L$ and L ask levels (A_ℓ, V_ℓ^a) sorted $A_1 < A_2 < \cdots < A_L$. Let the tick size be $\tau > 0$. Define the tick distance from the best quote on each side separately:

$$d_\ell^b = \max\bigl\{\mathrm{round}\bigl((B_1 - B_\ell)/\tau\bigr), 0\bigr\}, \qquad d_\ell^a = \max\bigl\{\mathrm{round}\bigl((A_\ell - A_1)/\tau\bigr), 0\bigr\}. \tag{2}$$

Assign an exponential weight to each level with decay parameter $\alpha > 0$:

$$w_\ell^b = e^{-\alpha d_\ell^b}, \qquad w_\ell^a = e^{-\alpha d_\ell^a}. \tag{3}$$

The effective side volumes are

$$\widetilde{V}^b = \sum_{\ell=1}^{L} w_\ell^b V_\ell^b, \qquad \widetilde{V}^a = \sum_{\ell=1}^{L} w_\ell^a V_\ell^a. \tag{4}$$

Define the volume-weighted effective prices on each side:

$$\widetilde{B} = \frac{\sum_{\ell=1}^{L} w_\ell^b B_\ell V_\ell^b}{\widetilde{V}^b}, \qquad \widetilde{A} = \frac{\sum_{\ell=1}^{L} w_\ell^a A_\ell V_\ell^a}{\widetilde{V}^a}. \tag{5}$$

The multi-level microprice is then given by

$$\mathrm{MP}_{\mathrm{multi}} = \frac{\widetilde{B}\widetilde{V}^a + \widetilde{A}\widetilde{V}^b}{\widetilde{V}^a + \widetilde{V}^b}. \tag{6}$$

Equation (6) reduces to (1) if $L = 1$ and $w_1^b = w_1^a = 1$. The estimator can optionally be clamped to $[B_1, A_1]$ to guarantee it lies inside the spread.

Interpretation

Equation (6) maintains the probabilistic interpretation of the microprice: $\widetilde{V}^a/(\widetilde{V}^a + \widetilde{V}^b)$ can be seen as the probability of the next trade hitting the bid, while $\widetilde{V}^b/(\widetilde{V}^a + \widetilde{V}^b)$ is the probability of hitting the ask. The use of $\widetilde{B}$ and $\widetilde{A}$ shifts the reference prices to account for nearby liquidity.

4 Worked Example

Consider an order book with tick size $\tau = 0.1$ and spread $A_1 - B_1 = 0.2$ (two ticks):

$$\begin{aligned} B_1 &= 100.0, \quad V_1^b = 200, & B_2 &= 99.9, \quad V_2^b = 500, \\ A_1 &= 100.2, \quad V_1^a = 50, & A_2 &= 100.3, \quad V_2^a = 50. \end{aligned}$$

Let $\alpha = 1.0$. The distances are $d_1^b = 0, d_2^b = 1$, giving weights $w_1^b = 1, w_2^b = e^{-1} \approx 0.3679$, similarly for the ask side. The effective volumes are

$$\widetilde{V}^b = 200 + 500e^{-1} \approx 383.94, \qquad \widetilde{V}^a = 50 + 50e^{-1} \approx 68.39.$$

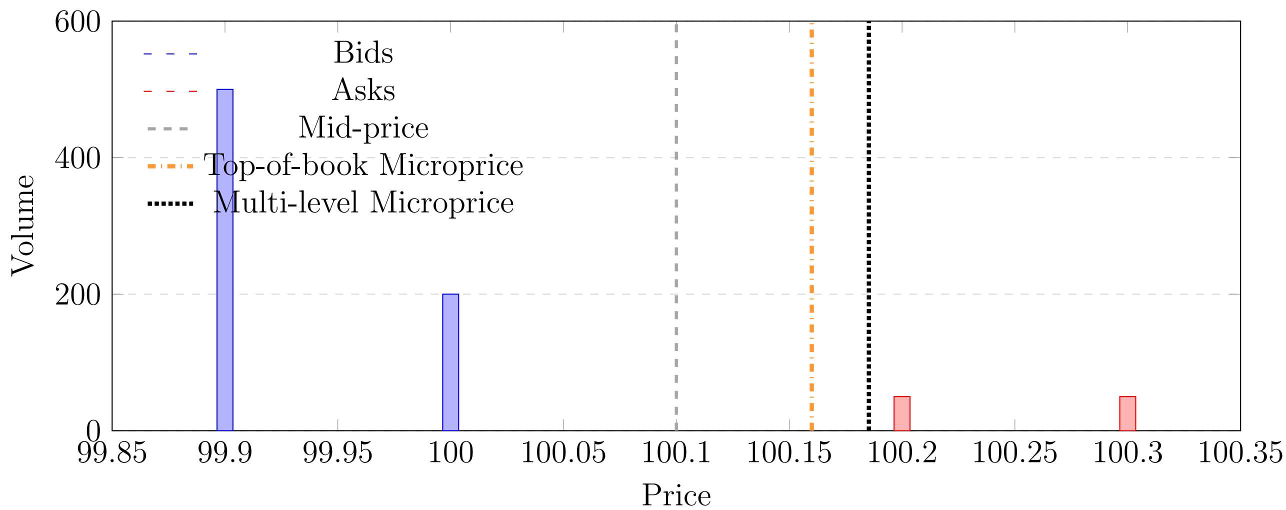
The effective prices are

$$\widetilde{B} \approx 99.9521, \qquad \widetilde{A} \approx 100.2269.$$

Substituting into (6) yields

$$\mathrm{MP}_{\text{multi}} \approx 100.1853.$$

For comparison, the classical microprice (1) gives $\mathrm{MP}_{\text{TOB}} = 100.16$ and the mid-price is 100.10. The multi-level microprice lies closer to the ask, reflecting the strong bid-side depth.



In this example, if we were a market maker, we would most likely improve the bid to 100.1 or join at 100 and step back on the ask side (assuming we don't hold any inventory).

5 Parameter Considerations and Properties

Decay parameter. The choice of α controls the contribution of distant levels. Small α approaches an unweighted average over available depth, while large α emphasizes only the nearest levels.

Truncation and stability. Truncating at $L = 3$–5 levels is typically sufficient, since deeper levels contribute negligibly once exponentially discounted. Guardrails include dropping levels with zero volume and reverting to the mid-price if $\widetilde{V}^a + \widetilde{V}^b$ is too small.

Reduction property. When only the best levels are used, the estimator reduces to the classical microprice, ensuring consistency.

Scale invariance. Measuring distances in ticks renders the estimator robust to nominal price level changes.

6 Conclusion

The multi-level, distance-weighted microprice extends the classical microprice by incorporating nearby depth in a principled, smoothly decaying manner. It yields a more informative fair value estimate, improving quoting, inventory control, and execution decisions in high-frequency trading strategies.

References

- Cartea, A., Jaimungal, S., & Penalva, J. (2015). *Algorithmic and High-Frequency Trading.* Academic Press.