Models

LOCAL

- Nodes can simulate any algorithm locally with constant slowdown.

CONGEST

- Each node can send one $O(\log n)$ message to each neighbor.

Bounds

Coloring

Type	Rounds
Rooted Tree	$O(log^* n)$
General Graph with $O(\Delta^2)$ Colors (Linial's Algorithm)	$O(\log^* n)$
General Graph with $O(\Delta + 1)$ Colors	$O(\Delta \log \Delta + \log^* n)$
$O(\log \log n)$ coloring of graph with $\Delta = O(1)$	$O(1)$

Unrooted tree to rooted tree: $O(\log n)$ *(?)*

Algorithms

Type	Rounds	Message Complexity
Minimum Spanning Tree (MST)	$O(n \log n)$	$O(m \log n)$
	$O(n)$	$O(m + n \log n)$
Maximal Independent Set (MIS, Luby's Algorithm)	$O(\log n)$, WHP	
Maximal Independent Set (MIS)	$poly(\log n)$, deterministic	
Maximal Independent Set (MIS) on Directed Cycle	$O(\log^* n)$	
Approximatie Min Cut	$\Omega(D)$	
Network Decomposition	$O(poly(\log n))$	
Comput Diameter	$O(n)$	$O(\log n)$

Δ-Cover-Free Family: No set in the family is a subset of the union of Δ other sets

Use Cases: Color Reduction, Interferring Radio Transmissions

Network Decomposition into Strong diameter clusters with $C = O(\log n)$, $D = O(\log n)$ can be done in $(\log^8 n)$ rounds.

Provided a network decomposition exists, then:

- *We can compute a* $\Delta + 1$ *coloring in* $O(CD)$ *time*

Assuming an Algorithm which computes MIS in $f(n) rounds exists, then:$

- *We can compute a* $\Delta + 1$ *coloring in* $f(n(\Delta + 1))$ *rounds*
- *We can compute a maximal matching in* $f(n(\Delta + 1)) rounds$

Shared Memory

Type	Worst Case Running Time
Ivy	$\log n$
Arrow	D (Diameter of Tree)

Labeling

Type	Label Size
Adjacency Tree	$2 \log n$
Adjacency General Graph	$\Omega(n)$
Ancestor Tree	$O(2 \log n)$
Distance Tree	$O(n \log n)$
Distance Tree (Heavy-Light Decomposition)	$O(\log^2 n)$

Wireless Protocols

Algorithm	Time
Leader Election ALOHA	$O(1)$ expected, $O(\log n)$ WHP

Definitions

Radius $r(u)$ of node u in G is the maximum of distances from u to all other nodes. The radius $r(G)$ is the minimum of the radii of all nodes in G.

If the radius is 1, there must be a node connected to all other nodes.

Diameter

Questions

Identify Cut Edges

For every edge e in spanning tree T, if there is edge e' such that $e' + T$ creates cycle, then e is not a cut edge.

Identify Connected Components

Each node forms its own component, with self as leader. Every node v selects neighbor u in different component, sends this information to leader. Leader merges the two components, informs about new leader. In every step, a new component contains two old components, thus taking $O(\log n)$ rounds.

Scheduling

Assign K tasks to n nodes such that each node has at least $\log n$ tasks assigned to it but not to any of its neighbors.

Asynchronous Broadcast

- Maximal running time of async broadcast: Radius, i.e. minimal distance to each node.
- Echo: Maximal length back to origin.

Sorting Networks

Math

Markov Inequality $P[X \geq a] \leq \frac{E[X]}{a}$

Chernoff Bound $P[X > (1+\delta)\mu] < (\frac{e^\delta}{(1+\delta)^{(1+\delta)}})^\mu$

E Approximation $(1-x)^t \leq e^{-xt}$

$(1-1/x)^x = 1/e$

Communication Complexity

Equality

$CC(EQ) = \Omega(n)$

Disjointness

$$DISJ(x,y) := \begin{cases} 0 & \exists i : x_i = y_i = 1 \\ 1 & \text{else} \end{cases}$$

$CC(DISJ) = \Omega(n)$